2.5

# 1. Evaporation of drizzle below cloud base

## *a. Sedimentation-evaporation model*

Because drizzle drops are small (precipitation rate and radar reflectivity are dominated by drops in the range $20 < r < 300\mu$m), their evaporation below cloud base is expected to change the reflectivity-rain rate relationship. We use a simple model of the sedimentation-evaporation process to simulate the evaporation of drizzle drops below cloud base. The model does not simulate the effects of coalescence or breakup, but these effects can be shown to be of little consequence below cloud for the sizes and concentrations of drizzle drops (Pruppacher and Klett 1997). In the drizzle drop size range ($r > 20\mu$m) the evaporation rate of a drop is linearly proportional to the degree of subsaturation and inversely proportional to its radius. The constant of proportionality is a fairly weak function of temperature and pressure. We can model profiles of the size distribution $N(r, z)$ of a population of sedimenting, evaporating drops via the equilibrium relationship

$$w_T(r)\frac{\partial N(r,z)}{\partial z} = -\frac{\partial}{\partial r}\left(N(r,z)\frac{Dr}{Dt}\right) \tag{1}$$

where $w_T(r)$ is the terminal velocity of a drop of radius $r$. We use an exponential distribution, truncated at at a lower radius of $r_0 = 20\mu$m, as the cloud base boundary condition

$$N(r, z_{CB}) = \frac{N_{exp}}{\overline{r_{exp}}}\exp\left(-r/\overline{r_{exp}}\right) \qquad (r > r_0) \tag{2}$$

where $N_{exp}$ is the total drop concentration and $\overline{r_{exp}}$ the mean radius of the complete (untruncated) exponential distribution. We then calculate $N(r, z)$ at any height below cloud. Run-averaged droplet size distributions in stratocumulus cloud have been shown to be well described using an exponential distribution (Wood 2002). We also assume here that the relative humidity is unity at cloud base and decreases at a rate consistent with a dry adiabatic temperature profile and constant specific humidity below cloud base. Radiosonde ascent profiles from the Ronald Brown support this assumption. We use a single power law terminal velocity relationship $w_T(r) = A_T r^{b_T}$ with $A_T = 5.98 \times 10^{-4}$ and $b_T = 1.53$ (for $r$ in microns and $w_T$ in m s$^{-1}$) that gives good accuracy over the drop radius range $20 < r < 300\mu$ m. This range encompasses the range of drizzle drop sizes generally observed in stratocumulus clouds. Finally, we make the assumption that the drops are falling below cloud in still air, i.e. there are no turbulent motions. Turbulent motions *are* responsible for the growth and development of the drizzle drop size distribution within cloud (Nicholls 1987; Baker 1993; Austin et al. 1995). However, no direct observational evidence exists as to how the sizes and concentrations of drizzle drops are correlated with vertical wind fluctuations in the cloud-base and subcloud regions.

Integrations of Eqn. 1 are obtained for the temperature range 270-290 K and for cloud base $r > 20\mu$m volume radii $r_{v,D(CB)}$ from 30-100 $\mu$m, which covers the range of observations. We then calculate the precipitation rate normalised with its cloud base value as a function of depth below cloud base. The rate of decrease in precipitation rate $R$ with height is lower when $r_{v,D(CB)}$ is larger because the larger drops require a longer distance to fall before evaporation, and that $r^3_{v,D(CB)}$ is a very good normalising variable for height. We plot $R/R(z_{CB})$ against $(z - z_{CB})/r^3_{v,D(CB)}$ for observations from 12 flights beneath drizzling stratocumulus (Wood 2002) and for the range of model conditions in Fig. 1. Although there is considerable spread in the observations, the modeled precipitation rate decrease below cloud base is in quite good agreement. The function

$$c(\chi) = R(z)/R(z_{CB}) = \exp\left\{-k_{es}\chi(z)\right\} \tag{3}$$

where $\chi(z) = [(z_{CB} - z)/r^2_{v,D(CB)}]^{1.5}$, provides a good parameterization of the model results. For the observed temperature and pressure range $k_{es} = 39.9\mu\text{m}^3\ \text{m}^{-1.5}$, and $R(z)/R(z_{CB})$ is shown in Fig. 1 by a dashed line. [1] The spread in observational values is a result of two main factors: (a) there is considerable spatial variability in the cloud base height; (b) turbulent mixing in the cloud-subcloud layer can alter the drizzle drop profiles. The latter was also noted by Nicholls (1987) who compared subcloud drop distributions with the results of a numerical model. Nicholls' model assumes still air beneath cloud base and results in drop concentrations decreasing faster with height below base than those

[1] The author notes that Eqn. 1 can be solved analytically by separation of variables but that the incorporation of the exponential distribution boundary condition at $z = z_{CB}$ would lead to unwieldy mathematical solutions.

measured. However, we find that the simple sedimentation-evaporation model can reproduce fairly well the observed decrease in precipitation rate below cloud base.

We can generalise the observational results to say that around 50% of the drizzle flux in the cases studied here evaporates within 60-100 m of cloud base and 80% evaporates within 150-250 m below cloud base. We expect this to be a general result for drizzling stratocumulus. The model also allows us to calculate changes to the relationship between the precipitation rate and the radar reflectivity $Z(z)$ as the drizzle evaporates. For the range of cloud base volume radii observed in drizzling stratocumulus we find from the model that

$$\frac{Z(z)}{Z(z_{CB})} \approx \left(\frac{R(z)}{R(z_{CB})}\right)^{0.75} \tag{4}$$

so that the reflectivity falls off more slowly with height than does the precipitation rate. This is expected because the dominant contributions to the reflectivity are from larger droplets than those that dominate the precipitation rate. Figure 2 demonstrates how the radar reflectivity-precipitation rate relationship will change due to evaporation below cloud. For a $Z-R$ relationship at cloud base that obeys a power law, i.e. $Z(z_{CB}) = AR(z_{CB})^B$, combination of Eqns. 3 and 4 leads to

$$Z(z) = c(\chi)^{0.75} Z(z_{CB}) = A\, c(\chi)^{0.75-B} R(z)^B. \tag{5}$$

It is then clear that the effect of evaporation leads to a $Z-R$ relationship with an unchanging exponent $B$ but, since $B > 0.75$ and $c(\chi) < 1$, a prefactor $A_{Z-R}(z) = A\, c(\chi)^{0.75-B}$ that increases with increasing distance (increasing evaporation) below cloud base.

### b. *Observational estimation of cloud base drizzle drop size distribution*

The combination of the cloud base radar reflectivity and its profile below cloud base can be used, together with the results of the sedimentation-evaporation model, to estimate the parameters of the drizzle drop size distribution at cloud base, assuming that it is a two-parameter exponential. Droplets smaller than the threshold radius of $r_0 = 20\mu$m contribute little to the precipitation rate, and so we make the additional assumption that $N(r, z_{CB}) = 0$ for $r < r_0$. Our method is as follows:

(1) We obtain $r_{v,D(CB)}, r > r_0$ by non-linear least-squares fitting the subcloud hourly-mean observed profile of $Z_z/Z_{CB}$ using Eqns. 3 and 4. The fitting procedure allows an estimation of the uncertainty in $r_{v,D(CB)}$ due to fitting uncertainty.

(2) The moments $M_n(r_0 < r < \infty)$ of order $n$ of a truncated exponential distribution are related, using an integral recurrence relation, to those of the complete distribution $M_n(0 < r < \infty)$ via

$$f(n, r_0/\overline{r_{exp}}) = \frac{M_n(r_0 < r < \infty)}{M_n(0 < r < \infty)} = \exp\left(-\frac{r_0}{\overline{r_{exp}}}\right) \sum_{i=0}^{n} \left(\frac{\overline{r_{exp}}}{r_0}\right)^i \frac{1}{i!} \tag{6}$$

where $r_0/\overline{r_{exp}}$ represents the degree with which the truncation will affect the moments. Large values of $r_0/\overline{r_{exp}}$ indicate that much of the distribution is contained in the smaller, truncated, sizes. The total drizzle liquid water content in the truncated exponential $q_{L,D}$ is related to that in the non-truncated distribution $q_{L,exp}$ through $q_{L,D} = f(3, r_0/\overline{r_{exp}}) q_{L,exp}$. A similar relation exists for the number concentrations, namely $N_{exp} = f(0, r_0/\overline{r_{exp}}) N_D$. The relations between drizzle drop concentrations, volume radii, and drizzle liquid water content for the truncated and untruncated distributions are

$$q_{L,D} = \frac{4\pi\rho_w}{3} N_D r_{v,D(CB)}^3 \tag{7}$$

$$q_{L,exp} = \frac{4\pi\rho_w}{3} N_{exp} r_{v,D(CB),exp}^3 \tag{8}$$

where $r_{v,D(CB),exp} = 6^{1/3}\overline{r_{exp}}$ is the volume radius of the drizzle drops for the untruncated distribution. Taking the ratio of Eqns. 7 and 8, and eliminating parameters pertaining to the untruncated distribution other than $\overline{r_{exp}}$, we obtain

$$6 f(3, r_0/\overline{r_{exp}}) \overline{r_{exp}}^3 = f(0, r_0/\overline{r_{exp}}) r_{v,D(CB)}^3. \tag{9}$$

Equation 9 must be solved to obtain $\overline{r_{exp}}$ given the best-fit value of $r_{v,D(CB)}$. This is carried out using the bisection method since $\overline{r_{exp}}$ is a monotonically increasing function of $r_{v,D(CB)}$ and therefore Eqn. 9 has a single root.
Because $f(6, r_0/\overline{r_{exp}}) > 0.97$ for all $r_0/\overline{r_{exp}} < 2$ (the observed range), the drizzle drop concentration $N_{exp}$ for the equivalent untruncated distribution is found from the observed cloud base reflectivity

$$N_{exp} = \frac{Z(z_{CB})}{2^6\, 6!\, \overline{r_{exp}}^6} \tag{10}$$

Once $\overline{r_{exp}}$ and $N_{exp}$ are found, it is a simple matter to obtain any of the moments of the truncated exponential distribution through Eqn. 6, since

$$M_n(0 < r < \infty) = n! N_{exp} \overline{r_{exp}}^n. \tag{11}$$

Precipitation rate $R$ is obtained using

$$R = \frac{4\pi\rho_w}{3} A_T M_d(r_0 < r < \infty) \tag{12}$$

where the drop terminal speed $w_T(r)$ over the range $r_0 < r < \infty$ is $w_T(r) = A_T r^d$, with $d = 1.52$ and $A_T = 5.67 \times 10^5$ m$^{-0.52}$ s$^{-1}$, which represents the best fit for the range $20 < r < 400\mu$m to the terminal velocity relations given in Pruppacher and Klett (1997). Tests indicate that using a linear $r - w_T(r)$ relation does not appreciably affect the results.

We obtain the cloud base precipitation information on hourly mean profiles of reflectivity, which are shown, following the fitting procedure to obtain $r_{v,D(CB)}$, using the normalized height $z_{CB} - z)/r^2_{v,D(CB)}$ in Fig. 3. Figure 4 shows time series of cloud base drizzle drop concentration, drizzle volume radius, and precipitation rate measured using the millimeter cloud radar (MMCR) on the NOAA Ron Brown during the EPIC 6-day observational period (DOY 289-295, 2001) at the IMET buoy (85W, 20S). Figure 5 shows correlations between drizzle drop concentration and precipitaton rate and between drizzle volume radius and precipitation rate. The squared correlation coefficients $r^2$ between the logarithms of these variables are 0.57 ($N_D, R$) and 0.25 ($r_{v,D}, R$) respectively, indicating that precipitation rate fluctuations are controlled more by fluctuations in drizzle drop concentration than by fluctuations in the volume radius. Figure 5 also shows that there is little correlation between $r_{v,D}$ and $N_D$. This results in the low values of the exponent $B$ in the $Z - R$ power law at cloud base:

$$Z_{CB} = A R_{CB}^B \tag{13}$$

For exponential distributions of cloud droplets

$$Z = 2^6\, 6! N_{exp} \overline{r_{exp}}^6 \tag{14}$$

$$R = \frac{4\pi\rho_w \Gamma(4+d)}{3} N_{exp} \overline{r_{exp}}^{3+d} \tag{15}$$

it is possible to show that, provided that there is no correlation between $N_{exp}$ and $\overline{r_{exp}}$, the exponent $B$ in Eqn. 13 is given by

$$B = \left\{ \frac{1 + 6^2 \gamma_{N_{exp}} / \gamma_{\overline{r_{exp}}}}{1 + (3+d)^2 \gamma_{N_{exp}} / \gamma_{\overline{r_{exp}}}} \right\}^{\frac{1}{2}} \tag{16}$$

where $\gamma_x = (\overline{x}/\sigma_x)^2$. Thus, the exponent is determined by the relative amount of variance in the drizzle drop number concentration and the mean radius. The two limiting cases are $B = 1$ (precipitation rate variations are entirely modulated by variations in the number concentration) and $B = 6/(3+d) = 4/3$ (precipitation rate variations are entirely modulated by variations in the mean radius). Equation 16 couches the $Z - R$ exponent as a physically significant parameter that is related to whether changes in drizzle droplet number concentration or changes in the size of the drops have most influence over changes in the precipitation rate.

For truncated distributions, Eqns. 15 for $Z$ and $R$ are modified through Eqn. 6, and no simple dependence upon the product of $N_{exp}$ and constant powers of $\overline{r_{exp}}$ can be written. However, for the observations,

$$Z = \alpha N_D r_{v,D}^{b_1} \tag{17}$$

$$R = \beta N_D r_{v,D}^{b_2} \tag{18}$$

with $b_1 = 7.5$ (instead of 6) and $b_2 = 5.2$ (instead of 4.52) (see Fig. 5). The equation for the exponent is then

$$B = \left\{ \frac{1 + b_1^2 \gamma_{N_D} / \gamma_{r_{v,D}}}{1 + b_2^2 \gamma_{N_D} / \gamma_{r_{v,D}}} \right\}^{\frac{1}{2}} \tag{19}$$

For the observations, we find that the ratio $\gamma_{N_D}/\gamma(r_{v,D}) = 0.018$, and therefore an estimated value of the exponent $B = 1.16$.

*c. Potential sources of error in the derived drizzle parameters*

Two categories of uncertainty lead to errors in the derived drizzle parameters. First, there are a number of issues related to the calibration and possible saturation of the cloud radar that must be addressed [****see earlier section****]. The second source of error arises from simplifying assumptions made in the method of estimation of drizzle parameters from the profile of reflectivity.

Calibration errors of 2 and 4 dBZ in radar reflectivity lead to rain rate errors of roughly 50 and 250% in derived precipitation rate.

An important assumption made in the model is that the vertical air motions have negligible effect upon the profiles of precipitation rate and reflectivity, and their dependence upon the volume radius of the drizzle drops at cloud base. It is well understood (e.g. Nicholls 1987) that the terminal velocity of drizzle drops is comparable to the vertical ascent/descent rates for turbulent eddies, and that this motion has an important effect upon the production of drizzle by recirculating small drizzle drops within the cloud. It is less certain how this may impact upon the precipitation rate profiles below cloud where many of the smaller drops have been evaporated. Because our analysis method uses hourly mean profiles, the mean vertical motion over this period will be negligible because the typical duration of a 1000 m scale eddy as it passes over the ship is $\sim 2$ minutes. However, a non-zero correlation between $r_{v,D(CB)}$ and vertical wind would alter the profile (Eqn. 3), and therefore the derived precipitation rates. Vali et al. (1998) presents the only observations published to date of correlations between radar reflectivity and vertical wind from three cases of marine stratus/stratocumulus. In the subcloud region these correlations are of the order -0.25. The correlations are negative because in the subcloud layer there is a positive gradient of reflectivity with height and a downward turbulent transport of drizzle drops from the more reflective cloud layer. This correlation results in profiles of precipitation rate that decrease less rapidly with height (for a given mean $r_{v,D(CB)}$) than profiles with no correlation. Therefore, using a still-air value of $k_{es}$ to derive the cloud base volume radius via Eqn. 3 results in an overestimate of $r_{v,D(CB)}$ if negative correlations between reflectivity and vertical wind are neglected. We simulate the effect of such a negative correlation between refectivity and vertical wind using an ensemble of 100 realizations of the sedimentation-evaporation model with a range of updrafts and downdrafts. We use a constant vertical wind standard deviation $\sigma_w = 0.6$ m s$^{-1}$, representative of turbulence in relatively deep cloud-topped marine boundary layers. Observations from the MMCR suggest that in the sub-hour timescale, the standard deviation of the radar reflectivity is approximately 5-10 dBZ. We initialize the model size distributions to give an ensemble of parcels with these statistical properties. We also assume that the updrafts/downdrafts are coherent through the depth of the subcloud layer. Using these ensembles, we find that the approximate overestimate of $r_{v,D(CB)}$ depends upon the magnitude of $r_{v,D(CB)}$, decreasing as $r_{v,D(CB)}$ increases, with errors of 30, 15 and 5% for $r_{v,D(CB)}$=40, 55 and 75 $\mu$m respectively. These overestimates in $r_{v,D(CB)}$ propagate to overestimates in cloud base precipitation rate of 67, 35 and 12% respectively. These errors, while non-negligible, are somewhat lower than the errors caused by uncertainty in the MMCR calibration.

# References


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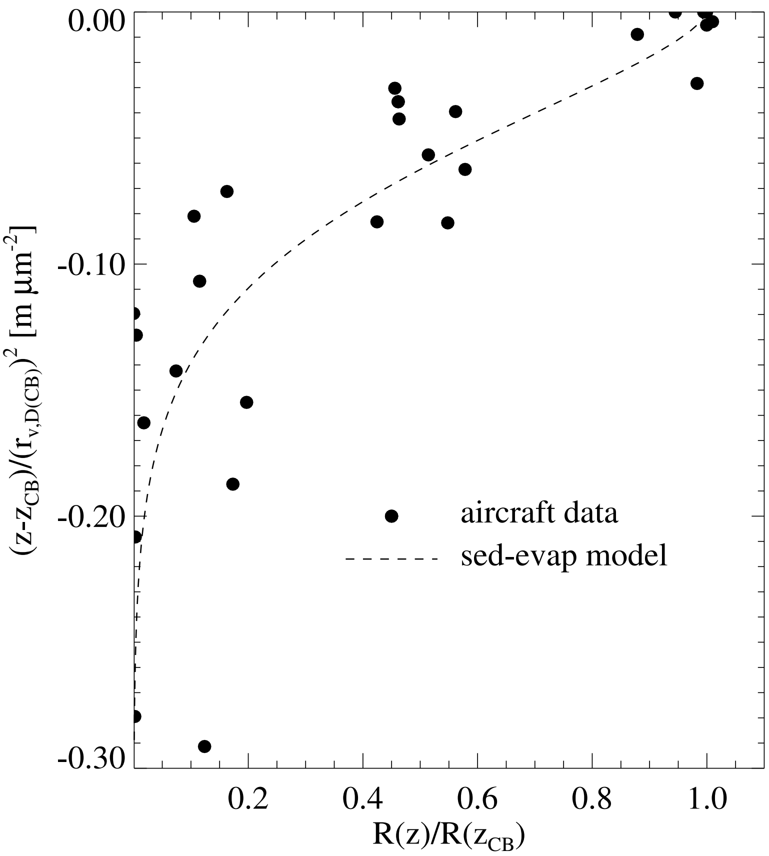


Figure 1: Sub-cloud precipitation rates normalised with cloud base values plotted as a function of height below cloud base normalised with the $r > 20\mu$m mean volume radius of the size distribution at cloud base. The dashed line shows the parameterization of Eqn. 3 based upon the sedimentation evaporation model which provides a reasonable fit to the aircraft observations.

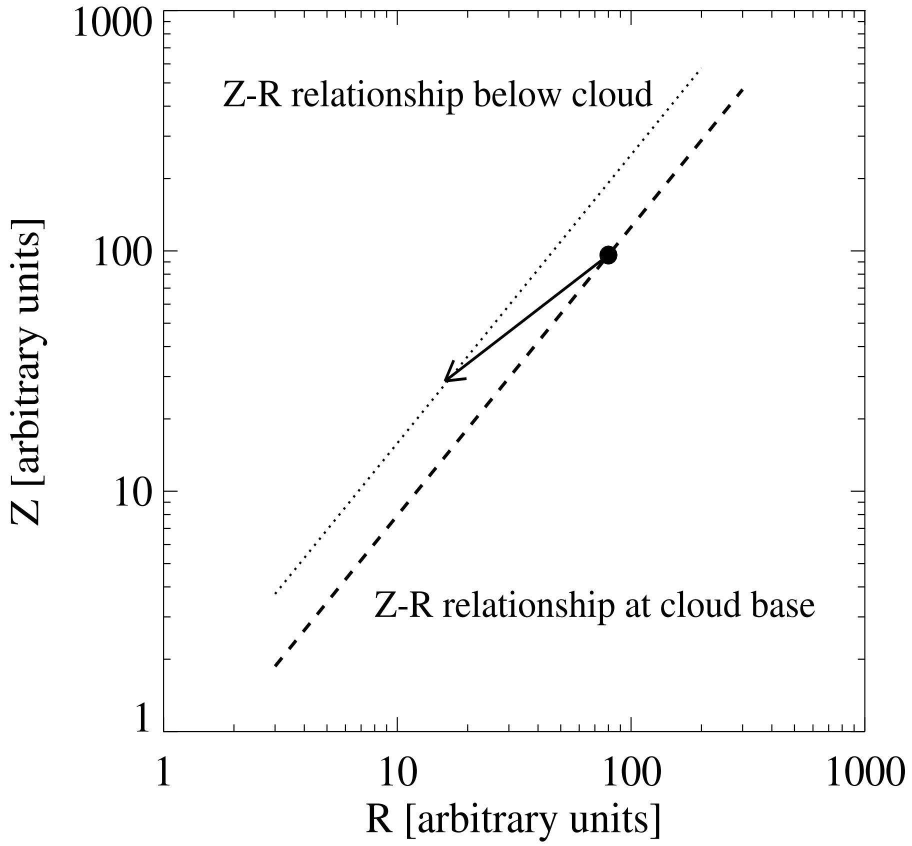


Figure 2: Schematic showing how evaporation changes the reflectivity-precipitation rate relationship. Aircraft observations in cloud show a relationship of the form $Z = aR^b$ with $b \approx 1.2$ (dashed line). From Eqn. 4, we can see that the reflectivity decreases sublinearly with the precipitation rate decrease as shown by the arrow, for an arbitrary size distribution. This results in a reflectivity-precipitation rate relationship below cloud that has an unchanging exponent $b$ but a constant $a$ that increases with depth below cloud base (dotted line).

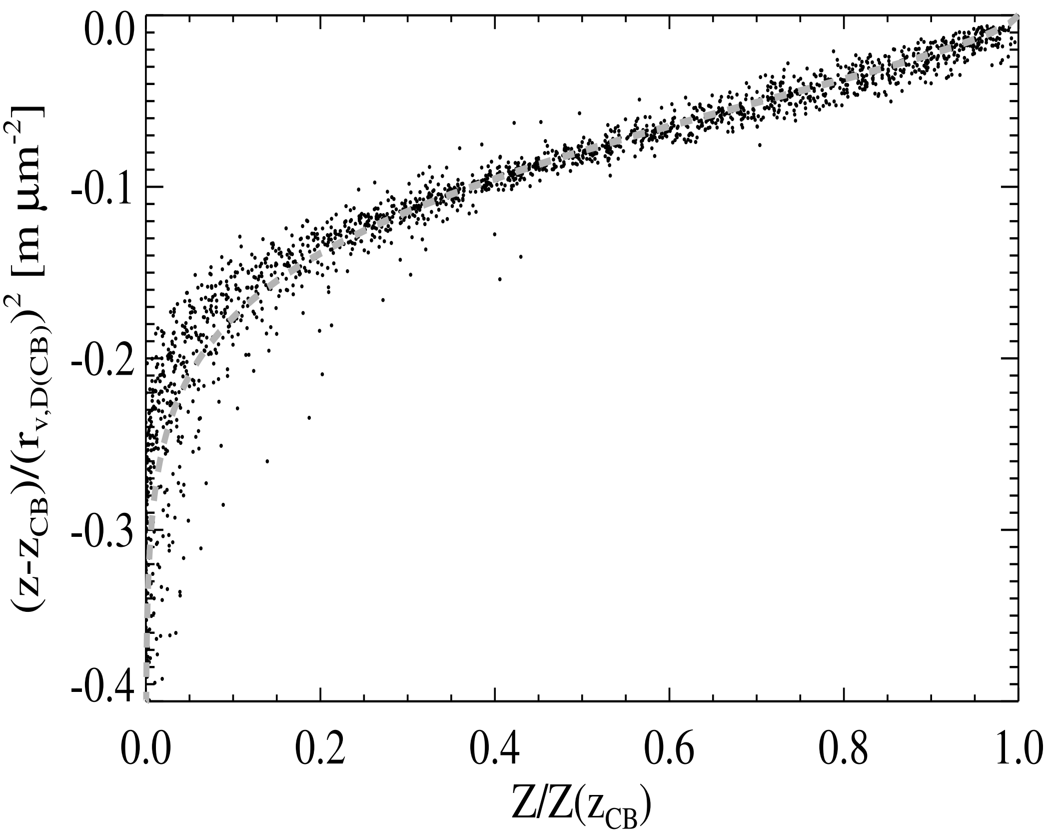


Figure 3: Profiles of radar reflectivity $Z$ below cloud normalized with the maximum value $Z_{CB}$ against the normalized height variable $(z_{CB} - z)/r^2_{v,D(CB)}$ from hourly MMCR data (solid circles, 144 profiles). The light gray line shows the parameterization using Eqns. 4 and 3.

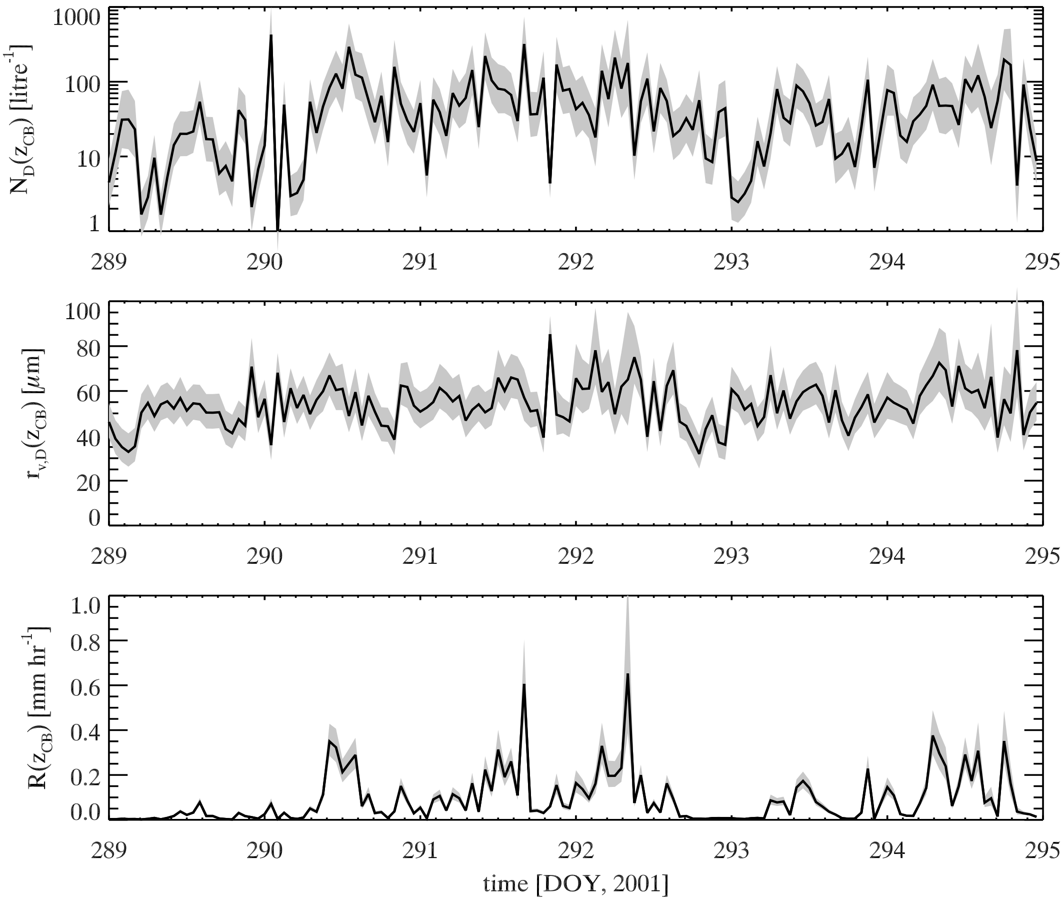


Figure 4: Time series of observationally estimated cloud base drizzle drop concentration $N_D(z_{CB})$, volume radius $r_{v,D}(z_{CB})$, and precipitation rate $R(z_{CB})$ estimated using the MMCR data on the NOAA Ron Brown during the EPIC period at the IMET buoy.

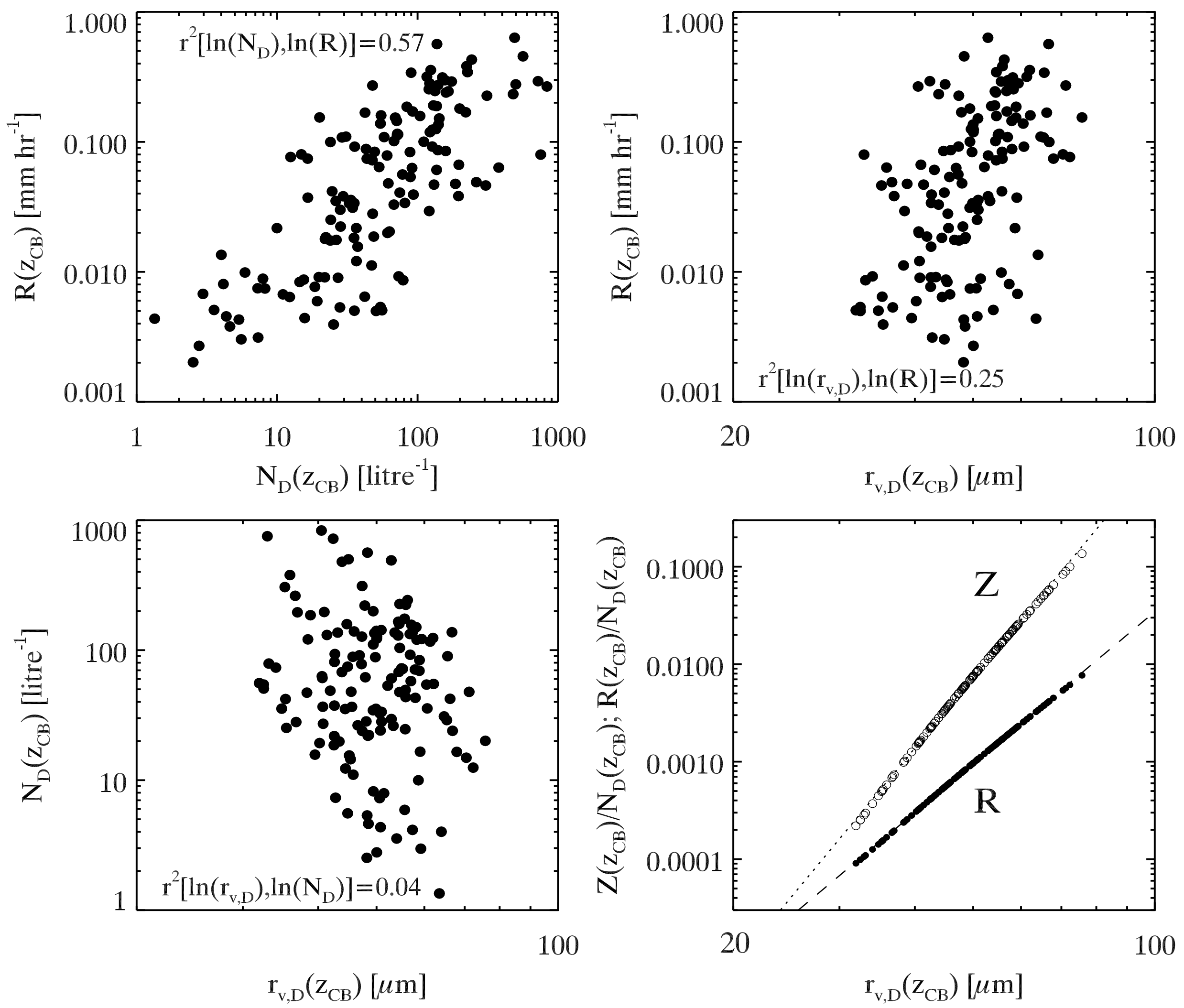


Figure 5: Correlations between drizzle drop concentration and precipitation rate (upper left), between drizzle drop volume radius and precipitation rate (upper right), and between drizzle drop volume radius and drop concentration (lower left). Also shown are the relationships between volume radius and cloud base reflectivity and rain rate normalized with drizzle drop concentration.