

Comments on: “On the parameterization of the autoconversion process.

Part I: Analytical formulation of the Kessler-type parameterizations”

ROBERT WOOD* and PETER N. BLOSSEY

University of Washington, Seattle, Washington

October 6, 2004

* *Corresponding author address:* Dr. Robert Wood, Atmospheric Sciences, University of Washington, Seattle, WA; *e-mail:* robwood@atmos.washington.edu.

In a recent paper Liu and Daum (2004) (henceforth LD04) derive analytical expressions for the rate of autoconversion of cloud droplets to embryonic raindrops for a range of assumptions about the dependence of the collection kernel upon cloud droplet size. Their analytical derivations are used to provide a physical formalism for some of the autoconversion parameterizations that are widely used in a broad range of numerical models (e.g. Kessler 1969; Tripoli and Cotton 1980; Baker 1993; Boucher et al. 1995).

The basis for LD04 is the integral (their Equation 3), here written in mass terms

$$P = \int_0^\infty \left\{ \int_0^\infty K(x, x') x' n(x') dx' \right\} n(x) dx, \quad (1)$$

where P is defined as the autoconversion rate in LD04, x and x' are the masses of two coalescing droplets, $n(x)$ is the number of droplets in the mass range x to $x + dx$, and $K(x, x')$ is the collection kernel for the coalescing droplets.

Most bulk precipitation schemes used in numerical models partition liquid water into separate cloud and rain classes using a threshold particle radius r_0 , or equivalently mass $x_0 = (4\pi\rho_w/3)r_0^3$, to distinguish the classes. The choice of r_0 is taken to be 20 μm in this paper. The autoconversion rate is then strictly defined as the rate at which liquid water is transferred from the cloud class to the rain class by coalescence of two cloud droplets. We term this the *cloud-to-rain autoconversion rate* (denoted here as A).

Autoconversion results from the collision of two cloud droplets with masses x, x' (each less than x_0) which form a rain droplet of mass $x + x' > x_0$. Therefore, the autoconversion may be

computed as:

$$A = \frac{1}{2} \underbrace{\int_0^{x_0} \int_0^{x_0}}_{x+x' > x_0} (x + x') K(x, x') n(x) n(x') dx dx' \quad (2)$$

where the factor of $1/2$ accounts for the double counting of the mass involved in collisions of x and x' . Since the kernel is symmetric, i.e. $K(x, x') = K(x', x)$, the contributions to the integral from the masses x and x' are identical, and the integral may be re-written in terms of x' (and without the factor of $1/2$) as

$$A = \int_0^{x_0} \left\{ \int_{x_0-x}^{x_0} K(x, x') x' n(x') dx' \right\} n(x) dx. \quad (3)$$

Note that the limits of integration in the inner integral reflect the constraint that $x + x' > x_0$. Equation (3) gives the true cloud-to-rain autoconversion and may also be derived from the stochastic collection equation, although that derivation is omitted here for brevity.

Equations (1) and (3) are identical apart from the limits of integration, which result in (3) being more difficult to solve in an analytic sense because the lower limit of the integral over x' is a function of x . Because of the different limits of integration (see figure 1), P and A have very different physical interpretations. Equation (1) represents the total rate of coalescence across all sizes, i.e. the total amount of droplet mass involved in coalescence in one second. This includes the coalescence of cloud droplets that result in a droplet smaller than r_0 . In contrast, (3) is rate of coalescence only of droplets that cross the x_0 threshold. Clearly, $A \leq P$ and represents the true autoconversion rate required by bulk parameterization schemes. While the use of threshold criteria for autoconversion and the strong weighting of the collection kernel $K(x, x')$ toward larger x and x' mimic to some degree the changes in the limits of integration from (1) to (3),

the integral in equation (1) still over-predicts the true cloud-to-rain autoconversion substantially under realistic conditions, as is demonstrated below.

If we set the upper limits of integration to x_0 for both integrals in (1), we interpret the LD04 equation as being the total rate of mass coalescence of cloud ($r < r_0$) droplets, a rate we label as P_{x_0} . We ask how different the values of A and P_{x_0} are for physically realistic cloud droplet size distributions and collection kernel $K(x, x')$. To achieve this we first use the collision kernel data of Hall (1980) and Stokes flow terminal velocity from Pruppacher and Klett (1997), assuming a coalescence efficiency of unity, to define $K(x, x')$. Our observational dataset comprises data from 12 flights of the UK Met Office C-130 in marine boundary layer (MBL) clouds (see Wood 2004a, for a complete description). Droplet size distributions ($r < 20 \mu\text{m}$) are measured at a number of levels in each cloud. The cloud liquid water contents and droplet concentrations span the range commonly observed in stratiform MBL clouds.

For each of the size distributions in cloud, we derive P_{x_0} and A as described above. The integrals (1) and (3) are evaluated using the trapezoidal rule over logarithmically-equispaced radius/mass bins. The results are shown in figure 2 and are in general agreement with the comparisons in Wood (2004b), where a time-dependent SCE solver was used to determine the cloud-to-rain autoconversion rates for the observed size distributions. We find that the 10th and 90th percentiles of P_{x_0}/A are 3.8 and 112 with the median value being 9.3. Inclusion only of size distributions that satisfy the critical radius threshold criterion of Liu et al. (2004) leads to a median value of P_{x_0}/A equal to 7.3 with 10th and 90th percentiles of 3.2 and 15.7. Thus, the LD04 autoconversion rate P results in an over-prediction of the cloud-to-rain autoconversion

rate by almost an order of magnitude even if the LD04 parameterization for the kernel $K(x, x')$ is perfect. Figure 2(b) shows P_{x_0}/A against the mean volume radius r_v . The dashed line indicates the results for a modified gamma distribution of cloud droplets (e.g. Austin et al. 1995) with the spectral width parameterized using Wood (2000). Only for spectra with r_v close to r_0 does the LD04 integral (1) approximate the cloud-to-rain autoconversion rate.

One could argue, based upon the kinetic potential of nucleation theory developed by McGraw and Liu (2003) that the autoconversion rate should represent the mass transfer through coalescence across a threshold radius that has physical significance, rather than one that is somewhat arbitrarily chosen to represent the size at which the importance of coalescence exceeds that of condensational growth. However, because the physically-based threshold radius is a complex function of cloud turbulence and thermodynamics (McGraw and Liu 2003), it is not possible to examine this with the data we have available. However, we note that the physically-based threshold radius is almost always in the range $20 < r < 30 \mu\text{m}$, and tests show that values of r_0 larger than $20 \mu\text{m}$ result in even poorer agreement between P_{x_0} and A .

To conclude, we have shown that the integral used in LD04 to determine the autoconversion rate does not give the cloud-to-rain autoconversion rate A , i.e. the rate of transfer of mass across a particular radius threshold by coalescence. Instead it gives the total rate of mass coalescence among cloud droplets and is typically much larger than A . This result therefore suggests that while LD04 provides a solid theoretical basis for the Kessler-type parameterizations, this class of parameterization does not predict the rate of conversion of mass from cloud droplets to embryonic raindrops as is commonly required by bulk precipitation schemes in numerical

models.

1. Acknowledgments

The authors wish to thank the staff of the Meteorological Research Flight and the C-130 aircrew and ground crew for their dedication in collecting the data presented in this study. We would also like to thank Chris Bretherton for helpful discussions.

References

- Austin, P., Y. Wang, R. Pincus, and V. Kujala: 1995, Precipitation in stratocumulus clouds: observations and modelling results. *J. Atmos. Sci.*, **52**, 2329–2352.
- Baker, M. B.: 1993, Variability in concentrations of cloud condensation nuclei in the marine cloud-topped boundary layer. *Tellus*, **45B**, 458–472.
- Berry, E. X.: 1967, Cloud droplet growth by collection. *J. Atmos. Sci.*, **24**, 688–701.
- Boucher, O., H. LeTreut, and M. B. Baker: 1995, Precipitation and radiation modeling in a general circulation model: Introduction of cloud microphysical processes. *J. Geophys. Res.*, **100**, 16395–16414.
- Hall, W. D.: 1980, A detailed microphysical model within a two-dimensional dynamic framework: Model description and preliminary results. *J. Atmos. Sci.*, **37**, 2486–2507.

- Kessler, E.: 1969, On the distribution and continuity of water substance in atmospheric circulation. *Meteorol. Monogr.*, **10**, 84pp.
- Liu, Y. and P. H. Daum: 2004, On the parameterization of the autoconversion process. part i: Analytical formulation of the kessler-type parameterizations. *J. Atmos. Sci.*, **61**, 1539–1548.
- Liu, Y., P. H. Daum, and R. McGraw: 2004, An analytical expression for predicting the critical radius in the autoconversion parameterization. *Geophys. Res. Lett.*, **31**, L06121, 10.1029/2003GL019117.
- McGraw, R. and Y. Liu: 2003, Kinetic potential and barrier crossing: A model for warm cloud drizzle formation. *Phys. Rev. Lett.*, **90**, 018501–1–4.
- Pruppacher, H. R. and J. D. Klett: 1997, *Microphysics of clouds and precipitation*. Kluwer Academic Publishers, 976 pp.
- Tripoli, G. J. and W. R. Cotton: 1980, A numerical investigation of several factors contributing to the observed variable density of deep convection over South Florida. *J. App. Meteorol.*, **19**, 1037–1063.
- Wood, R.: 2000, Parametrization of the effect of drizzle upon the droplet effective radius in stratocumulus clouds. *Quart. J. Roy. Meteorol. Soc.*, **126**, 3309–3324.
- 2004a, Drizzle in stratocumulus: Part I. Vertical and horizontal structure. *J. Atmos. Sci.*, submitted.
- 2004b, Drizzle in stratocumulus: Part II. microphysical aspects. *J. Atmos. Sci.*, submitted.

Figure 1: The left and right panels of this diagram depict the combinations of cloud droplets whose merging will yield a droplet whose mass (at left) or radius (at right) exceeds the threshold x_0 or r_0 , respectively. The shaded region in the left panel, $x + x' > x_0$ and $x, x' < x_0$, corresponds to the domain of integration in equation (3). In the right panel, the shaded region corresponds to $r^3 + r'^3 > r_0^3$ and $r, r' < r_0$.

Figure 2:(a) P_{x_0} against A for spectra from aircraft flights. Open circles represent the subset of spectra which do not reach the sixth moment critical radius criterion for autoconversion in LD04; (b) The ratio P_{x_0}/A against mean volume radius r_v for observations (circles, with open circles same as (a)). Dashed line shows P_{x_0}/A obtained assuming a modified gamma droplet size distribution (see text).

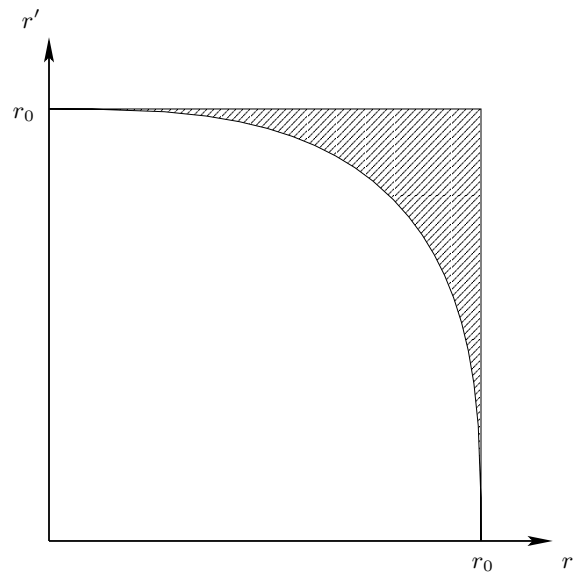
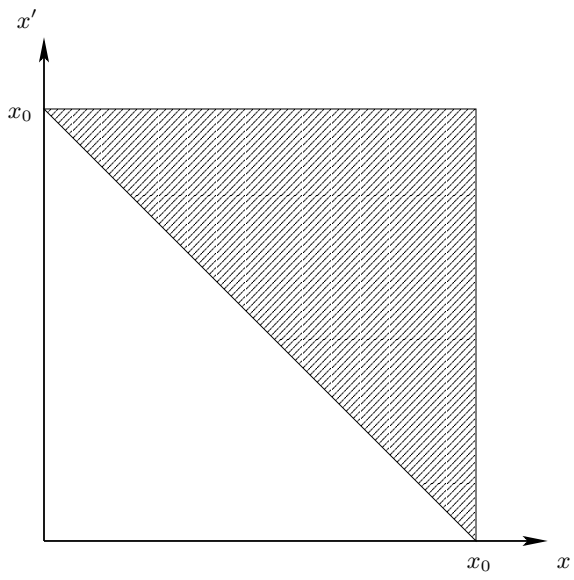


Figure 1:

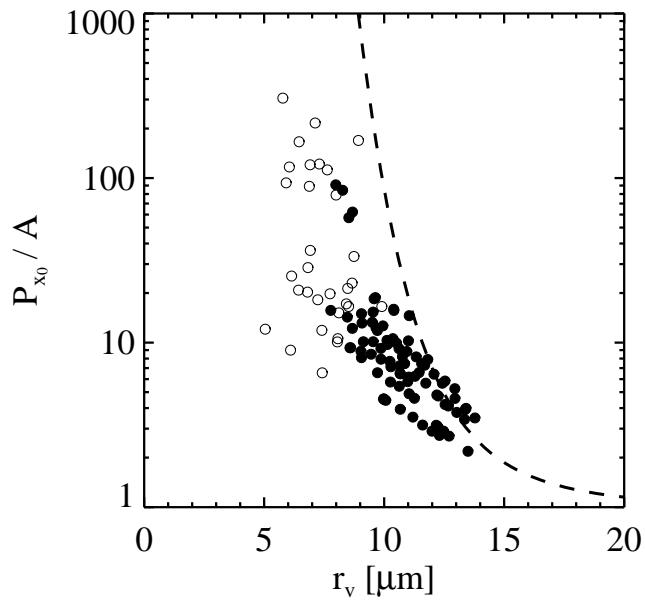
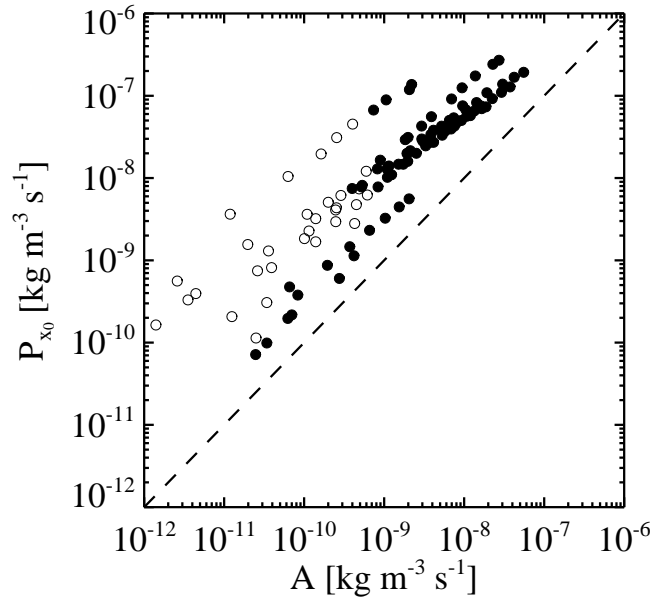


Figure 2: