Basic Convective Cloud Dynamics
Cumulus and cumulonimbus
Cumulus

Fair weather type
Cumulus congestus
Cumulus congestus
Cumulonimbus

Rain
More often: Cumulus congestus develops an anvil.
Cumulonimbus with Anvil

- Hail
- Rain
Tornadic Cumulonimbus

Tornado
All Cumulus and Cumulonimbus

Buoyancy phenomena

\[ B = -g \frac{\rho}{\rho_o} > 0 \]

Vorticity

Turbulent

Condensation level
Basic equations for vertical acceleration, mass continuity, & buoyancy

\[ \frac{dw}{dt} = -\frac{1}{\rho_e(z)} \frac{dx'}{dz} + B \]

\[ \nabla \cdot \rho_e(z) \frac{dx'}{dz} = 0 \]

\[ B = -8 \frac{\rho'}{\rho_e(z)} = \frac{\rho'}{\rho_e} \left[ \frac{\tau'}{\tau_e} + 0.009 \frac{q'}{\rho_e} - \frac{q'}{\rho_e} - \frac{q'}{\rho_e} \right] \]
Based on classic model of continuous & homogeneous entrainment
1-D Lagrangian model

Temperature equation

\[ \frac{dT_e}{dz} = -\frac{q}{C_p} - \frac{L}{C_p} \frac{dq_{vc}}{dz} + \lambda \left[ (T_e - T_c) + \frac{L}{q_f} (q_{ve} - q_{vc}) \right] \]

Predicts parcel temperature & buoyancy
1-D Lagrangian model

Momentum equation

\[ \frac{1}{r} \frac{d}{dz} \left( \frac{1}{r} \omega_c^2 \right) = -\frac{1}{\rho e} \frac{d}{dz} \rho e' + \frac{1}{\rho e} \omega_c^2 (-u_0) \]

Kessler warm cloud microphysics

\[ q_{vc} = q_{vs}[T_c, p(Z)] \]

\[ w_c \frac{\partial q_c}{\partial z} = -w_c \frac{\partial q_{vs}}{\partial z} - A - K - w_c \lambda q_c \]

\[ w_c \frac{\partial q_r}{\partial z} = A + K + F - w_c \lambda q_r \]
Entrainment

Turbulent jet

\[ \Lambda = \frac{2}{b} \]

\[ \frac{\partial w_c}{\partial x} = \frac{3}{b} w_c \]
Entrainment

Thermal

\[ \frac{dW_c}{dt} = \beta - D_R - \frac{\varepsilon}{6} v \]
Entrainment

Starting plume

\[
\frac{dw_c}{dt} = B - D_R - \frac{3}{5} w_c^2
\]
Entrainment

Raymond & Blyth’s Model of discontinuous, inhomogeneous entrainment
Three-dimensional Vorticity

Vorticity equations under Boussinesq conditions

\[ \frac{dn}{dt} = B_y + n u_x + \xi u_y + \xi z \]
\[ \frac{d\xi}{dt} = -B_x + \xi \nabla_y + \xi \nabla_z + \eta u_x \]
\[ \frac{d\eta}{dt} = 5w_z + 5w_y + \eta u_x \]

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**generation**

**stretching**

**tilting**
Three-dimensional Vorticity

Generation of horizontal vorticity by buoyancy
Three-dimensional Vorticité

Stretching of vertical vorticité

Horizontal convergence
Three-dimensional Vorticity

Tilting of horizontal vorticity into the vertical
Linearize vertical vorticity equation around the basic state:

\[ \vec{\omega} = \left[ u'(z), 0, 0 \right] \]

At a level in the cloud where the cloud is moving with the basic state velocity:

\[ \frac{\partial \vec{v}}{\partial x} = \vec{u}_y \cdot \vec{u}_x \]
Three-dimensional Vorticity

Linear process leads to vorticity couplet in an convective cloud that develops in a sheared environment
The pressure perturbation field in a convective cloud is governed by:

\[ \nabla^2 p' = F_B + F_D \]

\[ F_B = \frac{2}{3} z (c_B) \]

\[ F_D = -\nabla \cdot (\rho_0 \nabla \cdot \nabla \nabla) \]
Pressure Perturbation

Pressure gradient force required by the buoyancy field

Note! These are lines of force, not air motions
The pressure perturbation field in a convective cloud is governed by:

\[ \nabla^2 \rho' = F_B + F_D \]

\[ F_B = \frac{2}{3} z (\zeta \cdot \hat{n}) \]

\[ F_D = -\nabla \cdot (\zeta \cdot \nabla \hat{n}) \]

When vortices form in storms, this term requires a low pressure at the center of each vortex. These low pressure centers affect the storm dynamics by producing a pressure field in the storm that is different from that produced by buoyancy alone.