Parametrization of the effect of drizzle upon the droplet effective radius in stratocumulus clouds

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Abstract

A method is presented to parametrize the effects of drizzle upon the droplet effective radius in stratocumulus clouds. The cloud droplet size distribution in stratocumulus is represented by the sum of a modified Gamma distribution to represent the small (r <20 μm) droplets and an exponential Marshall-Palmer type distribution to represent the large (drizzle) droplets. Using this approach a relationship is derived to account for the effect of drizzle upon k, the cube of the ratio between the volume and effective radius. Observational evidence from flights in a range of different airmass types is presented to validate the approach. The results suggest that the value of k pertaining to the small droplets is better parametrized as a function of volume radius rather than of droplet concentration. The results also suggest that as the ratio of liquid water content contained in the large droplets to that in the small droplets increases beyond 0.05, the value of k decreases significantly. This results in an underprediction of the effective radius if commonly used parametrizations for k are used.

Keywords: stratocumulus, effective radius, droplet size distributions, drizzle

1 Introduction

The accurate representation of stratocumulus cloud radiative properties in large-scale numerical models is fundamental to the understanding of the climatic effects of these clouds (Slingo 1990, Jones et al. 1994). Determination of these properties relies on knowledge of processes controlling:

(a) the spatial distribution of liquid water content within a cloud system;
(b) the partitioning of liquid water content between droplets of different sizes.

Here, we will be concerned with the microphysical aspects (b) and address the problem of the parametrization of the effective radius r_e in stratocumulus clouds, defined as

$$r_e = \frac{\int_{r=0}^{\infty} N(r) r^3 dr}{\int_{r=0}^{\infty} N(r) r^2 dr}, \tag{1}$$

where $N(r)$ is the droplet concentration between radii r and $r + dr$. The importance of the effective radius is that for a cloud column the cloud optical thickness τ is given (e.g. Stephens 1978) to a very good degree of accuracy by

$$\tau \approx \frac{3}{2} \int_{z=0}^{h} \frac{q_L(z)}{\rho_w r_e(z)} dz, \tag{2}$$

where z is the height, q_L is the liquid water content, ρ_w is the density of liquid water and h is the height of the top of the cloud. It is clear from (2) that the parametrization of r_e is central to the accurate prediction of the cloud optical properties.

Computational limitations are such that climate models have not yet evolved to the state where accurate size-resolved microphysics can be incorporated. It is therefore necessary to attempt to represent

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cloud microphysics using a few key parameters (e.g. cloud water content, droplet number, spectral width, rain water content etc.). The effective radius is one key parameter because, as was demonstrated above, it couples cloud microphysics with radiative processes. There has been considerable impetus in recent years to develop parametrizations of effective radius that can be used within a climate model framework. Bower and Choularton (1992), Pontikis and Hicks (1992), Martin et al. (1994) and Gultepe et al. (1996) have all presented parametrizations which relate the effective radius to the liquid water content q_L and the droplet concentration N_0 via an equation of the form

$$r_e = \left(\frac{3q_L}{4\pi\rho_w N_0 k}\right)^{\frac{1}{3}} \tag{3}$$

where k differs between the different parametrizations, and is shown in table 1. It should be noted that k is the cube of the ratio of the volume radius and effective radius:

$$k = \left(\frac{r_{vol}}{r_e}\right)^3 \tag{4}$$

where $r_{vol} = (3q_L/(4\pi\rho_w N_0))^{\frac{1}{3}}$. The Bower and Choularton (1992) parametrization, derived from observations of stratocumulus, cumulus and hill cap-clouds makes the assumption that the droplet effective radius is equal to the volume radius, i.e. that $k = 1$ in (3). The Martin et al. (1994) parametrization was derived from observations in marine stratocumulus cloud only and suggests that two values of k should be used: $k = 0.67$ for stratocumulus in polluted airmasses and $k = 0.80$ for stratocumulus in marine airmasses. The Pontikis and Hicks (1992) parametrization was derived from observations of warm trade-wind cumuli and suggest that $k = 0.86$. Pontikis and Hicks (1992) and Martin et al. (1994) show that for zero skewness the value of k is a decreasing function of spectral dispersion. Because there are generally more coherent updraughts in cumuli, one would expect the spectral dispersion to be lower than in stratocumulus, which might be one of the reasons for the higher values of k in the Pontikis and Hicks (1992) observations. The parametrization of Pontikis (1996) does not consider values of k directly, but instead opts to diagnose a cloud-mean effective radius from the liquid water path, rather than the local liquid water content.

Importantly, none of the above parametrizations of effective radius considers the effect of drizzle upon the relationship between volume and effective radius. The microphysical measurements used to derive values of k were restricted to radii less than approximately 25 μm, whereas there are a number of well-documented cases where the amount of liquid water content contained in droplets larger than this is appreciable (e.g. Nicholls and Leighton, 1986; Austin et al., 1995; Boers et al., 1996; Gerber 1996; Hudson and Yum, 1997). In this study the relationship between volume radius and effective radius is examined for stratocumulus clouds in a range of different airmass types and with a range of drizzle contents. Section 2 describes a method by which the droplet size distribution in stratocumulus cloud is described using a two-distribution method. The small droplet spectrum is modelled using a three-parameter modified Gamma function (Austin et al. 1995); the large droplet spectrum (drizzle drops) is modelled using a two-parameter exponential distribution. Section 3 details *in-situ* observations of stratocumulus clouds using *The Met. Office* C-130 aircraft to verify a relationship predicted between the ratio of the drizzle liquid water to cloud liquid water content and the ratio of the volume to effective radius.

2 Two-function representation of the droplet size distribution

a. Representation of small droplets

The small droplet size distribution in stratocumulus (r less than approximately 20-30 μm) is well represented using a modified Gamma function (e.g. Austin et al. 1995) of the form

$$N_s(r) = 3N_{0,s}\frac{(1+\alpha)^{(1+\alpha)}}{\Gamma(1+\alpha)}\left(\frac{r^{(3\alpha+2)}}{r_{vol,s}^{(3\alpha+3)}}\right)\exp\left(-[1+\alpha]\frac{r^3}{r_{vol,s}^3}\right), \tag{5}$$

where $N_{0,s}$ is the total droplet concentration of the small droplets described using (5), Γ is the Gamma function, α is a variable parameter and $r_{vol,s}$ is the droplet volume radius given by

$$r_{vol,s} = \left(\frac{3q_{L,s}}{4\pi\rho_w N_{0,s}} \right)^{\frac{1}{3}}, \tag{6}$$

where ρ_w is the density of liquid water. The form (5) is a three-parameter distribution controlled by α and any two of $N_{0,s}$, $r_{vol,s}$ and $q_{L,s}$. The effect of α upon the distribution shape is demonstrated in Fig. 1 which shows three distributions from (5) with identical $N_{0,s}$ and $q_{L,s}$ but with values of α equal to 0.75 (solid), 1.5 (dotted) and 2.5 (dashed). The effect of α is clearly visible: larger values of α result in narrower spectra for the same $N_{0,s}$ and $q_{L,s}$. Note that $\alpha > 0$, since $\Gamma(0)$ is undefined.

To derive the small droplet effective radius $r_{e,s}$ from (5) we note that

$$\int_{r=0}^{\infty} N_s(r) r^2 dr = \frac{\Gamma\left(\alpha + \frac{2}{3}\right) r_{vol,s}^2}{\alpha^{\frac{2}{3}} \Gamma(\alpha)} \tag{7}$$

and therefore that

$$r_{e,s} = r_{vol,s} \frac{\alpha^{\frac{2}{3}} \Gamma(\alpha)}{\Gamma(\alpha + \frac{2}{3})}. \tag{8}$$

The ratio between the cubes of the volume and effective radii for small droplets is denoted as k in Martin et al. (1994). Because later we will examine the effect of larger droplets upon this parameter it is defined here as k_s (small droplets only). Observed values of k_s were generally larger for the clean airmasses (lower $N_{0,s}$; mean k_s=0.80) than for polluted airmasses (higher $N_{0,s}$; mean k_s=0.67). Using the modified Gamma distribution we obtain

$$k_s = \left(\frac{r_{vol,s}}{r_{e,s}} \right)^3 = \left[\frac{\Gamma\left(\alpha + \frac{2}{3}\right)}{\Gamma(\alpha)\, \alpha^{\frac{2}{3}}} \right]^3. \tag{9}$$

Figure 2 shows k_s plotted as a function of α using (9). The observations from Martin et al. (1994) show that approximately two thirds of all the cases (1σ) examined had values of k_s between 0.60 and 0.87, which suggests a range $0.55 < \alpha < 2.35$ is representative of small droplet size distributions in stratocumulus cloud. The possible limitation of the approach of Martin et al. (1994) is that each observed value of k was calculated using linear regression of $r_{vol,s}^3$ against $r_{e,s}^3$ in a profile. It is not possible to examine any variability of k with height in the cloud and such observed k_s values represent average values through the cloud deck.

b. Representation of large droplets

Observations lend support to large droplets (r >20-30 μm) in stratocumulus having a near-exponential size distribution (e.g. Ichimura 1980). Here we will represent the large droplet size distribution N_l by an equation of the form

$$N_l(r) = 6^{\frac{1}{3}} \frac{N_{0,l}}{r_{vol,l}} \exp\left(-6^{\frac{1}{3}} \frac{r}{r_{vol,l}} \right), \tag{10}$$

where $N_{0,l}$ is the total concentration in the large droplet size distribution and $r_{vol,l}$ is the corresponding large droplet volume radius. Thus any two of $N_{0,l}$, $r_{vol,l}$ and $q_{L,l}$ are sufficient to define completely the large droplet distribution. The effective radius calculated for the large droplets is

$$r_{e,l} = \frac{3}{6^{\frac{1}{3}}} r_{vol,l} \tag{11}$$

which leads to

$$k_l = \left(\frac{r_{vol,l}}{r_{e,l}} \right)^3 = \frac{2}{9} \approx 0.22. \tag{12}$$

Thus, if a size distribution is represented exactly by an non-truncated exponential distribution (i.e. $0 < r < \infty$) then the ratio of the volume radius to effective radius is a constant.

c. Combined droplet size distribution

In this study we denote the entire droplet spectrum ($N(r)$) by a linear sum of the Gamma distribution (5) and the exponential distribution (10):

$$N(r) = N_s(r) + N_l(r) \tag{13}$$

from which parameters such as volume radius r_{vol} and r_e corresponding to the entire droplet spectrum are derived as

$$r_{vol} = \left(\frac{N_{0,s}\, r_{vol,s}^3 + N_{0,l}\, r_{vol,l}^3}{N_{0,s} + N_{0,l}} \right)^{\frac{1}{3}} \tag{14}$$

and

$$r_e = \left(\frac{N_{0,s}\, r_{vol,s}^3 + N_{0,l}\, r_{vol,l}^3}{\left[N_{0,s}\Gamma(\alpha) r_{vol,s}^2 / \alpha^{\frac{2}{3}} \Gamma(\alpha + \frac{2}{3}) \right] + 6^{\frac{1}{3}} r_{vol,s}^2 N_{0,l}/3} \right). \tag{15}$$

The value of k relating the volume and effective radii for the entire droplet size distribution is found by combination of (14) and (15):

$$k = \frac{N_{0,s}}{N_{0,s} + N_{0,l}} \left(\frac{\Gamma(\alpha + \frac{2}{3})}{\alpha^{\frac{1}{3}} \Gamma(\alpha)} + 6^{\frac{1}{3}} \frac{N_{0,l}\, r_{vol,l}^2}{N_{0,s}\, r_{vol,s}} \right)^3 \Bigg/ \left(1 + \frac{N_{0,l}\, r_{vol,l}^3}{N_{0,s}\, r_{vol,s}^3} \right)^2 \tag{16}$$

or alternatively in terms of liquid water contents and by using the expressions (9) and (12) for k_s and k_l respectively one obtains

$$k = k_s \frac{N_{0,s}}{N_{0,s} + N_{0,l}} \left(1 + \left[\frac{k_l}{k_s} \right]^{\frac{1}{3}} \frac{r_{vol,s}}{r_{vol,l}} \phi \right)^3 \Bigg/ (1 + \phi)^2 \tag{17}$$

where $\phi = q_{L,l}/q_{L,s}$. Equation (17) provides a means for the prediction of the effect of large drops upon the ratio between volume and effective radius. However, it is not immediately clear how observational values of $q_{L,s}$, $q_{L,l}$, $r_{vol,s}$ and $r_{vol,l}$ would be defined. For example, variables with subscripts s will be calculated using the droplet spectrum ($0 < r < r_{thresh}$), where r_{thresh} is a chosen threshold radius which partitions the size distribution into small and large drops. What value should be chosen for r_{thresh}? Physically, large drops are those which have been formed largely via the coalescence process. It would therefore be a sensible choice to select an r_{thresh} somewhere around 20 μm, because drops smaller than this grow primarily by condensational growth and drops larger primarily by coalescence (e.g. Jonas 1996).

In this study we make the choice r_{thresh}=20 μm. The implication therefore is that the Gamma function represents droplets grown via the condensational process and the exponential those grown via coalescence (i.e. drizzle drops). If this is to be a physically realistic partitioning then it should not be the case that the Gamma function contains significant quanitities of liquid water content in drops larger than 20 μm. Similarly, the exponential function should not contain a significant amount of liquid water in the small drop range. Because we do not truncate the distributions N_s and N_l we need to estimate the fraction of liquid water content contained in the model distribution N_s in drops smaller than 20 μm when the observed ranges for the parameters $r_{vol,s}$ and k_s are used. Similarly, the fraction of liquid water content in the model distributions N_l in drops larger than 20 μm is estimated using the observed range of $r_{vol,l}$. For all N_s distributions formed using observed values of $r_{vol,s}$ and k_s over 95% of the liquid water content is contained in drops smaller than 20 μm. For all N_l distributions examined, over 90% of the liquid water content is contained in drops larger than 20 μm. These high fractions confirm that the choice of r_{thresh}=20 μm is a suitable partioning.

3 Observations

Observations presented here were all made using The Met. Office C-130 aircraft in marine stratocumulus clouds within a wide range of airmass types. The geographical regions sampled include off-coastal California in the East Pacific high pressure region, around the Azores Islands in the mid-Atlantic, the ocean areas north of the Canary Islands and ocean areas around the United Kingdom (The North Sea and Eastern Atlantic Ocean). The microphysical instrumentation used here consists of a PMS[2] Forward Scattering Spectrometer Probe (FSSP) and a PMS 2D cloud probe (2D-C). The FSSP measures droplets in the range $0.25 < r < 24$ μm, using 15 bins evenly spaced in radius. The 2D-C measures droplets in the range $12.5 < r < 400$ μm, using 31 bins evenly spaced in radius. Standard widely used corrections have been applied to the FSSP data (see Martin et al. 1994). Corrections to the 2D-C data include removal of particles touching the edge of the field of view and removal of zero-image particles (ones triggering probe but too small to leave an image). A summary of such corrections is given in Moss and Johnson (1994). For purposes of producing combined size distributions data from the smallest bin (centred on r=12.5 μm) of the 2D-C probe data were discarded because this bin has a poorly defined depth of field leading to considerable uncertaintly in the sample volume (see Baumgardner and Korolev 1997). After discarding the first size bin, there remains some overlap between the largest FSSP size bins and the smallest 2D-C bin. In the overlap region the combined drop concentration is formed from a linear average of the FSSP and 2D-C concentrations. The final combined size distributions are linearly interpolated onto a grid with equally-spaced radius bins (bin spacing 1.5 microns). The parameters $q_{L,s}$, $r_{vol,s}$ and k_s calculated from the measured distribution from data in size bins with radii less than 20 μm. The large drop parameters $q_{L,l}$ and $r_{vol,l}$ are calculated from data in size bins with radii greater than 20 μm. All size distributions used here are calculated from either in-cloud straight and level flight or shallow ascent/descent profiles. An averaging time of 30 seconds is used to minimise statistical counting errors especially for the drizzle-sized droplets. The maximum vertical distance travelled during any 30 second period is never greater than 75 m.

Because the amount of in-cloud data from the different clouds varied considerably, a maximum limit as to how many 30 second samples could be used from any one flight was imposed. This limit is set equal to the number of 30 second samples available from the flight with the shortest in-cloud time. If a flight contained more samples than this limit then samples were selected randomly from the available pool. This procedure ensures that the contribution from each cloud is weighted evenly.

a. Validity of two-function size distribution

Figure 3 shows size distributions measured using the FSSP (filled circles) and 2D-C (filled triangles) probes for two different clouds. Each spectrum is derived from 30 s of straight and level flight. Fig. 3(top) is from a heavily drizzling cloud in a clean airmass around the UK (flight A648) while Fig. 3(bottom) is from a cloud with no discernible drizzle leaving the cloud-base in a polluted airmass around the UK (flight A641). The thick dashed curve represents the two-function size distribution described above, defined using the parameters α (from k_s), $N_{0,s}$, $q_{L,s}$, $N_{0,l}$ and $q_{L,l}$. The values for the parameters were derived from the observed size distributions (α, $N_{0,s}$ and $q_{L,s}$ from the $r < 20\mu$m drops; $N_{0,l}$ and $q_{L,l}$ from the $r > 20\mu$m drops). For the two cases the two-function distribution represents a reasonably good fit to the observed data, visually demonstrating the validity of the approach. There is some discrepancy between the observed and modelled size-distributions, but the effects of this upon derived spectral parameters is fairly small. Also shown by vertical dashed lines in Fig. 3 is the effective radius derived from the small ($r < 20\mu$m) drops $r_{e,s}$ and from the entire measured drop size distribution. In the heavy drizzling case the effect of including the large drops is marked, increasing the effective radius considerably. In the polluted case there are insufficient large drops so that the effective radius derived from the small drops is almost identical to that calculated from the entire size distribution.

a. Prediction of k_s

First, we examine the relationships between the parameter k_s and the parameters $N_{0,s}$ and $r_{vol,s}$. Figure 4 shows these relationships for all the in-cloud data which indicates that the parameter $r_{vol,0}$ is a better predictor of k_s than is the droplet concentration $N_{0,s}$.

Of course, the relationship (6) between the volume radius and the droplet concentration indicates that at a given liquid water content $q_{L,s}$ the droplet concentration is inversely proportional to the third power of the volume radius. Hence, many of the cases with large values of $r_{vol,s}$ correspond to small $N_{0,s}$. However, Fig. 5 shows that for a single thick cloud deck (very little drizzle), the value of k_s increases with increasing height in the cloud. The cloud is polluted with mean $N_{0,s}$=300 cm^{-3} and accumulation mode aerosol concentrations of approximately 1000 cm^{-3}). Although the mean value of k_s towards the base of the cloud is around 0.6 typical of the polluted cases studied by Martin et al (1994), the mean value of k_s

[2]Particle Measuring Systems Inc., Boulder, Colorado, USA

close to the top of the cloud is around 0.75, more typical of values reported for clouds in clean airmasses.

To quantify the fact that $r_{vol,s}$ is a better predictor of k_s we carried out polynomial regression on the $[r_{vol,s}, k_s]$ and $[N_0, k_s]$ paired data for polynomial orders 1 (linear) through 4 (quartic). We then calculated the RMS difference between the observed and fitted k_s for both pairings. Fig. 6 shows this RMS difference for the four polynomial orders. For all orders the RMS difference is substantially smaller for the $[r_{vol,s}, k_s]$ pairing. It is therefore concluded that k_s is better parametrized using $r_{vol,s}$ rather than $N_{0,s}$. In addition, no great advantage is achieved by increasing the order of the polynomial beyond second order. Although the $[r_{vol,s}, k_s]$ data are well fitted by a second order polynomial, this has the disadvantage of having a turning point at approximately $r_{vol,s}$=12 μm, which is unlikely to be physically realistic. Instead we find that a monotonically increasing function of $r_{vol,s}$ given by

$$k_s = 0.865 - \exp\left(-0.30\, r_{vol,s}\right) \tag{18}$$

gives RMS difference (observed-fitted) almost identical to that of the second order polynomial.

b. Effect of large droplets upon k and r_e

Equation (17) is used to derive the value of k from the value of k_s and the ratio of water contents in the large droplet and small droplet modes. The factor involving the ratio of number concentrations $N_{0,s}/(N_{0,s} + N_{0,l})$ is assumed to be unity which is a good approximation in almost all cases (Fig. 7, left). The value of k_s in the square brackets in Eqn. (17) is taken to be 0.7 so that it is possible to plot the ratio k/k_s as a function of $q_{L,l}/q_{L,s}$. The resulting function is quite insensitive to changes in this k_s value (for values of k_s within the range of observed values). Figure 7(right) shows the probability distribution of the ratio $r_{vol,s}/r_{vol,l}$ derived from the observations. Over 94% of the observations has 0.1< $r_{vol,s}/r_{vol,l}$ <0.3. Figure 8 shows k/k_s against $q_{L,l}/q_{L,s}$ for the entire dataset. Overplotted are curves obtained using (17) for three values of the ratio $r_{vol,s}/r_{vol,l}$=0.1, 0.2 and 0.3. The observations and predictions are in excellent agreement. The best agreement is obtained for $r_{vol,s}/r_{vol,l}$=0.2.

Figure 9 shows the predicted value of k using (17) against the observed value of k for data from all the clouds studied. In this case the values of k_s are calculated using the parametrization fit (18). Over 86% of the predicted values are within 10% of the observed values. The rms difference between predicted and observed is 0.056.

To ascertain the effect of the inclusion of drizzle-sized droplets in the calculation of the effective radius in stratocumulus, we compare the effective radius values calculated using the Martin parametrization with those using (18) and (17). The details of the parametrizations are:

(a) Martin et al. (1994). The volume radius is calculated from the observed liquid water content (q_L=$q_{L,s}$+$q_{L,l}$) and the total droplet concentration N_0. The value of k used is 0.67 (N_0 >150 cm^{-3}) and 0.8 (N_0 <150 cm^{-3}) where a concentration of 150 cm^{-3} is taken to be the threshold between clean and polluted airmasses. Denote this effective radius as r_e(Martin).

(b) This study. Parametrization presented here in Eqns. 18 and 17 to account for the effects of drizzle upon effective radius. We make the assumption that $r_{vol,s}/r_{vol,l}$=0.2 and that $N_{0,s}/(N_{0,s}+N_{0,l})$=1. An explicit formulation for the effective radius can be written as

$$\frac{r_e}{r_e(\text{Martin})} = \frac{(1+\phi)^{\frac{2}{3}}}{\left(1+0.2\left(\frac{k_l}{k_s}\right)^{\frac{1}{3}}\phi\right)} \tag{19}$$

where $\phi = q_{L,l}/q_{L,s}$, k_l=2/9 from Eqn. (12) and k_s is obtained as a function of the volume radius of the small drops via Eqn. (18). Expressing in terms of the effective radius parametrized using Martin et al. (1994) allows a comparison of the two schemes as a function of ϕ, which is presented in Fig. 10. The ratio of the effective radius parametrized using the method presented in this study to that using Martin et al. (1994) is plotted as a function of ϕ, for different values of k_s and $r_{vol,s}/r_{vol,l}$ to demonstrate the sensitivity to these parameters. The value of k_s does not make a very large difference to the ratio. The value of $r_{vol,s}/r_{vol,l}$ makes more difference especially at large ϕ. For the value of $r_{vol,s}/r_{vol,l}$ = 0.2 suggested in this study the inclusion of the effects of drizzle lead to an increase in the effective radius by some 30% when the liquid water content in the large drops is equal to that in the small drops.

Figure 11 shows the ratio of the observed to parametrized effective radius as a function of the ratio of the liquid water content in the large drops to that in the small drops. The observed effective radius was calculated from the entire droplet size distribution including both large and small drops. The parametrization *(b)* leads to much better prediction of the effective radius even at low drizzle water contents. Above $q_{L,l}/q_{L,s}$ >0.05 the Martin et al. parametrization starts to underpredict the effective radius. In heavy drizzle cases the Martin et al. (1994) parametrization the underestimation of effective radius is severe. Table 2 shows the mean value of r_e(pred)/r_e(obs) for the two parametrizations as a function of ϕ=$q_{L,l}/q_{L,s}$.

4 Conclusions

It has been demonstrated that in order to accurately parametrize the effective radius in stratocumulus it is important to take into account the amount of drizzle liquid water content. Large droplets (r >20 μm) are distributed approximately exponentially therefore have a different relationship between effective and volume radius. A parametrization has been suggested based upon a two-distribution model of the droplet spectrum. The smaller droplets are described by a modified Gamma function and the larger droplets by an exponential form. Such a description leads to a relationship between the ratio of the large to small droplet liquid water content, the ratio of small to large volume radii and the reduction in the k parameter associated with the presence of drizzle, thereby allowing prediction of effective radius as a function of the amount of liquid water content in the large droplet mode. The existing parametrization of Martin et al. (1994) works well when little drizzle is present in the cloud but underestimates the effective radius for higher drizzle ratios. It is suggested that $r_{vol,s}/r_{vol,l}$=0.2 and $N_{0,s}/(N_{0,s}+N_{0,l})$=1 are suitable approximations. The parametrization is intended for use in climate models which carry a prognostic drizzle water variable.

Given that the effect of drizzle upon stratocumulus cloud radiative properties is postulated to be climatologically important, it is crucial that the strong link between cloud microphysical and radiative properties is understood better. Inclusion of the effects of drizzle on the shortwave properties of stratocumulus would lead to model clouds more optically thin than those predicted using existing parametrizations of effective radius. For example, a 5% increase in the parametrized effective radius throughout a cloud layer resulting from a cloud with 5-10% of its liquid water in the large droplet mode (Fig 11 and Table 2) would lead to a decrease in optical depth of 5% (Eqn. 2). There is thus a need to include the effects of drizzle droplets when parameterizing the shortwave radiative properties of stratocumulus clouds in climate models.

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Tables

Table 1: Some existing parametrizations of effective radius which use a relationship between volume and effective radius

Parametrization	Cloud type studied	k
Bower and Choularton (1992)	stratocumulus, cumulus, cap-clouds	1 (r_e=r_{vol})
Pontikis et al. (1992)	trade-wind cumulus	0.86
Martin et al. (1994)	stratocumulus	0.67±0.07 (polluted)
		0.80±0.07 (clean)
Gultepe et al. (1996)	stratocumulus	0.72 (using all cases)

Table 2: Mean ratio of predicted to observed r_e as a function of the ratio of ϕ liquid water content contained in large and small drops.

	Mean r_e(pred)/r_e(obs) for ϕ=$q_{L,l}/q_{L,s}$ in range:				
Parametrization	0.001-0.01	0.01-0.05	0.05-0.1	0.1-0.5	0.5-5.0
(a) Martin et al (1994)	1.02	0.98	0.95	0.89	0.62
(b) This study	1.00	1.00	1.00	0.98	1.00

Figure captions

Figure 1. Effect of α upon form of modified gamma function representation of the small droplet size distribution. All three spectra have $q_{L,s}$=0.5 g m^{-3} and $N_{0,s}$=100 cm^{-3} and thus $r_{vol,s}$=10.61 μm.

Figure 2. Graph showing relationship between k_s and α for the modified Gamma distribution.

Figure 3. Observed and modelled droplet size distributions from two clouds: A heavily drizzling case (top) and a polluted case with no cloud-base drizzle (bottom). The observations are represented by symbols (FSSP, filled circles; 2D-C filled triangles) and the dashed line curves are the modelled two-function distribution. The vertical dashed lines represent the observed values of the effective radius. For the heavy drizzling case the effective radius derived from the small ($r < 20\mu$m) drops $r_{e,s}$ is much smaller than that derived from the entire spectrum indicating that drizzle-sized droplets are important in determining the optical properties of the cloud. This is not the case in the polluted case where the effective radius derived from the drops is very similar to that derived from the entire spectrum.

Figure 4. Plot of k_s against $r_{vol,s}$ (left) and k_s against $N_{0,s}$ (right) for all in-cloud data showing that the volume radius of the small drops is a better predictor of the value of k_s.

Figure 5. Plot of $r_{vol,s}$, $N_{0,s}$ and k_s agianst height for a cloud deck observed over the North Sea on 3 December 1998. Notice how the value of k_s increases with height along with $r_{vol,s}$ whereas the droplet concentration remains fairly constant with height.

Figure 6. Plot of RMS difference between the observed and fitted k_s for polynomial orders from 1-4. The solid bar corresponds to the prediction of k_s using $r_{vol,s}$ and the open bar is for that using N_0. The horizontal dotted line is the RMS difference between the observed k_s and that fitted using the exponential form in Eqn. (18).

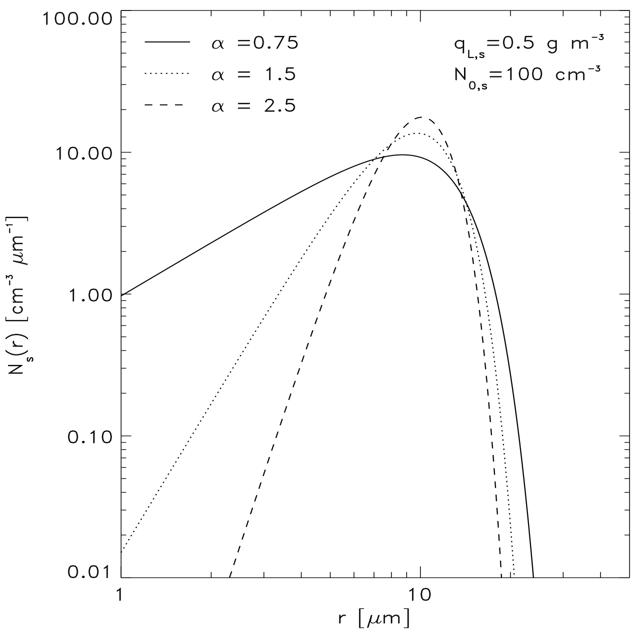
Figure 7. Left: Cumulative distribution of the ratio $N_{0,s}/(N_{0,s} + N_{0,l})$. More than 96% of values are larger than 0.99. Right: Distribution of the ratio $r_{vol,s}/r_{vol,l}$ from the entire dataset, where 94% of values are between 0.1 and 0.3.

Figure 8. Plot of k/k_s against $\phi = q_{L,l}/q_{L,s}$ for the entire dataset (filled circles). Overplotted are the curves using the two-distribution approximation to the size distribution (Eqn. 17) for values of $r_{vol,s}/r_{vol,l}$=0.1, 0.2 and 0.3. In addition, approximations have been made that $N_{0,s}/(N_{0,s} + N_{0,l})$=1 and that the value of k_s inside the square brackets in (17) is a constant equal to 0.7. Changing this value over the entire range of observed values makes little difference to the final curves obtained.

Figure 9. Observed and predicted values of k. The dotted line shows perfect agreement. The dashed lines indicated an error of 10%.

Figure 10. Ratio of the effective radius parametrized using the method presented in this study to that of Martin et al. (1994), plotted as a function of ϕ the ratio of the liquid water content in the large drops to that in the small drops. The different curves represent different values of k_s and $r_{vol,s}/r_{vol,l}$, designed to show the sensitivity of the parametrization to changes in these parameters. The parametrization suggested in this study ($r_{vol,s}/r_{vol,l}$=0.2) is denoted by the thickest lines.

Figure 11. Ratio of predicted to observed effective radius for entire spectrum plotted against ratio of liquid water in large droplets to that in small droplets. The prediction using the parametrization of Martin et al. (1994) (triangles) begins to underestimate the effective radius as $q_{L,l}/q_{L,s}$ rises above 0.05. The underprediction is as great as 50% in the highest drizzle cases. The prediction using the parametrization of Eqn. (17) (filled circles) predicts the effective radius very well even at hight drizzle water contents.



100.00
10.00
1.00
0.10
0.01
1
10
r [μm]
N_s(r) [cm⁻³ μm⁻¹]
α =0.75
α = 1.5
α = 2.5
q_L,s=0.5 g m⁻³
N_0,s=100 cm⁻³

Figure 1:

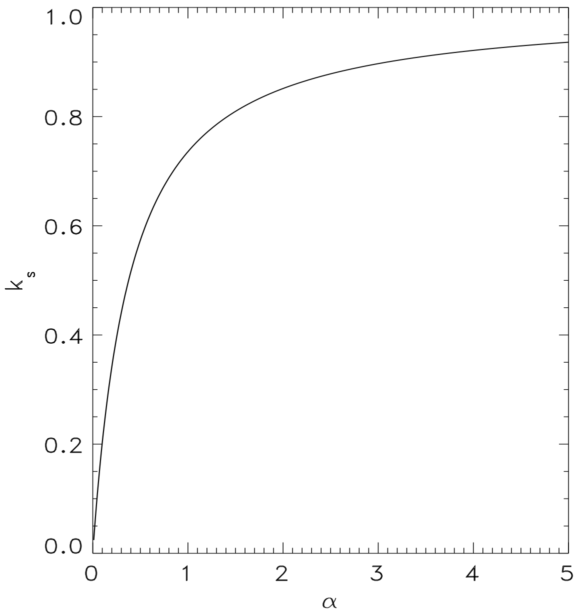


Figure 2:

Figure 3:

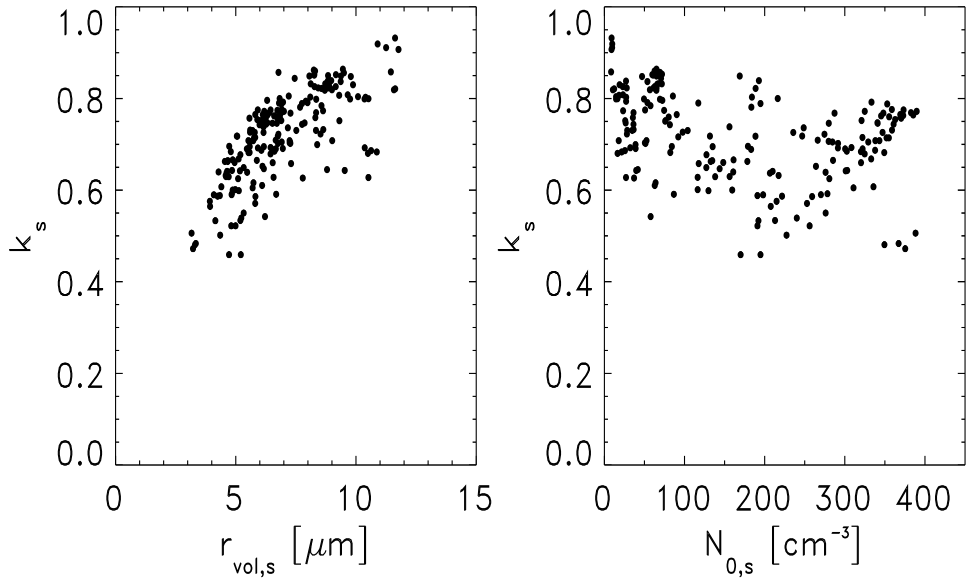
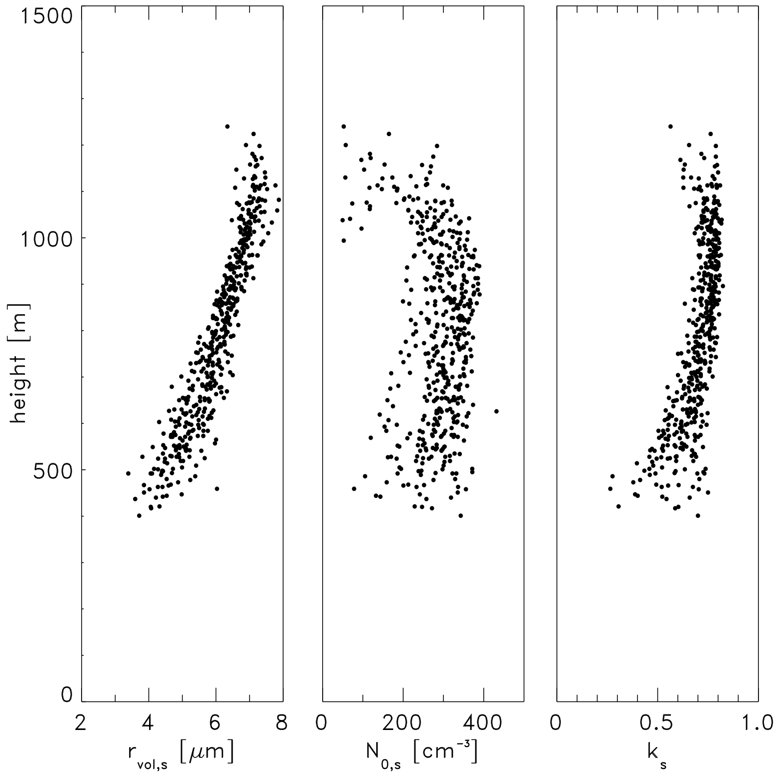

1.0
0.8
0.6
0.4
0.2
0.0
k_s
0
5
10
15
$r_{vol,s}$ [μm]
1.0
0.8
0.6
0.4
0.2
0.0
k_s
0
100
200
300
400
$N_{0,s}$ [cm^{-3}]

Figure 4:



1500
1000
500
0
height [m]
2
4
6
8
$r_{vol,s}$ [μm]
0
200
400
$N_{0,s}$ [cm^{-3}]
0
0.5
1.0
k_s

Figure 5:

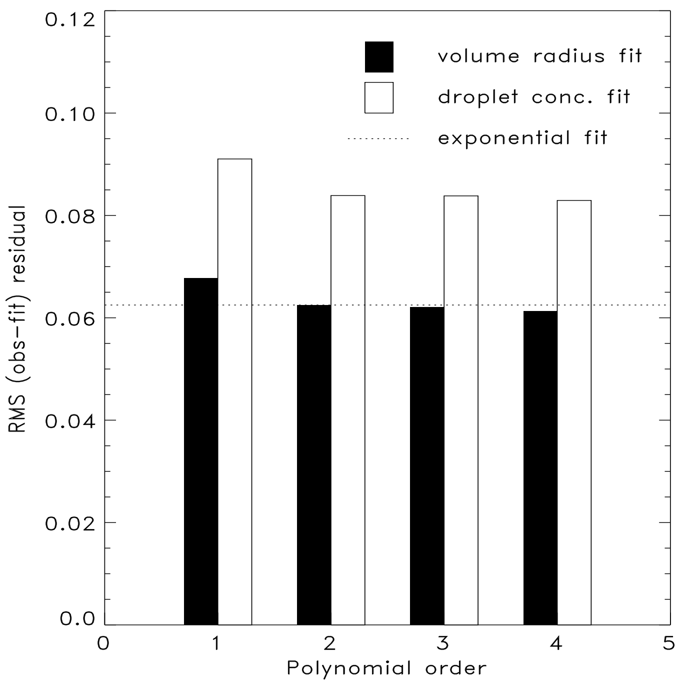

volume radius fit
droplet conc. fit
exponential fit
RMS (obs−fit) residual
0.12
0.10
0.08
0.06
0.04
0.02
0.0
0
1
2
3
4
5
Polynomial order

Figure 6:

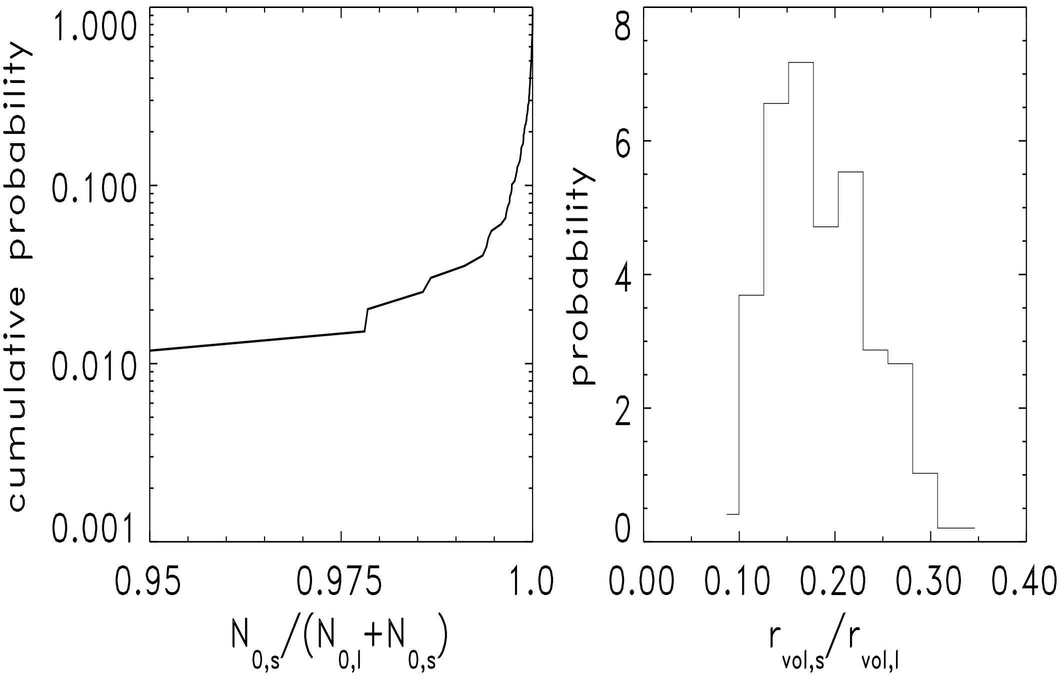
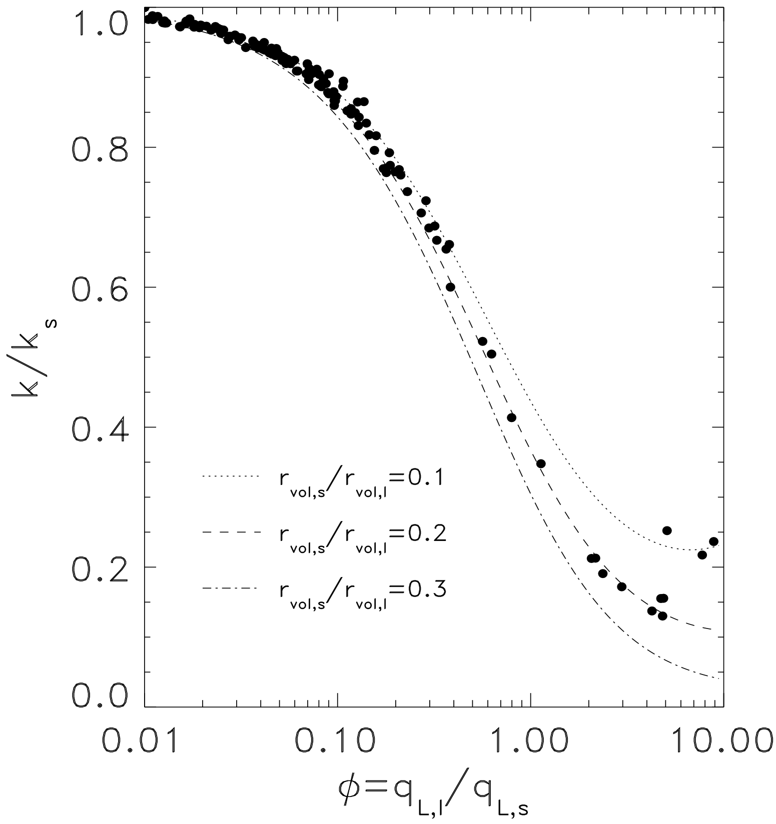

1.000
0.100
0.010
0.001
cumulative probability
0.95
0.975
1.0
N0,s/(N0,l+N0,s)
8
6
4
2
0
probability
0.00
0.10
0.20
0.30
0.40
rvol,s/rvol,l

Figure 7:



1.0
0.8
0.6
0.4
0.2
0.0
k/k_s
0.01
0.10
1.00
10.00
φ=q_L,l/q_L,s
r_vol,s/r_vol,l=0.1
r_vol,s/r_vol,l=0.2
r_vol,s/r_vol,l=0.3

Figure 8:

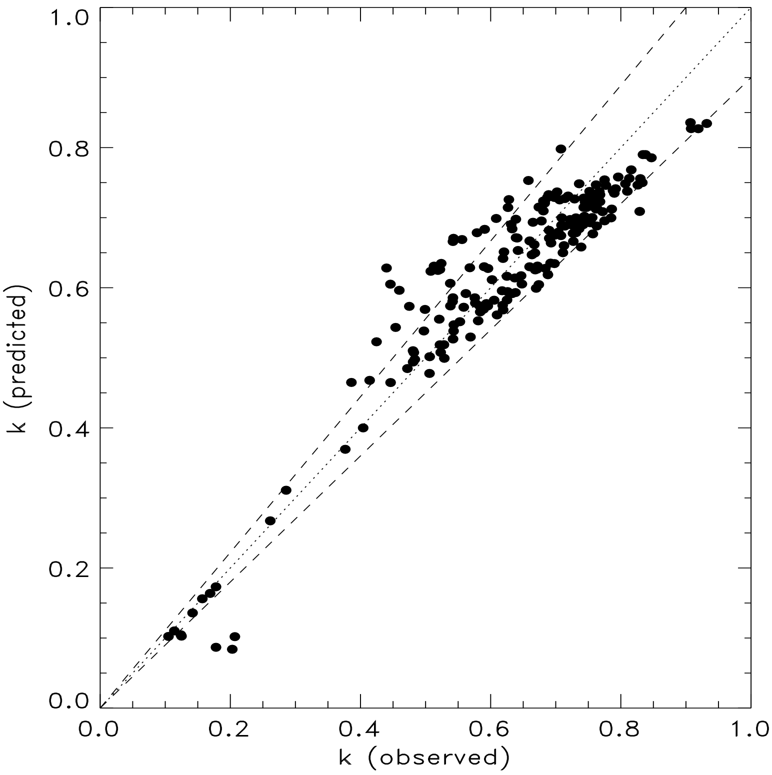
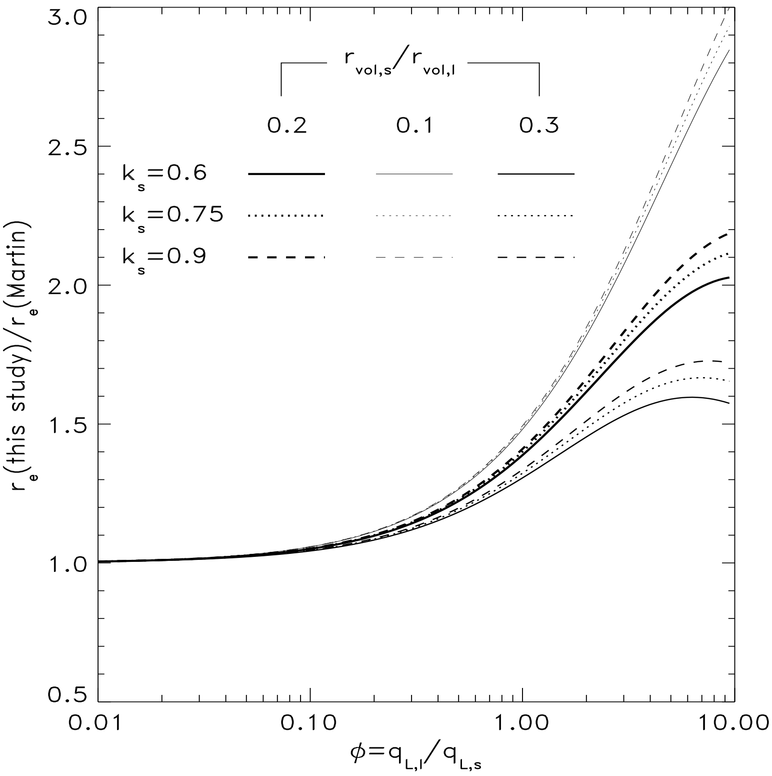
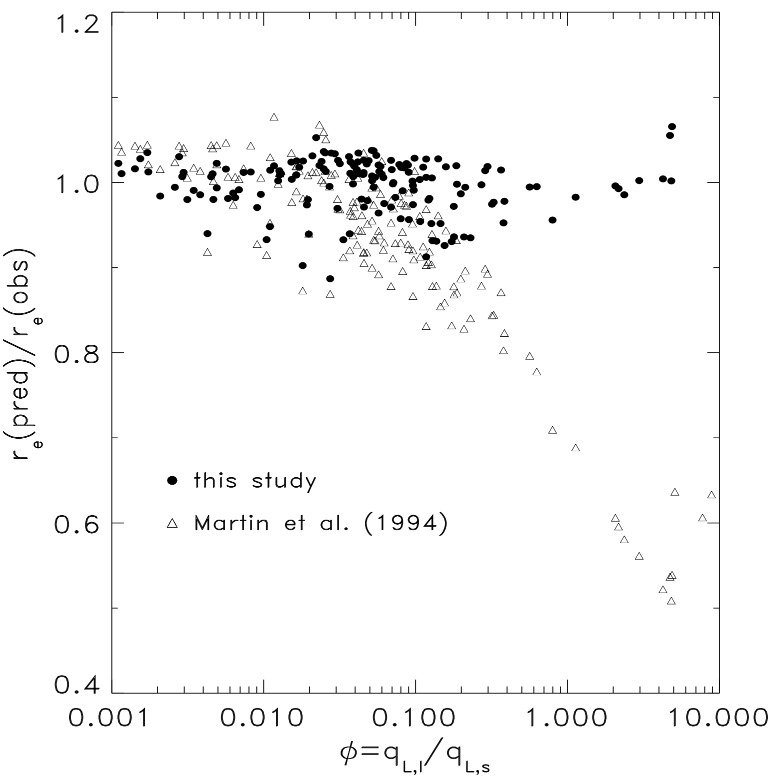


Figure 9:



$r_{vol,s}/r_{vol,l}$
0.2
0.1
0.3
k_s=0.6
k_s=0.75
k_s=0.9
3.0
2.5
2.0
1.5
1.0
0.5
r_e(this study)/r_e(Martin)
0.01
0.10
1.00
10.00
$\phi=q_{L,l}/q_{L,s}$

Figure 10:



1.2
1.0
0.8
0.6
0.4
0.001
0.010
0.100
1.000
10.000
r_e(pred)/r_e(obs)
$\phi=q_{L,l}/q_{L,s}$
this study
Martin et al. (1994)

Figure 11: