Understanding the Importance of Microphysics and Macrophysics for Warm Rain in Marine Low Clouds. Part II: Heuristic Models of Rain Formation

ROBERT WOOD, TERENCE L. KUBAR, AND DENNIS L. HARTMANN

University of Washington, Seattle, Washington

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ABSTRACT

Two simple heuristic model formulations for warm rain formation are introduced and their behavior explored. The first, which is primarily aimed at representing warm rain formation in shallow convective clouds, is a continuous collection model that uses an assumed cloud droplet size distribution consistent with observations as the source of embryonic drizzle drops that are then allowed to fall through a fixed cloud, accreting cloud droplets. The second, which is applicable to steady-state precipitation formation in stratocumulus, is a simple two-moment bulk autoconversion and accretion model in which cloud liquid water is removed by drizzle formation and replenished on a externally specified time scale that reflects the efficacy of turbulent overturning that characterizes stratocumulus.

The models’ behavior is shown to be broadly consistent with observations from the A-Train constellation of satellites, allowing the authors to explore reasons for changing model sensitivity to microphysical and macrophysical cloud properties. The models are consistent with one another, and with the observations, in that they demonstrate that the sensitivity of rain rate to cloud droplet concentration \( N_d \) (which here represents microphysical influence) is greatest for weakly precipitating clouds (i.e., for low cloud liquid water path and/or high \( N_d \)). For the steady-state model, microphysical sensitivity is shown to strongly decrease with the ratio of replenishment to drizzle time scales. Thus, rain from strongly drizzling and/or weakly replenished clouds shows low sensitivity to microphysics. This is essentially because most precipitation in these clouds is forming via accretion rather than autoconversion. For the continuous-collection model, as cloud liquid water content increases, the precipitation rate becomes more strongly controlled by the availability of cloud liquid water than by the initial embryo size or by the cloud droplet size. The models help to explain why warm rain in marine stratocumulus clouds is sensitive to \( N_d \) but why precipitation from thicker cumulus clouds appears to be less so.

1. Introduction

A mounting body of observational and modeling evidence suggests that light amounts of warm rain (on the order of 1 mm day\(^{-1}\)) falling from low clouds can influence the dynamics and structure of the marine boundary layer (MBL; Paluch and Lenschow 1991; Ackerman et al. 1993; Stevens et al. 1998; Comstock et al. 2005; Savic-Jovcic and Stevens 2008; Xue et al. 2008; Wang and Feingold 2009a,b). This influence can in turn impact the cloud cover and/or thickness and therefore the cloud albedo (e.g., Albrecht 1989; Savic-Jovcic and Stevens 2008). Indeed, over the cold regions of the eastern subtropical and tropical oceans, observations of marine stratocumulus sheets reveal a striking relationship between the mode of mesoscale cellular convection and the occurrence of drizzle, with open cells frequently associated with strong drizzle and closed cells less frequently so (Stevens et al. 2005; Comstock et al. 2007; Wood et al. 2008).

Over warmer regions of the tropics, cold pools driven by evaporating precipitation also appear to play a fundamental role in the transition from shallow to deep convection (Tompkins 2001; Khairoutdinov and Randall 2006; Kuang and Bretherton 2006). Thus, evidence points to important connections between the formation of precipitation and the organization of shallow convection and ultimately cloud albedo. It is critical that we gain an understanding of the factors controlling the ability of warm clouds to precipitate.

A conventional wisdom, going back to early pioneering studies (e.g., Byers and Hall 1955), is that precipitation occurrence and amount are largely dictated by the macrophysical properties of clouds (e.g., thickness,
liquid condensate amount, cloud dynamics, etc.). Recently this wisdom is being challenged by evidence suggesting that details of the cloud microphysical properties (e.g., cloud condensation nuclei concentrations, cloud droplet concentration, etc.) may also exert some control over the formation of warm rain. Modeling (e.g., Liou and Ou 1989; Albrecht 1989; Savic-Jovcic and Stevens 2008; Xue et al. 2008; Wang and Feingold 2009a,b) and observational studies (Ferek et al. 2000; Pawlowska and Brenguier 2003; Comstock et al. 2004; van Zanten et al. 2005; Wood 2005a) suggest that $N_d$ may significantly alter the precipitation efficiency of shallow marine clouds. Systematic precipitation closure attempts using multiple case studies demonstrate microphysical control over precipitation rate for stratocumulus clouds (see summary in Geoffroy et al. 2008). However, in the more strongly precipitating trade cumulus regime, such a clear link between $N_d$ and precipitation rate (Nuijens et al. 2009) has not been observed.

Some evidence suggests that the lack of microphysical sensitivity in shallow convection is associated with an increasing dominance of the accretion process over autoconversion as precipitation rates increase (Stevens and Seifert 2008). Because the accretion rate is almost independent of $N_d$ (it depends only on the product of the cloud and rain mixing ratios) whereas autoconversion is strongly sensitive to $N_d$ (Beheng 1994; Khairoutdinov and Kogan 2000; Liu and Daum 2004; Wood 2005b), an increased role of accretion may dampen the sensitivity of precipitation to $N_d$.

In Part I of this study (Kubar et al. 2009, hereafter Part I) observations from the A-Train satellites were used to determine characteristics of warm rain formation in marine low clouds and to investigate the cloud macrophysical and microphysical factors that influence it. In most regions studied, greater drizzle intensity (higher radar reflectivity) is associated with significant increases in cloud top height and cloud liquid water path (LWP) but with decreases in cloud droplet concentration $N_d$ that are more modest. This is particularly true for regions over the remote oceans that are relatively pristine. In polluted regions off the East Asian coast and over the Gulf of Mexico, higher liquid water contents are required to give the same drizzle intensity as clouds over pristine regions, consistent with a reduction in precipitation efficiency due to higher cloud droplet concentrations.

Here, we introduce two simple heuristic models to attempt to understand the behavior seen in the observations introduced in Part I of this study and to attempt to reconcile the apparently different impacts of microphysics in controlling precipitation amounts in shallow stratocumulus compared with the more strongly precipitating trade cumulus regime. Section 2 describes the model physics, section 3 presents some of the essential model behavior, and section 4 describes the observations and how the models are compared with them, with section 5 detailing the results of the comparisons. A discussion of the findings is contained in section 6 and conclusions from the study are given in section 7.

2. Model formulations

a. Continuous collection model

We derive a simple continuous collection (CC) model that determines the precipitation rate at the base of an idealized warm cloud resulting from drops falling and accreting cloud water. It is primarily aimed at reproducing the instantaneous precipitation falling from a cloud that is assumed not to change during the course of the precipitation event. The model inputs are the cloud thickness (or alternatively its liquid water path) and the cloud-top effective radius (or alternatively the cloud droplet concentration).

The assumptions made are detailed below.

1) ASSUMED MACROPHYSICAL CLOUD STRUCTURE

The cloud consists of a layer with a constant specified cloud droplet concentration $N_d$ and a cloud liquid water content $\rho q_l$ that increases with height above cloud base $z$ at a specified rate $\rho dq_l/dz = \Gamma$, where $q_l$ is the cloud water mixing ratio and $\rho$ is the air density. The liquid water gradient $\Gamma$ is given by $\Gamma = \gamma_{\text{ad}} \Gamma_{\text{ad}}$, where $\Gamma_{\text{ad}}$ is the thermodynamically determined increase for an adiabatic parcel ascent and $\gamma_{\text{ad}}$ is the adiabaticity factor. We parameterize $\gamma_{\text{ad}} = z_0/(z_0 + z)$ where $z_0$ is a scaling parameter, set to 500 m, which matches very well with liquid water content observations in warm marine clouds (Rangno and Hobbs 2005) with depths of 1–4 km. Thus, shallow clouds tend to be closer to adiabatic than deeper ones, consistent with arguments for the dilution of entraining plumes (e.g., Bretherton et al. 2004) and with observations (e.g., Rauber et al. 2007).

2) ASSUMED MICROPHYSICAL CLOUD STRUCTURE

The cloud droplet size distribution $n(r)$ is assumed to be a gamma distribution (Austin et al. 1995; Wood 2000) with a spectral width determined as a function of the mean volume radius $r_v = (3\rho q_l/\pi \rho \rho_{\text{d}} N)^{1/3}$ using the parameterization of Wood (2000) and accounts implicitly for the narrowing of the size distribution due to condensational growth and its broadening (in a fractional sense) due to increased cloud condensation nuclei (CCN) concentration.
3) PRECIPITATION EMBRYOS

At the cloud top, a small subset of the largest cloud droplets (specified number concentration \( N_D \)) are considered to be precipitation embryos that subsequently are allowed to fall through the cloud layer and grow by coalescing with smaller cloud droplets. Here we specify \( N_D \), which is a free parameter, in accordance with observed concentrations of precipitation drops in warm precipitating clouds and use this to determine the minimum size \( r_- \) of the drizzle embryos using

\[
\int_{r_-}^{\infty} n(r) \, dr = N_D. \tag{1}
\]

We then use \( N_D \) and \( r_- \) and the assumed cloud droplet size distribution to determine the mean mass-weighted radius of the embryos \( R_{\text{emb}} \), that is,

\[
R_{\text{emb}} = \left[ \frac{1}{N_D} \int_{r_-}^{\infty} r^3 n(r) \, dr \right]^{1/3}. \tag{2}
\]

An alternative would be to specify \( r_- \) as a free parameter and then use (1) to determine \( N_D \). We experimented with this approach and found that the salient findings were not markedly different. For simplicity, we do not report further on these experiments.

4) COALESCENCE GROWTH OF DRIZZLE DROS

The drizzle embryos, which are represented by a single-sized embryo with an initial radius \( R_{\text{emb}} \), then fall through the depth of the cloud continuously collecting cloud droplets and growing in radius at a rate that is approximately the product of the cloud liquid water content, the collector drop fall speed, and a collection efficiency (Rogers and Yau 1989). The cloud droplets collected are assumed to have a size equal to \( r_c \) at that level. The radius growth rate of the falling drizzle drop \( R \) with respect to height is taken from the continuous collection model (see, e.g., Rogers and Yau 1989):

\[
\frac{dR}{dz} = \left(1 + \frac{r_c}{R}\right)^2 \left[1 - \frac{v_f(r_c)}{v_f(R)}\right] \rho_g E(R, r_c) \frac{4\rho_w}{\nu_f}, \tag{3}
\]

where \( \nu_f \) is the terminal velocity of a falling drop, \( E(R, r_c) \) is the collection efficiency of the falling drizzle drop and the collected cloud droplets, and \( \rho_w \) is the density of liquid water. Terminal velocities for the cloud droplets \( [v_f(r_c)] \) are determined using the Stokes flow relations given in Pruppacher and Klett (1997), whereas for collector drops we use a power-law relation suitable for drizzle drops (Comstock et al. 2004, detailed below). Collection efficiencies are taken from Hall (1980). The negative sign in (3) indicates that the drizzle drops grow downward. The drops are assumed to be falling in still air.

5) DETERMINING BULK DRIZZLE CHARACTERISTICS

At any level in cloud \((0 < z < h)\), the drizzle liquid water mixing ratio \( q_{\text{LW}} \) and precipitation rate \( P \) and Rayleigh radar reflectivity factor \( Z \) are determined using \( R \):

\[
q_{\text{LW}} = \frac{4\pi\rho_w}{3\rho} N_D R^3, \tag{4}
\]

\[
P = \frac{4\pi\rho_w}{3} \alpha_f N_D R^{3+\delta}, \tag{5}
\]

\[
Z = 2^6 N_D R^6. \tag{6}
\]

Here, we use the approximate formulation for the terminal fall speed \( v_f(R) = \alpha_f R^2 \) with \( \alpha = 2.2 \times 10^5 \text{ m}^{-0.4} \text{ s}^{-1} \) and \( \delta = 1.4 \), values taken from (Comstock et al. 2004).

CC model—Free parameters

The key free parameters of the CC model, as shown in Table 1, are the cloud thickness \( h \) (or equivalently LWP insofar as it is uniquely related to \( h \)) and the cloud droplet concentration \( N_D \). Experimentation shows relatively weak sensitivity of the precipitation characteristics (when expressed as a function of liquid water path) to the choice of the liquid water adiabaticity scaling parameter \( z_0 \). This is because the total condensate through which the collector drop falls is much more important than the details of the vertical organization of the condensate. We use a constant value of \( \Gamma_{\text{ad}} = 2 \times 10^{-6} \text{ kg m}^{-4} \).

The cloud-top effective radius \( r_c^e \), which can be estimated from satellite measurements, is determined
uniquely from the mean volume radius at the top of the cloud \( r_{cb} \), which is a function of cloud-top liquid water content and \( N_d \).

b. Steady-state bulk autoconversion/accretion model

The second model we introduce—the steady-state (SS) model—is a highly simplified model of precipitation formation in a stratiform layer cloud in which there exists (in steady state) a balance between a loss of cloud water through collision–coalescence and its replenishment by assumed turbulent motions that drive the cloud back toward an adiabatic layer. It is primarily designed as a heuristic model to reproduce the equilibrium behavior of precipitation in stratiform boundary layer clouds. The model uses a two-moment bulk microphysical formulation (two moments for cloud and two moments for precipitation), with prognostic equations for the cloud water, precipitation water, and the precipitation drop concentration. The assumptions made are listed below.

1) Prognostic Bulk Microphysical Equations

The cloud consists of a single vertically homogeneous layer of thickness \( h \). The cloud state is characterized by a cloud droplet concentration \( N_d \), which is an external, time-invariant parameter, and a cloud liquid water mixing ratio \( q_l \), which evolves in time. The precipitation is characterized by a rain mass mixing ratio \( q_r \) and a rain drop concentration \( N_D \), both of which evolve until a steady state is reached. The following equations describe the evolution of the three prognostic variables:

\[
\frac{dq_l}{dt} = \frac{\rho (q_{ad} - q_l)}{\tau_{rep}} - A_c - K_c, \tag{7}
\]

\[
\frac{dq_r}{dt} = A_c + K_c - S_q, \tag{8}
\]

\[
\frac{dN_D}{dt} = \frac{A_c}{m_{emb}} - S_N. \tag{9}
\]

Here \( q_{ad} \) is the adiabatic mean liquid water mixing ratio for the layer (determined as \( \frac{1}{2} \Gamma_{ad} h \); see section 2a above), to which \( q_l \) relaxes with a replenishment time scale \( \tau_{rep} \). The autoconversion rate \( A_c \) is a specified function of the cloud state \( q_l, N_d \) (see assumption 2 below). The accretion rate is specified as \( K_c = \gamma \rho^2 q_l q_r \), with \( \gamma = 4.7 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1} \) as in the formulation of Tripoli and Cotton (1980). Observations suggest that diversity across different accretion rate formulations is modest compared with that across autoconversion rate formulations (Wood 2005b), and we find that our findings are barely altered by using a different formulation (not shown).

The sedimentation rates for rainwater \( S_q \) and rain number \( S_N \) are specified as \( S_q = 2 \rho q_v v_{T,q}/h \) and \( S_N = 2 N_d v_{T,N}/h \) respectively, where \( v_{T,q} \) and \( v_{T,N} \) are the fall speeds for the third and zeroth moment of the rain size distribution, each of which is specified as a linear function of the rain drop volume radius \( r_{v,D} = (3 \rho q_v/(4 \pi \rho_r N_D))^{1/3} \) using the parameterization of Khairoutdinov and Kogan (2000). Finally, \( m_{emb} \) is the mass of a drizzle drop embryo formed by autoconversion, which we assume to have a radius of 22 \( \mu \text{m} \), consistent with observations in stratus-cumulus (Wood 2005b). The results are not highly sensitive to the exact choice of this radius.

2) Sensitivity to Autoconversion

Available expressions for the autoconversion rate differ markedly in their sensitivities to \( q_l \) and \( N_d \) (Wood 2005b), so we investigate the sensitivity of our findings to these differences by using a number of different autoconversion parameterizations. We use the formulations of Khairoutdinov and Kogan (2000), Liu and Daum (2004) (as modified by Wood 2005b), Beheng (1994), and Seifert and Beheng (2001); these parameterizations are referred to hereafter as KK, LD, BEH, and SB.

3) Steady-State Solutions

The model is run with \( q_l(t = 0) = 0 \) until a steady state has been reached.

4) Determining Bulk Drizzle Characteristics

The model precipitation rate, implicitly assumed to be that at the base of the cloud layer, is \( P_{CB} = \rho q_v v_{T,q} \). The model estimates a Rayleigh radar reflectivity factor due to precipitation \( Z \) from the two moments \( q_l \) and \( N_D \) by assuming an exponential size distribution truncated at the assumed threshold for cloud and drizzle drop (\( r = 20 \mu \text{m} \)).

Steady-state model—Free parameters

The free parameters of the steady-state autoconversion and accretion model are the cloud thickness \( h \), the cloud droplet concentration \( N_d \), and the cloud water replenishment time scale \( \tau_{rep} \) (see Table 1 for values used here). We use a constant value of \( \Gamma_{ad} = 2 \times 10^{-6} \text{ kg m}^{-4} \).

3. Model Behavior

a. Sensitivity of precipitation rate to LWP and \( N_d \)

Before attempting to address the question of how faithfully our two models are able to capture the salient features of the A-Train observations, it is useful to explore some of the essential model behavior.

Figure 1 shows the cloud base precipitation rate \( P_{CB} \) as a function of the cloud LWP and the cloud droplet
concentration $N_d$ from the CC and SS models (the latter with the Khairoutdinov and Kogan autoconversion parameterization). Both models show that the highest precipitation rates are associated with clouds with high LWP and also with low $N_d$, and both show similar regions of phase space where $P_{\text{CB}}$ is effectively zero. The models are in the best agreement for $N_d$, $100 \text{ cm}^{-3}$. At higher $N_d$ there is a significantly stronger dependence of $P_{\text{CB}}$ on $N_d$ in the CC model than in the SS model. The Khairoutdinov and Kogan autoconversion parameterization was derived from bin-resolved large-eddy simulation for $N_d$ typical of clean marine conditions (Khairoutdinov and Kogan 2000), with almost all $N_d$ values below $120 \text{ cm}^{-3}$ and so we might not expect it to perform well at high $N_d$. However, similar discrepancies exist between the CC and SS models at $N_d > 100 \text{ cm}^{-3}$ for the other autoconversion parameterization schemes used in the SS model. Thus, differences in $P_{\text{CB}}$ produced by the CC and SS models are not primarily driven by subtleties in the autoconversion parameterizations used.

Both models show a steepening of the $P_{\text{CB}}$ isolines (isohyets) as $P_{\text{CB}}$ increases (Fig. 1), demonstrating an increase in the relative dependence on LWP compared with $N_d$ at large $P_{\text{CB}}$. We can understand this behavior in the CC model using a minimal analytical CC model (see the appendix), which demonstrates that as LWP increases the key process limiting the rain rate becomes the availability of cloud liquid water to accrete onto growing raindrops rather than the initial size of the precipitation embryos or their efficiency as collector drops to accrete cloud liquid water. However, at high droplet concentrations typical of polluted conditions ($N_d$ of a few hundred per cubic centimeter or more), the initial embryo size can still be an important limiter of precipitation formation even at LWP typical of a few hundred g m$^{-2}$. Insofar as the CC model is a reasonable replicator of reality, these results have important implications for the sensitivity of warm rain formation to cloud microphysics.

It should be noted however, that the CC model precipitation isolines are much more tightly packed for high LWP and high $N_d$ than those from the SS model. This behavior is more or less reproduced by the minimal CC model (see Fig. A2 and associated discussion in the appendix) and is caused by the extreme sensitivity of the collection efficiency to droplet size for droplets of around $10 \mu\text{m}$ and smaller (see Fig. A1). The SS model does not reproduce this behavior because its bulk autoconversion formulations are not particularly designed to capture this sensitivity. In the application of autoconversion in many models, a threshold function is frequently applied that is usually a function of a characteristic cloud droplet radius (Liu et al. 2005). This is designed to replicate (in most cases somewhat crudely) the strong sensitivity of collection efficiency at small droplet sizes. Because these threshold functions are typically highly arbitrary, used as tuning parameters, and affect a relatively limited portion of the phase space we explore in this study, we do not consider their impacts further here.

b. Sensitivity to model parameterizations

1) CC MODEL

The CC model is sensitive to the number concentration $N_D$ of precipitation embryos (see model assumption 3 in section 2a), which is set equal to $100 \text{ L}^{-1}$, a number toward the upper end of observed concentrations of
drops with radii larger than 20 μm in precipitating warm clouds (Hudson and Svensson 1995; Comstock et al. 2004; Wood 2005a). We find that a doubling of $N_D$ results in an increase of 50%–80% in precipitation rate, an increase that is weaker than linear because a higher $N_D$ is partly compensated by a smaller mass-weighted embryo radius $R_{emb}$ (see model assumption 3 in section 2a). Nevertheless, there is little doubt that the need to specify $N_D$ remains a significant limitation of the CC model.

The CC model precipitation rate is only weakly sensitive to $z_0$ when expressed as a function of LWP. Increases of only 5%–20% in $P_{CB}$ are found for a given [LWP, $N_d$] pair as $z_0$ increases from 250 to 1000 m.

2) SS MODEL

The SS model results display some sensitivity to the autoconversion parameterization used. Figure 2 shows that the greatest fractional sensitivity of precipitation rate to autoconversion parameterization is generally at low values of $P_{CB}$ (i.e., at low LWP and high $N_d$). For example, at LWP = 100 g m$^{-2}$ and $N_d = 50$ cm$^{-3}$, $P_{CB}$ values are 0.05, 0.15, 0.13, and 0.5 mm day$^{-1}$ (i.e., an order of magnitude spread) for the four parameterizations (LD, KK, BEH, SB) respectively, whereas at LWP = 400 g m$^{-2}$ and $N_d = 20$ cm$^{-3}$, $P_{CB}$ values are 13, 31, 45, and 55 mm day$^{-1}$, respectively. For reference, the CC model rates are 0.2 and 38 mm day$^{-1}$ respectively for the low and high LWP cases.

Thus, there is a somewhat weaker sensitivity of $P_{CB}$ to autoconversion parameterization at high $P_{CB}$. However, more striking is that the sensitivity of $P_{CB}$ to changes in LWP and $N_d$ is much less dependent upon the autoconversion parameterization at high $P_{CB}$. This can be seen by the spacing and the orientation of the $P_{CB}$ iso-lines in Fig. 2, which are less autoconversion-dependent to the lower right of the panels.

In other words, although the autoconversion impacts the precise value of $P_{CB}$, the role of the autoconversion parameterization in determining the sensitivity of $P_{CB}$ to changes in, for example, aerosols, is diminished for strongly precipitating clouds. We return to this in the discussion.

The SS model $P_{CB}$ is also sensitive to the cloud liquid water replenishment time scale $\tau_{rep}$, and Fig. 3 shows
that sensitivity to $\tau_{\text{rep}}$ is strongest at high $P_{\text{CB}}$. This is because at low $P_{\text{CB}}$ the clouds are close to adiabatic because sedimentation is inefficient at removing cloud water. As the precipitation efficiency increases (i.e., the time scale for precipitation removal becomes comparable with $\tau_{\text{rep}}$), increasing the replenishment rate of cloud liquid water (decreasing $\tau_{\text{rep}}$) permits a larger total rate of conversion of cloud to rain and a larger $P_{\text{CB}}$. However, although Fig. 3 shows that there is sensitivity of the precipitation rate to $\tau_{\text{rep}}$, it is primarily LWP and $N_d$ that determine the precipitation sensitivity. The results for the other autoconversion parameterizations are qualitatively very similar (not shown).

We should note that the SS model does not attempt to parameterize the effect of turbulence on recycling of drizzle drops themselves, only the replenishment of the cloud water that feeds the drizzle. The former is known to be important for the formation of the largest drizzle drops in precipitating stratocumulus (Nicholls 1989; Baker 1993; Austin et al. 1995), but including this effect here would add an additional level of complexity that we choose to defer to future study.

4. Comparing the models with observations

a. Observations

How well do our models reproduce behavior seen in nature? To assess this we first examine some previous precipitation closure studies, before turning to observations of radar reflectivity from the CloudSat satellite and visible/near-infrared estimates of cloud liquid water path and collocated cloud droplet concentration from Moderate Resolution Imaging Spectroradiometer (MODIS) on NASA’s Aqua satellite.

Precipitation closure studies (Pawlowska and Brenguier 2003; Comstock et al. 2004; vanZanten et al. 2005; Wood 2005a; Geoffroy et al. 2008; Brenguier and Wood 2009) attempt to determine the sensitivity of precipitation rate to macrophysical and microphysical cloud properties by exploring numerous case studies spanning a range of different conditions. Currently, systematic exploration is limited to cases in marine stratocumulus, where the precipitation rates are mostly lower than 1 mm day$^{-1}$, the LWP is 200 g m$^{-2}$ or less, and $N_d$ is $\sim$200 cm$^{-3}$ or less. In general, these studies all show a strong sensitivity of precipitation rate to cloud liquid water path and a weaker but inverse sensitivity to cloud droplet concentration [see, e.g., Geoffroy et al. (2008) for an integration of the existing studies].

Assuming linearly increasing cloud liquid water content with height in cloud, aircraft closure studies suggest a dependence on LWP$^{1.5-2}$/Nd, and a ship-based study is more consistent with (LWP/Nd)$^{1.75}$. Figure 1 shows lines of constant LWP/Nd and LWP$^2$/Nd, which span the observed range of sensitivity. Both the SS model and CC model isohyets for the region of (LWP, Nd) phase space to which they apply are broadly consistent with the closure studies. Unfortunately, no closure studies exist with which to evaluate the dependencies at higher precipitation rates associated with deeper marine low clouds.

Extensive details regarding the CloudSat/MODIS observations and their limitations can be found in Part I of this paper. In this part of the study we attempt to reproduce, using the CC and SS models, the observed dependency of the column maximum radar reflectivity on the cloud LWP and Nd. We use the eight different regions over the subtropical and tropical Pacific Ocean and Gulf of Mexico defined in Part I (see Table 2 here and also Fig. 5 in Part I), together with a region that covers the oceanic region between 30°S and 30°N and 100°E and 70°W.

For each region joint probability distribution functions (PDFs) of LWP and effective droplet concentration$^1$ ($Nd_{\text{eff}}$) are constructed from the MODIS retrievals for all the matched CloudSat/MODIS data with detectable radar reflectivity and retrieved cloud properties. A selection of statistical properties is shown in Table 2, which are discussed in some detail in Part I. In Part I we demonstrated that the radar reflectivity (which for reflectivities greater than $-15$ dBZ is inferred to be precipitation)

$^1$ The effective droplet concentration $Nd_{\text{eff}}$ is retrieved from the MODIS estimates of cloud optical thickness and cloud-top effective radius under the assumption that the cloud liquid water content follows an adiabatic vertical profile. See Part I for further details.
The cloud particles that in the model are the initial source of the precipitation also scatter and frequently dominate the reflectivity signature when the precipitation rates are lower than a few tenths of a millimeter per day. The assumed gamma distribution is used to determine the cloud reflectivity, which is largest at cloud top. Because of this and because Mie scattering is not important for cloud drops, the reflectivity from cloud drops does not require corrections. Finally, the model’s column maximum reflectivity is determined as the maximum of that due to cloud and precipitation.

We should note here that in the CC model the reflectivity from cloud droplets only exceeds that due to precipitation for cases with very light amounts of precipitation (less than a few tenths of a millimeter per day or reflectivities lower than $-10$ dBZ). Such rates are not likely to have a significant dynamical impact and so for most relevant precipitation rates the radar reflectivity is primarily providing information about precipitation and not cloud.

c. Treatment of SS model data

The SS model radar reflectivity due to precipitation (which is implicitly assumed to maximize at cloud base) is corrected for Mie scattering and attenuation as a function of LWP in the same way as for the CC model. The mean volume radius of the drizzle drops $r_{v,d}$ is used to determine a Mie scattering correction, and the LWP is used to determine the attenuation.

The cloud contribution to the reflectivity (which is assumed to maximize at cloud top despite the model lacking an explicit vertical dimension) is determined by assuming a gamma distribution for the cloud droplet size distribution, with inputs $q_c$, $N_c$, and a parameterized spectral width in exactly the same manner as for the CC model (see assumption 2 in section 2a above). The reflectivity from cloud droplets does not require correction for Mie scattering.


d. Comparison with A-Train data

The CC model is run for 10 < LWP < 1000 g m\(^{-2}\) and 10 < \(N_d\) < 1000 cm\(^{-3}\). The SS model is run for 100 < \(h\) < 3000 m, which ensures LWP in the range of 10–1000 g m\(^{-2}\), and 10 < \(N_d\) < 1000 cm\(^{-3}\). We also use the observationally derived joint PDFs of LWP and \(N_{\text{eff}}\) to determine statistics of the population of clouds in each of our eight focus regions.

To ensure consistency with the observations, which assume adiabatic \(N_{\text{eff}}\), the CC model receives a value of \(N_d\) corrected for the subadiabatic nature of the assumed model clouds. This is done by determining, for each value of LWP, a cloud thickness that is consistent with the assumed vertical liquid water structure (see assumption 1 in section 2a above). This cloud thickness is then used to estimate the cloud top adiabaticity factor, and this is used to correct \(N_d\). This ensures consistency in the assumed vertical structure between the CC model and the observations. No such correction is made to the SS model because the adiabaticity is an intrinsic model variable.

The statistics we determine from the observations and model are

(i) the fraction of observed clouds that are precipitating (i.e., have a column maximum corrected reflectivity \(Z_{\text{model}} > -15 \text{ dBZ}\));
(ii) the fraction of clouds with moderate drizzle (corrected reflectivity \(Z_{\text{model}} > 0 \text{ dBZ}\));
(iii) the fraction of clouds with heavy drizzle (\(Z_{\text{model}} > 7.5 \text{ dBZ}\));
(iv) the median reflectivity of all clouds; and
(v) the median reflectivity of precipitating (\(Z_{\text{model}} > -15 \text{ dBZ}\)) clouds.

The observed values of (i)–(v) are presented by region in Table 1 of Part I of this study.

Given the \(Z-R\) relationship of Comstock et al. (2004), and taking into account the attenuation/Mie corrections appropriate for 94 GHz, a cloud base reflectivity of \(-15 \text{ dBZ}\) corresponds to approximately 0.25–0.5 mm day\(^{-1}\), whereas 0 dBZ corresponds to \(\sim 2-5 \text{ mm day}^{-1}\) and 7.5 dBZ corresponds to \(\sim 20-40 \text{ mm day}^{-1}\), but we should caution that the attenuation and Mie corrections do upset the uniqueness of the relationship between reflectivity at 94 GHz and rain rate, especially for the heaviest precipitation rates explored in this study. For more accurate study of these rates it would be preferable to use an attenuation-based retrieval such as in Haynes et al. (2009).

5. Comparison results

a. Comparison in the LWP, \(N_{\text{eff}}\) plane

Figure 4 shows the column maximum 94-GHz radar reflectivity as a function of LWP and \(N_d\) from the continuous collection model and for the entire set of A-Train observations within 30°S–30°N, 100°E–70°W. In general, both models reproduce critical aspects of the observations with some fidelity. Especially good is the ability to reproduce the threshold between precipitating and nonprecipitating clouds (i.e., the \(-15\text{-dBZ}\) contour), although the CC model is less skillful at reproducing the observed dBZ structure at high values of LWP and \(N_{\text{eff}}\) than is the SS model. Both models quite successfully capture the broad change in contour slope as the clouds transition from nonprecipitating to precipitating.

\[\text{Figure 4. Cloud-base 94-GHz radar reflectivity (solid black lines) from (a) the CC model and (b) the SS model with the KK autoconversion overlaid on A-Train data (colors, with white labels) as a function of the cloud LWP and cloud droplet concentration } N_d. \text{ For the observations the adiabatic droplet concentration } N_{\text{eff}} \text{ is used. Model inputs are corrected as described in section 4c.}\]
The CC model appears to be more successful than the SS model at capturing the weakening microphysical dependency at high $P_{CB}$ that is seen in the observations and reproduced well by the CC model. However, a comparison of Figs. 1 and 4 suggests that this reflects a different relationship between $P_{CB}$ and $Z$ in the SS model than in the CC model because $P_{CB}$ isolines in the SS model do become more vertical at increased $P_{CB}$ even though the reflectivity isolines do not. However, the $P_{CB}$ isolines in the CC model at high LWP and low $N_d$ are still steeper than the SS model for all autoconversion parameterizations other than SB (see Fig. 2).

b. Comparison of precipitation characteristics by region

For each of the eight regions described in Table 2 we use the observed joint PDF of LWP and $N_{eff}$ to produce model estimates of the metrics introduced in section 4d. These are compared with the observed values in Figs. 5 and 6. In general, both models successfully reproduce the fraction of observed/screened$^2$ clouds that are precipitating (those with reflectivities $>15$ dBZ), and also those with dBZ $>0$ (moderate drizzle). Thus, to a good degree, both models can determine important characteristics about the distribution of light and moderate drizzle for warm, relatively homogeneous clouds given their LWP and cloud droplet concentration.

However, the models cannot accurately reproduce the fraction of clouds with heavy drizzle (dBZ $>7.5$),

FIG. 5. Comparison of the (left) CC and (right) SS models with observed fraction of clouds with reflectivities greater than (a),(b) $-15$, (c),(d) 0, and (e),(f) 7.5 dBZ for the eight regions given in Table 2. The KK autoconversion parameterization is used in the SS model.

We should note that the observed drizzling fraction reported here should not be confused with the fraction of all clouds that are drizzling; rather, it is the fraction of clouds selected as being detectable by both CloudSat and MODIS, and being appropriately screened as being optically thick and relatively homogeneous as detailed in Part I.
although the CC model displays some skill. This is perhaps not surprising for the SS model given that it is primarily designed to simulate precipitation in overturning stratocumulus clouds, which rarely exhibit precipitation rates greater than 20 mm day\(^{-1}\) (Comstock et al. 2004).

It is interesting that the observations show good relationships between the fraction with moderate drizzle and the fraction with any drizzle, and between the fraction with heavy drizzle and that with moderate drizzle (Fig. 7) for the observations and for the CC model. The observed relationships are reproduced fairly well assuming that the distribution of reflectivity in each region is lognormal with a fixed geometrical standard deviation of 14 dB\(Z\) (and a variable mean). It is not known whether there is any fundamental significance to the approximately constant standard deviation, but it implies that the spread of precipitation rates (in a fractional sense) in a given region is fairly universal despite considerable differences in the mean precipitation rate. The CC model is able to capture the essence of these relationships with some skill.

The models also perform reasonably well in reproducing the median reflectivities for the different regions (Fig. 6). However, both models underestimate the precipitating fraction (\(>-15\) dB\(Z\)) and the median reflectivity (especially the CC model) for the Asian coast and Gulf of Mexico regions, which are the two regions with the highest median \(N_{\text{eff}}\) and LWP. For the CC model, this could have been anticipated given the relatively poor comparison of the model and observed reflectivity structure in the (LWP, \(N_{\text{eff}}\)) plane for high LWP and \(N_{\text{eff}}\) (see the previous section and Fig. 4). It is shown later (Fig. A3; see appendix) that this is the region of phase space where the characteristic droplet radii are around 10 \(\mu m\) or smaller, the approximate size at which the collection efficiency changes rapidly with droplet size (e.g., Rogers and Yau 1989). This certainly explains the strong sensitivity of \(P_{\text{CB}}\) to droplet size in this region for the CC model. The SS model also underestimates the median reflectivity for the ITCZ/SPCZ region, which may reflect its unsuitability for predicting precipitation in strongly drizzling cumuliform clouds.

6. Discussion

a. Sensitivity of warm rain to microphysics and macrophysics

We have seen that the observed distribution of radar reflectivity and its dependence on cloud macrophysical and microphysical properties can be reproduced with some skill using the simple heuristic models presented here. It thus gives us some confidence that we can use the model to infer aspects of the sensitivity of warm rain
to changes in macrophysical and microphysical cloud properties.

In Fig. 8 we show, for the CC model, the sensitivity of the distribution of reflectivity in each region to an across-the-board increase of 50% in LWP and a 50% decrease in $N_{eff}$. The behavior of the SS model (not shown) is very similar. Increased LWP results in a shift to the right of the reflectivity PDFs by a substantial amount (roughly 4–5 dB_Z, equivalent to around a factor of 2.5–3 increase in reflectivity), whereas the reduction in $N_{eff}$ induces significantly more modest increases in reflectivity. Further, the microphysical sensitivity decreases as the reflectivity increases (from 1–2 dB_Z around 0 dB_Z to less than 1 dB_Z at 5–10 dB_Z), consistent with the behavior seen in Fig. 4; this can be explained theoretically (see appendix) as being due to a shift to an accretion-limited regime at high reflectivity. Note that a shift of $10 \log_{10}(1.5) = 1.76$ dB_Z is equivalent to a 50% increase in reflectivity.

To confirm that these results are not strongly dependent upon our assumptions about the reflectivity–rain rate relationship, we show a similar plot for the modeled cloud base precipitation rate $P_{CB}$ distributions for the northeast Pacific region (Fig. 9). At high precipitation rates, the sensitivity of $P_{CB}$ to changes in LWP is much greater than to changes in $N_{eff}$. This behavior is repeated for the other regions (not shown).

These model results, when taken together with the observed tendency for the reflectivity to become more strongly dependent on LWP as the reflectivity increases (Fig. 4), suggest a markedly diminished ability for cloud microphysics to influence precipitation rates in clouds that are dominated by accretion. Moreover, the ability to unequivocally attribute observed variability in precipitation rates to microphysical changes will therefore be more difficult for clouds with higher precipitation rates. This finding largely explains why, although there is strong evidence for $N_d$-limited precipitation in drizzling stratocumulus (Pawlowska and Brenguier 2003; Comstock et al. 2004; vanZanten et al. 2005; Wood 2005a; Geoffroy et al. 2008), there is no such evidence for substantial $N_d$ limitation of precipitation in clouds with markedly greater liquid water paths than those found in stratocumulus (Levin and Cotton 2008). Existing observational studies supporting such microphysical impacts are largely flawed in part because they have
inferred precipitation occurrence from measures such as cloud-top effective radius (see Part I of this study) that have little direct connection to precipitation rate.

b. Steady-state precipitation

For the steady-state model, in which a balance is reached between loss of cloud water by conversion to drizzle and replenishment via turbulent updrafts (time scale \( \tau_{rep} \); see section 2b), we can define an additional time scale \( \tau_{driz} \) for the conversion of cloud to drizzle:

\[
\tau_{driz} = \frac{\rho q_l}{A_c + K_c}.
\]

A remarkable behavior of the steady-state model is that the microphysical susceptibility of the cloud base
precipitation rate \( S = d \ln P_{\text{CB}}/d \ln N_d \) is very strongly related to the ratio of the replenishment to depletion time scales \( \tau_{\text{rep}}/\tau_{\text{driz}} \). We examine \( S \) for model inputs spanning the [LWP, \( N_d \)] phase space shown in Fig. 2, with \( \tau_{\text{rep}} \) spanning 60 to 240 min, and with different autoconversion parameterizations. Figure 10 shows \( S \) normalized by the sensitivity of the autoconversion parameterization to changes in \( N_d \) (i.e., \( \beta = d \ln A_c/d \ln N_d \)) as a function of \( \tau_{\text{rep}}/\tau_{\text{driz}} \) and with this we find that \( S/\beta \) is almost independent of the autoconversion parameterization.

What this tells us is that clouds in steady state with high precipitation efficiency (low \( \tau_{\text{driz}} \)) and/or slow replenishment (high \( \tau_{\text{rep}} \)) have a lower sensitivity to \( N_d \) than those with lower precipitation efficiency and/or rapid replenishment. Observations in stratocumulus clouds (Wood et al. 2005a) suggest that replenishment time scales for liquid water in stratocumulus may be a few times the eddy turnover time scale, or around 1–2 h. This is consistent with the lifetimes of mesoscale drizzle cells that frequently dominate the dynamics of these clouds (Comstock et al. 2007). Given that typical liquid water contents in stratocumuli are \( \approx 0.5 \) g kg\(^{-1} \), and that \( A_c + K_c \) is on the order of \( 5 \times 10^{-9} \) to \( 5 \times 10^{-8} \) g m\(^{-3} \) s\(^{-1} \) (for precipitation rates of the order of 0.5–5 mm day\(^{-1} \)), this would put \( \tau_{\text{driz}} \) in the range \( 10^4 \) to \( 10^5 \) s and \( S/\beta \) in the range 0.2–0.8. Even drizzling stratocumulus clouds may therefore exhibit a sensitivity to \( N_d \) that is substantially weaker than the sensitivity of autoconversion itself. This may explain why precipitation closure observations in stratocumulus show sensitivities to \( N_d \) at the lower end of those in most autoconversion parameterizations.

The reasons for this behavior are again consistent with the increased importance of accretion as the precipita-

![Fig. 9](image-url)  
**Fig. 9.** As in Fig. 8, but showing the distributions of the cloud base precipitation rate \( P_{CB} \) from the CC model for the northeast Pacific region (see Table 2).

![Fig. 10](image-url)  
**Fig. 10.** Fractional sensitivity, for the SS model, of the cloud base precipitation rate to cloud droplet concentration \( S = d \ln P_{CB}/d \ln N_d \) normalized with the autoconversion sensitivity \( \beta \) as a function of the ratio of the replenishment to drizzle time scales \( \tau_{\text{rep}}/\tau_{\text{driz}} \). Each point represents a separate combination of the cloud thickness (100 < \( h < 3000 \) m), droplet concentration (10 < \( N_d < 1000 \) cm\(^{-3} \)), \( \tau_{\text{rep}} \) (30 < \( \tau_{\text{rep}} < 240 \) min), and the different colors are for different autoconversion parameterizations. For reference, \( \beta \) is \( -1.79, -1.0, \) and \( -3.3 \) for the KK, LD, and BEH autoconversion parameterizations, respectively. The SB parameterization is not included in this plot because the autoconversion rate also depends on the drizzle LWC for this model and so a single value of \( \beta \) cannot readily be defined. The gray line is the arbitrary fit through the data using the function \( S/\beta = (1 + 5 \tau_{\text{rep}}/\tau_{\text{driz}})^{-1} \).

c. **Representation in large-scale models**

Climate models generally show second aerosol indirect effects (AIEs) that are of comparable magnitude to the first AIE (Lohmann and Feichter 2005). This suggests that climate model precipitation is sensitive to cloud microphysical changes induced by increasing aerosol concentrations. Given that the representation of precipitation in climate models is at least as detailed as the steady-state model used here, is it necessary to be concerned about how climate models treat warm rain?

One of the key assumptions in the steady-state model is that precipitation is treated prognostically, whereas in most climate models the longer time steps required render a diagnostic formulation more efficient (e.g., Ghan and Easter 1992). Posselt and Lohmann (2008) show that a prognostic treatment of precipitation increases the relative contribution to precipitation from accretion compared with autoconversion in weakly precipitating clouds. Given that this appears from our model results to be a critical determinant of the sensitivity of...
precipitation to cloud microphysics, it may be plausible to hypothesize that diagnostic treatment of warm rain formation in climate models will overestimate the microphysical sensitivity and therefore the second AIE. Further work is required to establish if this is a pathological problem for climate models.

7. Conclusions

In this study, we have attempted to reproduce some of the salient observational results detailed in Part I of this study. These observations, for which important new data from the CloudSat satellite is combined with visible/near-IR data from MODIS, demonstrate that the radar reflectivity in precipitating low clouds is influenced both by variability in macrophysical quantities like cloud liquid water path and by microphysical quantities such as the estimated cloud droplet concentration \( N_{\text{eff}} \). The observations demonstrate that for clouds with higher reflectivities, the reflectivity is more sensitive to LWP observations demonstrate that for clouds with higher reflectivities, the reflectivity is more sensitive to LWP than to \( N_{\text{eff}} \). This general tendency is reproduced well by a continuous collection (CC) model of precipitation formation and can be explained by the increasing importance of accretion in controlling the precipitation amount when the precipitation rate is larger than a few millimeters per day.

The general behavior of the observations is replicated using a minimal analytical CC model that aids understanding. Because accretion is largely controlled by the availability of liquid water and is not strongly limited by reduced collection efficiency, this leads to a weakened dependence of precipitation on \( N_{\text{eff}} \) at high LWP in the accretion-dominated regime. Quantitatively similar behavior is obtained in a bulk microphysical steady-state precursor model, but there is strong quantitative sensitivity to the choice of autoconversion parameterization.

Finally, the observations suggest that the influence of anthropogenic aerosols (through their ability to act as CCN and influence the cloud droplet concentration) on warm rain is likely to be a strong function of the cloud microphysical and macrophysical state of the clouds into which the aerosols are being ingested. In other words, the susceptibility of warm rain to microphysical changes will likely depend on the type of clouds and therefore on the meteorological regime. It will be useful in future observational and modeling studies to examine this susceptibility as a function of the liquid water path and the cloud droplet concentration.

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APPENDIX

Minimal CC Model

The form of the relationship of LWP, \( N_{\text{d}} \), and \( P_{\text{CB}} \) in the CC model is strongly controlled by the dependence of the collection efficiency on the collector drop size. To demonstrate this, a minimal model is constructed that simplifies the warm rain model into an analytical form.

We begin with (3) and retain the \( R \) dependence only through the collection efficiency, that is, \( E(R, r_{\text{w}}) \). This is reasonable since \( \eta = (1 + r_{\text{w}}/R)^{4}[1 - v_{\text{w}}(r_{\text{w}})/v_{\text{T}}(R)] \) does not vary strongly with \( r_{\text{w}}/R \), whereas the collection efficiency drops dramatically for small droplets. We set \( \eta = 1.2 \), and assume a constant value of \( r_{\text{w}} \) through the cloud layer. We choose \( r_{\text{w}} \) as the liquid water weighted mean value of \( r_{\text{w}} \) for the cloud layer. It can be shown that this is \( 6/7 \) times the value at the cloud top \( r_{\text{w}} \) for clouds in which liquid water increases linearly with height.

We then integrate (3) from cloud top to cloud base:

\[
\int_{R_{\text{num}}}^{R_{\text{CB}}} E^{-1}(R, r_{\text{w}}) dR = \frac{\eta \text{LWP}}{4 \rho_{w}}. \tag{A1}
\]

We then parameterize \( E^{-1}(R, r) \), which can be thought of as a collection inhibition factor, as the product of a function depending upon the collector drop size to \( R \) and a function depending on the collected drop radius \( r_{\text{w}} \); that is,

\[
E^{-1}(R, r) = Q_{T}Q_{R} \approx \left[ 1 + \left( \frac{a}{R} \right)^{5} \right] \left[ 1 + \left( \frac{b}{r_{\text{w}}} \right)^{4} \right]. \tag{A2}
\]

with \( a = 30.6 \mu\text{m} \) and \( b = 6.27 \mu\text{m} \). Figure A1 shows \( E^{-1}(R, r) \) from Hall (1980) and from (A2). The parameterization captures the inhibition of coalescence for small values of the collected drop and particularly the collector drop. With the parameterization (A1) becomes

\[
\left[ R - \frac{a^{5}}{4} R^{4} \right]_{R_{\text{num}}}^{R_{\text{CB}}} = \frac{\eta \text{LWP}}{4 \rho_{w} \left[ 1 + \left( b/r_{\text{w}} \right)^{4} \right]} \tag{A3}
\]

We further approximate by assuming that the \( R^{-4} \) term is small compared to the first for the upper limit; that is, \( 1/4(a/R_{\text{CB}})^{4} \ll 1.1R_{\text{CB}} \), which is a good approximation for \( R_{\text{CB}} \gtrsim 30 \mu\text{m} \), which is the case for all cases with drizzle greater than a few hundredths of a millimeter per day. Then (A2) yields an expression for the growth of the embryonic drizzle drops from cloud top to base:
ΔR = R_{CB} - R_{emb} = \frac{ηLWP}{4\rho_w[1 + (b/r_v)^4]} - \frac{a^5}{4R_{emb}^4}, \quad (A4)

Given the assumptions made regarding the cloud droplet size distribution—that is, model assumption 2 (see section 2a)—a simple parameterization for $R_{emb}$ as a function of $r_v$ is possible, namely $R_{emb} \approx c(r_v + d)$, with $c = 1.4$ and $d = 4.0$ μm (note that the values of $c$ and $d$ will depend on the assumed precipitation embryo concentration $N_D$, here assumed to be 100 L$^{-1}$, and should be tuned if $N_D$ is varied). Then (A4) becomes

$$ΔR = R_{CB} - R_{emb} \approx \frac{ηLWP}{4\rho_w[1 + (b/r_v)^4]} - \frac{a^5}{4c^4(r_v + d)^4}. \quad (A5)$$

The precipitation rate at cloud base from the minimal model $P_{CB}$ is then given by

$$P_{CB} = \frac{4\pi\rho_w}{3} \alpha_f N_D R_{emb}^{3+δ} \quad (A6)$$

$$= \frac{4\pi\rho_w}{3} \alpha_f N_D \left\{ c(r_v + d) + \frac{ηLWP}{4\rho_w[1 + (b/r_v)^4]} \right\} + \frac{a^5}{4c^4(r_v + d)^4}. \quad (A7)$$

Equation (A7) expresses the cloud base precipitation rate as a function only of the cloud liquid water path and the mean volume radius at cloud top (uniquely determined as a function of LWP and $N_D$ for a given $z_0$). The first term in the parentheses represents a memory of the initial size of the embryonic drizzle drops. The second term represents the reservoir of available cloud water to be collected by the falling drizzle drops (with the denominator describing limits to their growth when the collected droplets are small and therefore collected inefficiently), and the third term is a suppression that represents inhibition by the limited initial size of the falling embryos themselves.

Figure A2 shows that the minimal model cloud base precipitation rate $P_{CB}$ agrees very well with that from the full CC model and is able to demonstrate the increasing importance of LWP in determining the precipitation rate at low $N_D$ and high LWP. Equation (A7) readily demonstrates sensitivity to both LWP and $N_D$ consistent with observations in stratocumulus clouds (Pawlowska and Brenguier 2003; Comstock et al. 2004; vanZanten et al. 2005), but for deeper precipitating trade cumulus clouds (which typically have higher local LWP and low $N_D$) the sensitivity to LWP increases and that $N_D$ decreases so that $P_{CB} \sim LWP^{3+δ}$.

To further indicate the utility of the minimal model, Fig. A3 shows the relative importance of the three terms in (A7) superimposed on the precipitation rate from the CC model. As the precipitation rate increases, the term involving the accretion of cloud water becomes dominant. This also demonstrates that the cloud-top effective radius alone does not serve as a particularly useful predictor of the tendency to precipitation or of the rate...
that a cloud of a given thickness might produce. Only for the very smallest $P_{CB}$ and for very low LWP are the precipitation isohyets even close to being parallel to the lines of constant effective radius.

For these thicker clouds, where $h$ is significantly larger than $z_0$, LWP is approximately linearly dependent upon the cloud thickness. This is consistent with an observed lack of a trend in the liquid water profile with height for clouds thicker than about 1 km (Rauber et al. 2007). Thus, the CC model suggests that the precipitation rate in trade cumuli would scale with the cloud thickness to the fourth or fifth power, with a much weaker dependence on $N_d$, a result that is at least qualitatively consistent with recent large-eddy simulations (Stevens and Seifert 2008).

REFERENCES


