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Apparent optical properties of spherical particles in absorbing medium

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Abstract

An apparent absorption efficiency for spherical particles in absorbing medium is introduced to take into account the non-exponential absorption of the near-field scattered radiation in the absorbing medium. The apparent extinction, which is the summation of the apparent scattering efficiency following previous studies and the apparent absorption efficiency, is the same as the actual extinction. These apparent optical properties are suited to radiative transfer equations.

Keywords: Light scattering in absorbing medium; Apparent optical properties; Radiative transfer

1. Introduction

The analytic expressions have recently been developed for the single-scattering properties of a spherical particle embedded in an absorbing medium [1–3], which include absorption (Q_a), scattering (Q_s), and extinction (Q_e) efficiencies along with the scattering phase function and the asymmetry factor [2]. Since the application of radiative transfer equations requires the scattering properties of the particles in the far field [4], Yang et al. [5] defined an apparent scattering efficiency ($\widehat{Q}_s$) using the far-field scattered radiation rescaled exponentially back to the particle surface [6]. The apparent extinction efficiency was then defined as the summation of the absorption efficiency [1–3] and the apparent scattering efficiency. In this note, we suggest that an apparent absorption efficiency ($\widehat{Q}_a$) needs to be introduced to take into account the non-exponential absorption of the near-field scattered radiation in the absorbing medium. Our apparent extinction efficiency, $\widehat{Q}_e$, which is $\widehat{Q}_s + \widehat{Q}_a$, is the same as the actual extinction efficiency, Q_e. Note that the extinction efficiency as well as the extinction cross-section in an absorbing medium does not depend on the reference plane or the particle location within the medium [7]. In Section 2, we briefly summarize the absorption, scattering, and extinction efficiencies from [2] of a spherical particle in an absorbing medium. We present the apparent optical properties in Section 3 along with some numerical results and discussions.

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2. Absorption, scattering, and extinction efficiencies

Following [2], the rates of energy absorbed (W_{a}) and scattered (W_{s}) by a spherical particle of radius a in an absorbing medium are

$$W_{\mathrm{a}} = \frac{\pi |E_0|^2}{\omega \mu_{\mathrm{t}}} \sum_{n=1}^{\infty} (2n+1) \mathrm{Im}(A_n), \tag{1a}$$

$$W_{\mathrm{s}} = \frac{\pi |E_0|^2}{\omega \mu} \sum_{n=1}^{\infty} (2n+1) \mathrm{Im}(B_n), \tag{1b}$$

where E_0 is the amplitude of the incident wave at the center of the sphere; ω the angular frequency; μ and μ_{t} the permeabilities of the host medium and the scatterer, respectively; Im( ) represents the imaginary part of the argument; and

$$A_n = \frac{|c_n|^2 \psi_n(\beta) \psi_n'^*(\beta) - |d_n|^2 \psi_n'(\beta) \psi_n^*(\beta)}{k_{\mathrm{t}}}, \tag{2a}$$

$$B_n = \frac{|a_n|^2 \xi_n'(\alpha) \xi_n^*(\alpha) - |b_n|^2 \xi_n(\alpha) \xi_n'^*(\alpha)}{k}. \tag{2b}$$

Here, $k = 2\pi m/\lambda_0$ and $k_{\mathrm{t}} = 2\pi m_{\mathrm{t}}/\lambda_0$ with λ_0 the wavelength in vacuum, and m and m_{t} the refractive indices of the host medium and the scatterer, respectively; $\alpha = ka$, $\beta = k_{\mathrm{t}}a$; ψ_n and ξ_n are the Riccati–Bessel functions associated with the spherical Bessel function and the Hankel function, respectively; the asterisk denotes the complex conjugate value; and the coefficients, a_n, b_n, c_n, and d_n are obtained using the boundary conditions at the particle–medium interface [2].

The rate of energy attenuated by the spherical particle is $W_{\mathrm{e}} = W_{\mathrm{a}} + W_{\mathrm{s}}$, which can be written in the form

$$W_{\mathrm{e}} = \frac{\pi |E_0|^2}{\omega} \sum_{n=1}^{\infty} (2n+1) \mathrm{Im}\left(\frac{A_n}{\mu_{\mathrm{t}}} + \frac{B_n}{\mu}\right). \tag{3}$$

The rate of energy incident on the sphere in an absorbing medium is

$$f = \frac{2\pi a^2}{(\eta a)^2} I_0 [1 + (\eta a - 1) \mathrm{e}^{\eta a}], \tag{4}$$

where $\eta = 4\pi m_{\mathrm{i}}/\lambda_0$ and $I_0 = (m_{\mathrm{r}}/2c\mu)|E_0|^2$. Here, c is the speed of light in vacuum, and m_{r} and m_{i} are the real and imaginary parts of the complex refractive index of the host medium, respectively. Therefore, the absorption, scattering, and extinction efficiencies are, respectively,

$$Q_{\mathrm{a}} = W_{\mathrm{a}}/f, \tag{5a}$$

$$Q_{\mathrm{s}} = W_{\mathrm{s}}/f, \tag{5b}$$

$$Q_{\mathrm{e}} = W_{\mathrm{e}}/f. \tag{5c}$$

Yang et al. [5] called the quantities defined in Eq. (5) as the inherent absorption, scattering and extinction efficiencies, which may not be suitable to be directly used in the radiative transfer equation.

3. Apparent absorption, scattering, and extinction efficiencies

Since the scattered wave in the far field is the relevant quantity for radiative transfer equations [4], it is necessary to derive the apparent scattering efficiency based on the far-field information [5]. Following [6], the total scattered energy rate in the far field is given by

$$\widehat{W}_{\mathrm{s}} = \frac{\pi |E_0|^2}{\omega \mu} \frac{2\pi m_{\mathrm{r}} \mathrm{e}^{-\eta r}}{\lambda_0 |k|^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2), \tag{6}$$

where r is the distance between the particle and the observational point. The apparent scattering efficiency of the scatterer is then defined in the form [5,6]

$$\widehat{Q}_{\mathrm{s}} = \widehat{W}_{\mathrm{s}}\, \mathrm{e}^{\eta(r-a)}/f. \tag{7}$$

Here, an exponential absorption is assumed to trace the radial far-field scattered radiation back to the particle surface [5,6]. Such defined apparent scattering efficiency is consistent with the scattering phase function and asymmetry factor based on the far-field scattered waves [2,5].

The scattered radiation is absorbed in the absorbing medium but it is not necessarily exponentially damped with the radial distance, especially in the region near the particle. The non-exponential absorption can be quantified by the difference between the actual and apparent scattering efficiencies. Here, we define an apparent absorption efficiency, $\widehat{Q}_{\mathrm{a}}$, to take into account the non-exponential absorption of the scattered radiation in the form

$$\widehat{Q}_{\mathrm{a}} = Q_{\mathrm{a}} + Q_{\mathrm{s}} - \widehat{Q}_{\mathrm{s}}. \tag{8}$$

The apparent extinction efficiency, $\widehat{Q}_{\mathrm{e}}$, is then given by

$$\widehat{Q}_{\mathrm{e}} = \widehat{Q}_{\mathrm{s}} + \widehat{Q}_{\mathrm{a}} = Q_{\mathrm{s}} + Q_{\mathrm{a}} = Q_{\mathrm{e}}. \tag{9}$$

Thus, the extinction of incident radiation based on the apparent optical properties is the same as the actual extinction. This is an important requirement to evaluate the attenuation of the incident radiation. It is noted

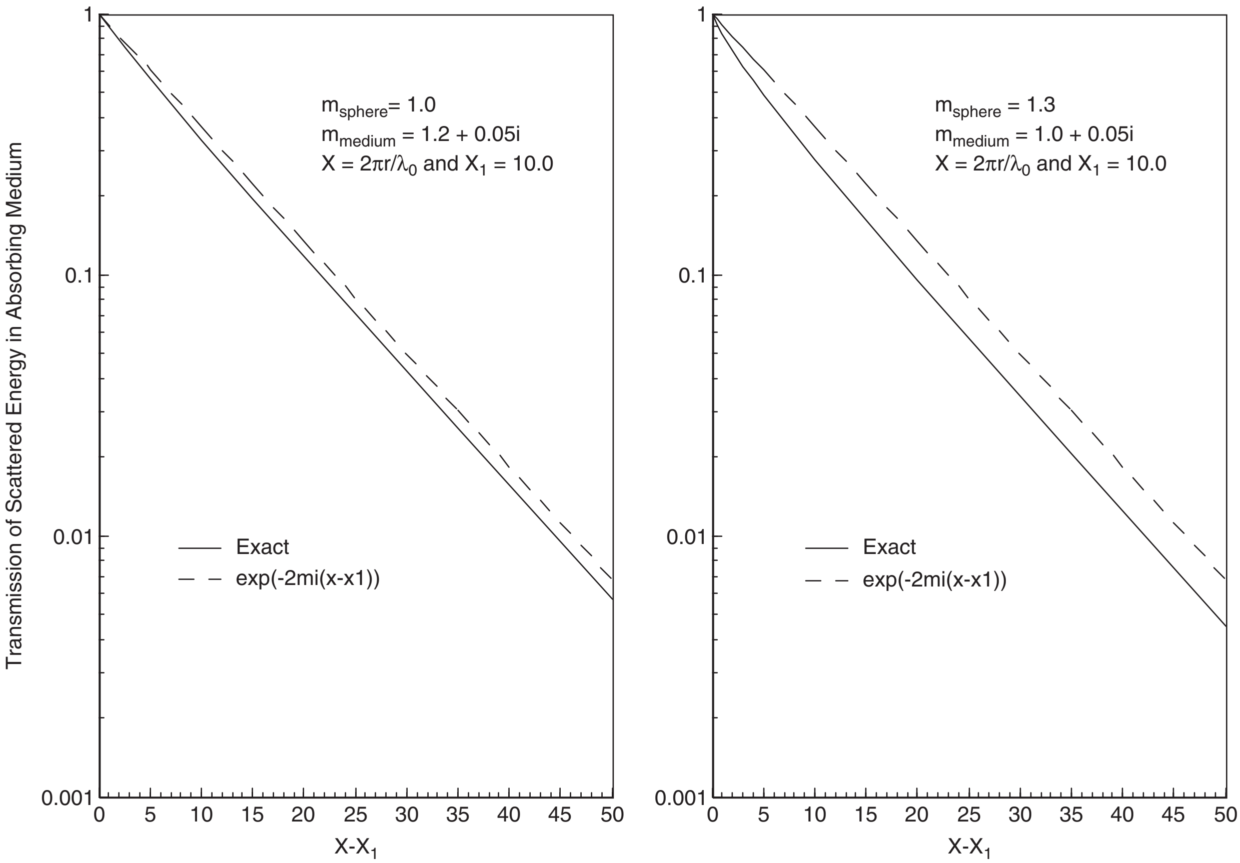

Fig. 1. Transmission of scattered energy in absorbing medium based on the exponential attenuation (dashed line) and exact simulation (solid line). In the left (right) panel, the real part of the refractive index for the sphere is smaller (larger) than that for the medium.

that in the formulation of apparent optical properties in [5], the non-exponential absorption of the scattered radiation is neglected, which would underestimate the absorption of scattered radiation in host medium. This point will be discussed further in the following along with numerical calculations.

Fig. 1 shows the transmission of scattered radiation as a function of the distance from the spherical surface. It is the scattered energy divided by that at the spherical surface. We use a m_t of 1.0 and m of 1.2 + 0.05i in Fig. 1 left panel and a m_t of 1.3 and m of 1.0 + 0.05i in Fig. 1 right panel. The size parameter of the sphere is 10. We see that the absorption estimated using the exponential attenuation is smaller than that from exact calculations. This result can be explained by the fact that the scattered light from the spherical surface can be in both radial and tangential directions. For example, the evanescent waves [8] related to the total external (Fig. 1 left panel) and internal (Fig. 1 right panel) reflections propagate to the far field along the tangential direction of the particle surface. Thus, the exponential attenuation along the radial direction would always underestimate the absorption of the scattered radiation in the host medium. It is also interesting to note that the exponential results deviate more from the exact values when the internal reflection is involved (Fig. 1 right panel).

Fig. 2 shows actual (inherent) and apparent scattering and absorption efficiencies of spherical particles with a refractive index of 1.34 + 0.01i embedded in a medium with a refractive index of 1.0 + 0.001i. The apparent scattering and absorption efficiencies defined in Eqs. (7) and (8), respectively, are different from the actual ones reported in [2]. With the increase of size parameter, the apparent scattering efficiency becomes smaller

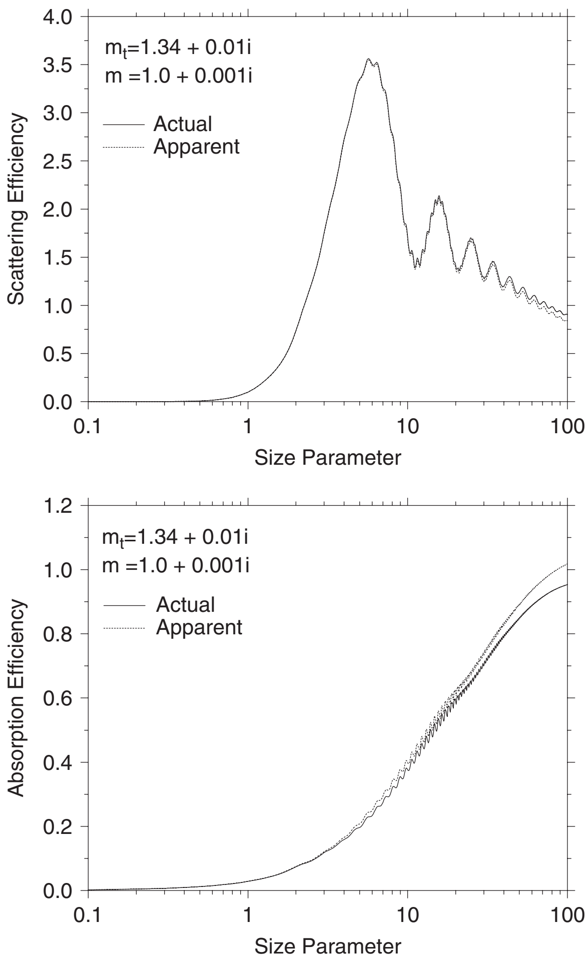

Fig. 2. Actual and apparent scattering and absorption efficiencies of spherical particles with a refractive index of 1.34 + 0.01i embedded in a medium with a refractive index of 1.0 + 0.001i as functions of size parameter ($2\pi\alpha/\lambda_0$).

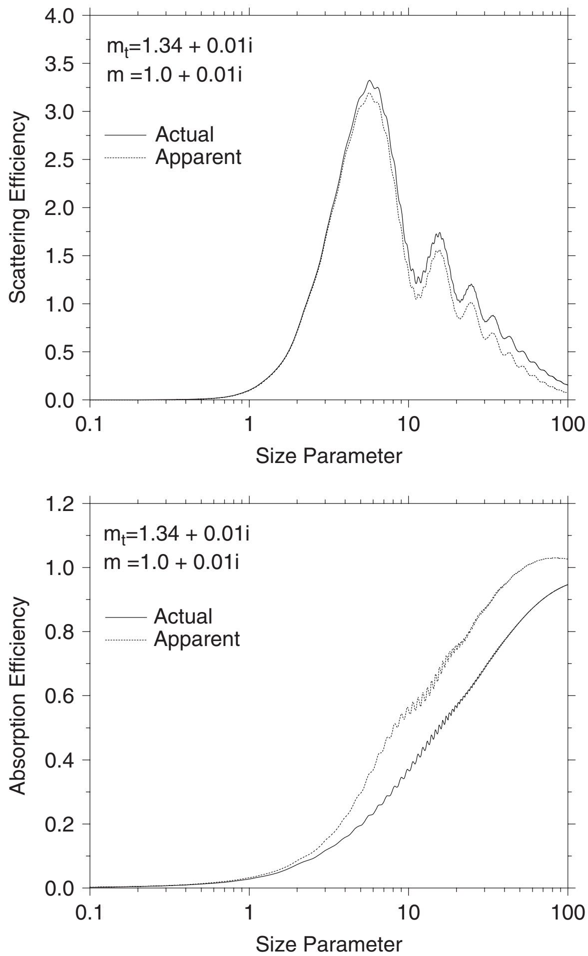

Fig. 3. Actual and apparent scattering and absorption efficiencies of spherical particles with a refractive index of 1.34 + 0.01i embedded in a medium with a refractive index of 1.0 + 0.01i ($2\pi\alpha/\lambda_0$).

than the actual scattering efficiency, whereas the apparent absorption efficiency is larger than the actual one because the non-exponential absorption of scattered radiation is added to the actual values. When the medium absorption is stronger, as shown in Fig. 3, the difference between the apparent and actual scattering and absorption properties becomes more significant. Following Yang et al. [5], the apparent optical properties presented in this study are suited to radiative transfer equations.

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