Two- and Four-Stream Combination Approximations for Computation of Diffuse Actinic Fluxes

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ABSTRACT

The δ-two- and four-stream combination approximations, which use a source function from the two-stream approximations and evaluate intensities in the four-stream directions, are formulated for the calculation of diffuse actinic fluxes. The accuracy and efficiency of the three computational techniques—the δ-two-stream approximations, the δ-two- and four-stream combination approximations based on various two-stream approaches, and the δ-four-stream approximation—have been investigated. The diffuse actinic fluxes are examined by considering molecular, aerosol, haze, and cloud scattering over a wide range of solar zenith angles, optical depths, and surface albedos. In view of the overall accuracy and computational efficiency, the δ-two- and four-stream combination method based on the quadrature scheme appears to be well suited to radiative transfer calculations involving photodissociation processes.

1. Introduction

Photochemical processes in the atmosphere are driven by solar radiation, which dissociates certain molecules into reactive atoms or free radicals. Models that simulate the chemistry of the atmosphere must accurately simulate the radiation processes that initiate photodissociation. The photodissociation rate is proportional to the actinic flux. In addition, for some biological applications such as exposure of small “bodies” suspended in air or in water (e.g., phytoplankton in the ocean), actinic fluxes are also used to compute the dose rate and total dose (Kylling et al. 1995).

A variety of approximate techniques are now commonly used for the calculation of actinic fluxes, including the variational method (Yung 1976), isotropic integration (Anderson and Meier 1979), the Isaksen–Luther method (Isaksen et al. 1977; Luther 1980; Thompson 1984; Madronich 1987; Dvortsov et al. 1992), various two-stream methods (Liou 1974; Coakley and Chylek 1975; Joseph et al. 1976; Meador and Weaver 1980; Toon et al. 1989; Kylling et al. 1995; Qi 1999; Lu et al. 2009), four-stream methods (Liou et al. 1988; Li and Dobbie 1998; Li and Ramaswamy 1996), and discrete ordinates methods (Stamnes et al. 1988).

Among these techniques, the variational method, isotropic integration, and Isaksen–Luther method assume that the phase function is isotropic or that the light
scattered by an entire layer is isotropic. Such approximations cannot be applied to the presence of clouds and aerosols, which have strong forward scattering. The δ-two-stream methods (Meador and Weaver 1980; Toon et al. 1989; Kylting et al. 1995), which were developed to calculate multiple scattering in aerosols and clouds, are widely used to simulate diffuse actinic fluxes. The δ-four-stream method is more accurate but also more computationally expensive. Even though computer speed is rapidly improving these days, saving computational cost is still very important in global climate models and in some remote sensing operational applications. Here we develop the δ-two- and four-stream combination approximations for the calculation of diffuse actinic fluxes, which are shown to be more accurate than the two-stream methods and more efficient than the δ-four-stream method. Note that the photodissociation rate is proportional to the total actinic flux that is summation of the direct and diffuse radiation, and the former is often dominated by the latter. Lary and Pyle (1991) found that a correct treatment of the diffuse radiative field is important in the modeling of ozone above 35 km. Considerable efforts were made to include the effects of diffuse radiation in photochemical models (Luther and Gelinas 1976; Fiocco 1979; Mugnai et al. 1979). In addition, Leighton (1961) showed that diffuse radiation is more effective than direct radiation by a factor of 2 cos(θ0) (Madronich 1987).

The δ-two- and four-stream combination approximations are developed based on the source function technique proposed by Davies (1980) and Toon et al. (1989). Fu et al. (1997) showed that they are suitable for the radiative flux and heating rate calculations in the infrared, with an accuracy close to the δ-four-stream method but a computational efficiency only about 50% more than the δ-two-stream methods. Unfortunately, for the solar radiation, when the single scattering albedo is equal to 1, these approaches do not necessarily yield conserved radiative fluxes (Toon et al. 1989). In the infrared and solar spectra, the approaches do yield a useful approximation to intensities and can be used to obtain quantities such as the geometric albedo that cannot be found with various two-stream approximations (Toon et al. 1989). However, little work has been done to apply the δ-two- and four-stream combination approximations to the calculations of actinic fluxes.

In section 2, we briefly introduce the δ-two-stream and δ-four-stream approximations and formulate the δ-two- and four-stream combination approximations based on various two-stream approaches. In section 3, we examine the accuracy and computational efficiency of these approximations for a wide range of cases. A summary and conclusions are given in section 4.

2. Theory and method
a. Basic equations

To obtain the diffuse actinic fluxes, we begin with the azimuthally averaged radiative transfer equation for the diffuse solar intensity \( I(\tau, \mu) \) in plane-parallel atmospheres (e.g., Liou et al. 1988):

\[
\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - \frac{\sigma}{4\pi} \pi E_0 e^{-\tau/\mu_0} P(\mu, -\mu_0) - \frac{\sigma}{2} \int_{-1}^{1} I(\tau, \mu') P(\mu', \mu') d\mu',
\]

(1)

<table>
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<th>Method based on</th>
<th>( G(\pm \mu) )</th>
<th>( H(\pm \mu) )</th>
<th>( z(\pm \mu) )</th>
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<td>( \frac{\sigma}{4\pi} [\pi E_0 P(\mu, -\mu_0) + 2(Z_+ + Z_-)] )</td>
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<td>( \frac{3^{1/2} \sigma}{4\pi} [R \alpha(\pm \mu) + 2 - \alpha(\pm \mu)] )</td>
<td>( \frac{\sigma}{4\pi} [\pi E_0 P(\mu, -\mu_0) + 3^{1/2}[Z_+ \alpha(\pm \mu) + 2Z_- - Z_- \alpha(\pm \mu)]] )</td>
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<tr>
<td>Hemispheric constant</td>
<td>( \frac{g_0 \sigma}{2\pi} (\alpha(\pm \mu) + R[2 - \alpha(\pm \mu)]) )</td>
<td>( \frac{g_0 \sigma}{2\pi} [R \alpha(\pm \mu) + 2 - \alpha(\pm \mu)] )</td>
<td>( \frac{\sigma}{4\pi} [\pi E_0 P(\mu, -\mu_0) + 2Z_- - Z_- \alpha(\pm \mu)]] )</td>
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</table>

Table 2. Summary of coefficients in the two- and four-stream combination approximations. In the table, \( \alpha(\pm \mu) = \int_{-1}^{1} P(\pm \mu, \mu') d\mu' \).
where $\mu$ is the cosine of the zenith angle; $\tau$ the optical depth; $\sigma$ the single scattering albedo; $P(\mu, \mu')$ the azimuthally averaged scattering phase function, defining the light incidence at $\mu'$, which is scattered in the direction $\mu$; $\pi E_0$ the direct solar irradiance perpendicular to the solar direct beam; and $\mu_0$ the cosine of the solar zenith angle. The quantity $P(\mu, \mu')$ can be written as (e.g., Liou et al. 1988)

$$P(\mu, \mu') = \sum_{l=0}^{N} \sigma_l P_l(\mu)P_l(\mu'),$$

(2)

where $P_l(\mu)$ is the Legendre function and $\sigma_l$ is the expansion coefficient of the scattering phase function in terms of the Legendre polynomials (e.g., Liou et al. 1988); $\sigma_0 = 1$ and $\sigma_1 = 3g$, where $g$ is the asymmetry factor.

The diffuse actinic fluxes and irradiance fluxes in the upward and downward directions are

$$F = 2\pi \int_{-1}^{1} I(\tau, \mu) \, d\mu,$$

(3)

$$E^\pm = 2\pi \int_{0}^{1} I(\tau, \pm\mu) \mu \, d\mu,$$

(4)

Next, we introduce various approximations for computation of the diffuse actinic fluxes.

**b. Two-stream approximations**

Two-stream approximations have been widely used for many years to rapidly solve radiative transfer problems. They avoid the complex and lengthy algorithms necessary for numerical solutions of the radiative transfer equation by yielding analytical solutions that are relatively easy to implement. Meador and Weaver (1980) have shown that various two-stream schemes can generally be expressed in the form

$$\frac{\partial E^+}{\partial \tau} = r_1 E^+_1 - r_2 E^-_1 - r_3 \pi E_0 \sin e^{-\tau/\mu_0},$$

(5)

$$\frac{\partial E^-}{\partial \tau} = r_2 E^+_1 - r_1 E^-_1 + (1 - r_3) \pi E_0 \sin e^{-\tau/\mu_0},$$

(6)

where $r_1$, $r_2$, and $r_3$ are coefficients that depend on the particular form of the two-stream schemes. Table 1 presents the values of $r_1$, $r_2$, and $r_3$ for some commonly used two-stream approximations (Meador and Weaver 1980; Thomas and Stamnes 2002).

The solution for Eqs. (5) and (6) can be written as
where 

\[ \mu^+ = (1 - \sigma)/(r_1 - r_2). \]  

The parameter \( \mu^+ \) is a constant such as ½ or \( \sqrt{3} \) rather than a frequency-dependent property (Toon et al. 1989).

c. Four-stream approximation

The four-stream approximation is based on the general solution to the discrete ordinates method for radiative transfer. It has been discussed in detail by Liou et al. (1988) and Fu and Liou (1993). Using Gaussian quadrature and the phase function expansion in Eq. (2), the four-stream approximation may be written as

\[
\frac{\mu_i}{\sigma} \frac{dI(\tau, \mu_i)}{d\tau} = I(\tau, \mu_i) - \frac{\sigma}{2 \pi} \sum_{l=0}^{3} \sigma_l P_l(\mu_i) 
\times \sum_{l=0}^{2} I(\tau, \mu_j) l_{P_l} \mu_{l} (\mu_j) a_j - \frac{\sigma}{4 \pi} \pi E_0 
\times \sum_{l=0}^{3} \sigma_l P_l(\mu_j) l_{P_l} (\mu_j) e^{-\tau_{l+1}} , \quad i = \pm 1, \pm 2, 
\]  

and the coefficients \( g_{1,2} \) are to be determined from boundary conditions.

In the two-stream approximations, the diffuse actinic flux is given by

\[
F_1(\tau) = 2\pi \int_{-1}^{1} I(\tau, \mu) d\mu = [E_1^+(\tau) + E_1^-(\tau)]/\mu^+, 
\]  

where

\[
E_1^+(\tau) = g_1 e^{-k(\tau_i - \tau)} + g_2 R e^{-k\tau} + Z_1 e^{-r(\tau_i - \mu)}, 
\]

\[
E_1^-(\tau) = g_1 R e^{-k(\tau_i - \tau)} + g_2 e^{-k\tau} + Z_1 e^{-r(\tau_i - \mu)}, 
\]

where

\[
k = (r_1^2 - r_2^2)^{1/2}, 
\]

\[
R = \frac{r_1 - k}{r_2} = \frac{r_2}{r_1 + k}, 
\]

\[
Z_+ = \frac{\sigma \pi E_0 (r_1 - 1/\mu_0) r_3 + r_3 (1 - r_1)}{k^2 - 1/\mu_0^2}, 
\]

\[
Z_- = \frac{\sigma \pi E_0 (r_1 + 1/\mu_0) (1 - r_3) + r_2 r_3}{k^2 - 1/\mu_0^2}, 
\]
Table 4a. As in Table 3a, but for the $\delta$-two-stream approximation and $\delta$-two- and four-stream combination approximation based on the quadrature scheme.

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The diffuse actinic fluxes and irradiance fluxes in the upward and downward directions are defined as

\[
F^\uparrow_2(\tau) = 2\pi \left[ \sum_{i=1}^{2} a_i I(\tau, \mu_i) + \sum_{i=1}^{2} a_i I(\tau, -\mu_i) \right], \quad (16)
\]

\[
E^\uparrow_2(\tau) = 2\pi \sum_{i=1}^{2} a_i \mu_i I(\tau, \pm \mu_i). \quad (17)
\]

**d. Two- and four-stream combination approximations**

For a homogeneous layer with an optical depth $\tau_1$, Eq. (1) is formally solved to obtain ($\mu > 0$)

\[
I(0, \mu) = I(\tau_1, \mu) e^{-\tau_1/\mu} + \int_{0}^{\tau_1} M(\tau', \mu) e^{-\tau'/\mu} d\tau'/\mu, \quad (18)
\]

\[
I(\tau_1, -\mu) = I(0, -\mu) e^{-\tau_1/\mu} + \int_{0}^{\tau_1} M(\tau', -\mu) e^{-\tau'/\mu} d\tau'/\mu. \quad (19)
\]

where $I(\tau_1, \mu)$ and $I(0, -\mu)$ are, respectively, the inward intensities at the bottom and top surfaces. In Eqs. (18) and (19), $M(\tau', \pm \mu)$ is the source function associated with multiple scattering and single scattering. It can be written as

\[
M(\tau, \pm \mu) = \frac{\sigma_0}{4\pi} \pi E_0 e^{-\tau/\mu} P(\pm \mu, -\mu_0) + \frac{\sigma_0}{2} \int_{-1}^{1} I(\tau, \mu') P(\pm \mu, \mu') d\mu'. \quad (20)
\]

In the two- and four-stream combination methods, we first solve the source function using various two-stream schemes. Then, we use Eqs. (18) and (19) to evaluate intensities in the four-stream directions. Here, the double Gaussian points and weights are used in the four-stream intensity and flux calculations.

Using various two-stream schemes, the source function is given by
The upward and downward diffuse fluxes at a given level $\tau$ for this method are defined by

$$E_\tau^\pm(\tau) = 2\pi \sum_{i=1}^{2} a_i \mu E_i(\tau, \pm \mu),$$  \hspace{1cm} (24)$$

where $\mu_1 = 0.211$ 324 8, $\mu_2 = 0.788$ 675 2, $a_1 = 0.5$, and $a_2 = 0.5$ in the two- and four-stream combination approximations.

The two- and four-stream combination approximations are significantly better at representing the angular characteristics of the radiance field than the two-stream approximations (Davies 1980). However, the methods are somewhat unsatisfactory, because where $\sigma = 1$, they do not necessarily yield flux conservation at solar wavelengths (Toon et al. 1989). We therefore extend it by defining

$$I'(\tau, \pm \mu) = I(\tau, \pm \mu)E_{\tau}^\pm/E_{\tau}^\pm,$$ \hspace{1cm} (25)$$

where $I(\tau, \pm \mu)$ is taken from Eqs. (22) and (23). In addition, $E_{\tau}^+$, $E_{\tau}^-$, and $E_{\tau}^\pm$ are calculated from Eqs. (7), (8), and (24), respectively. Note that $I'(\tau, \pm \mu)$ has the same angular characteristics as $I(\tau, \pm \mu)$ but retains flux
conservation when \( \sigma = 1 \). Using \( l'(\tau, \pm \mu) \) from Eq. (25), the diffuse actinic flux is given by

\[
F_3(\tau) = 2\pi \left[ \sum_{i=1}^{2} a_i l'(\tau, \mu_i) + \sum_{i=1}^{2} a_i l'(\tau, -\mu_i) \right]. \tag{26}
\]

The two- and four-stream combination methods are appealing because they combine the advantages of the speed of the two-stream approximations and the accuracy of the four-stream approximation.

In the solar radiative transfer, it is important to perform the delta adjustment to account for the forward-scattering peak (Joseph et al. 1976; Liou et al. 1988; Fu et al. 1997; Cuzzi et al. 1982). If a fraction of the scattering energy \( f \) is considered to be in the forward peak, the above solution can still be used, as long as the following transformations are applied to the optical properties:

\[
\tau' = \tau(1 - f \sigma), \tag{27}
\]

\[
\sigma' = (1 - f)\sigma(1 - f \sigma). \tag{28}
\]

For the \( \delta \)-two-stream and \( \delta \)-two- and four-stream combination approximations, \( l = 0, 1 \) and \( f = \sigma_4/5 \); for the \( \delta \)-four-stream approximation, \( l = 0, 1, 2, 3 \) and \( f = \sigma_4/9 \). The use of function adjustment would enhance the accuracy of approximate treatments of multiple scattering.

3. Computational results and discussion

Here we examine the accuracy of the \( \delta \)-two-stream, \( \delta \)-two- and four-stream combination, and \( \delta \)-four-stream approximations by comparing results with the exact values taken from Yung (1976) or computed from the discrete ordinates method (Stamnes et al. 1988).

The impact of Rayleigh scattering on photochemical processes in the stratosphere is important. We first compare the diffuse actinic fluxes at the top [labeled \( F(0) \)] and the bottom [labeled \( F(\tau_1) \)] in a conservative Rayleigh scattering atmosphere. Tables 3–5 show diffuse actinic fluxes using a solar flux \( (\pi E_0) \) of 1, with different optical depths, surface albedos, and solar zenith angles.
for each of the computational techniques. In these tables, the \( \delta \)-four-stream approximation is labeled Four and the \( \delta \)-two-and-four-stream combination approximations based on the Eddington, quadrature, and hemispheric constant schemes are labeled SFH4, SFO4, and SFH4, respectively. In addition, the \( \delta \)-two-stream methods based on the Eddington, quadrature, and hemispheric constant schemes are respectively labeled Eddington, Quadrature, and Hemispheric constant. The exact values are taken from Yung (1976). Relative errors (labeled Err) between these approximations and exact values are also given.

Results from the present \( \delta \)-two-stream method based on quadrature approximation in Tables 4a and 4b agree with those of the \( \delta \)-two-stream method given by Toon et al. (1989, Table 6 in their paper) and Kylling et al. (1995, Table III in their paper). Since the present \( \delta \)-two-stream method based on quadrature is the same as the method used by Toon et al. (1989) and Kylling et al. (1995), the slight differences between them are due to numerical rounding errors.

For \( F(0) \) and \( F(\tau_1) \), no significant differences are observed among the \( \delta \)-two- and four-stream combination approximations based on the Eddington, quadrature, and hemispheric constant schemes. The maximum relative errors in both \( F(0) \) and \( F(\tau_1) \) of the \( \delta \)-two- and four-stream combination approximations are \( \sim 37\% \), which are similar to those of the \( \delta \)-four-stream approximation (\( \sim 37\% \)) but much smaller than these of the two-stream approximations (\( \sim 65\% \)). For both \( F(0) \) and \( F(\tau_1) \) when the optical depth \( \tau_1 \geq 0.25 \), the \( \delta \)-two-stream, \( \delta \)-two- and four-stream combination, and \( \delta \)-four-stream approximations have errors less than \( \sim 43\%, \sim 16\%, \) and \( \sim 13\% \) respectively. In general, for a conservative Rayleigh scattering atmosphere, the accuracy of the \( \delta \)-two- and four-stream combination approximations based on various two-stream approaches is close to that of the \( \delta \)-four-stream approximation and much more accurate than that of the \( \delta \)-two-stream approximations.

Radiative transfer through ozone layer around 35 km is also important (Lary and Pyle 1991). Because of absorption by ozone, the single scattering albedo is not equal to 1 in the ultraviolet. The diffuse actinic fluxes in the two test cases with \( \bar{\omega} = 0.7 \) and \( \bar{\omega} = 0.9 \) for non-conservative Rayleigh scattering atmosphere are shown in Fig. 1. It shows that the accuracy of the \( \delta \)-two- and four-stream combination approximations based on various
two-stream approaches is close to that of the $\delta$-four-stream approximation and much more accurate than that of the $\delta$-two-stream approximations. It should be noted that it is necessary to integrate over part of the solar spectrum in the computations of photolysis rates. The actual errors may be significantly smaller than those in the monochromatic radiative transfer computations because of the compensating errors.

FIG. 1. Diffuse actinic flux $F$ for a nonconservative Rayleigh scattering atmosphere vs the optical depth $\tau$ for $\tau = 0.7$ and $0.9$. Here, the surface albedo $R$ equals 0, the solar flux $(\sigma T_0)$ equals 1, the optical depth $\tau_1 = 1$, and the cosine of the solar zenith angle $\cos \theta_0 = 0.5$. The $\delta$-four-stream approximation is labeled Four; the $\delta$-128-stream method is denoted Exact. The $\delta$-two- and four-stream combination approximations based on the Eddington, quadrature, and hemispheric constant schemes are SFE4, SFQ4, and SFH4, respectively. In addition, the $\delta$-two-stream methods based on the Eddington, quadrature, and hemispheric constant schemes are labeled as Eddington, Quadrature, and Hemispheric constant, respectively.
We also calculated the diffuse actinic flux by considering nonabsorbing aerosols. The asymmetry factor of aerosols is used to represent the phase function through the Henyey–Greenstein function. Figures 2–4 show the diffuse actinic fluxes calculated by the $d$-two-stream, $d$-two- and four-stream combination, $d$-four-stream, and $d$-128-stream approximations. The latter is labeled Exact. Figures 2–4 show that the $d$-two- and four-stream combination approximations based on the Eddington scheme are denoted SFE4 and Eddington, respectively.

![Figure 2](image_url)

**Fig. 2.** Diffuse actinic flux $F$ for the conservative scattering aerosol layer vs the optical depth $\tau$ : $\tau_I = (\text{top})$ (left) 0.02 and (right) 0.25; (middle) (left) 0.50 and (right) 1.00; and (bottom) (left) 2.50 and (right) 5.00. Here, the surface albedo $R$ is equal to 0.25, the phase function is the Henyey–Greenstein with $g = 0.65$, the solar flux ($\pi E_0$) is equal to 1, and the cosine of the solar zenith angle $\mu_0 = 0.5$. The $d$-four-stream approximate is denoted Four; the $d$-128-stream method is denoted Exact. In addition, the $d$-two- and four-stream combination and $d$-two-stream approximations based on the Eddington scheme are denoted SFE4 and Eddington, respectively.
constant schemes for all cases. When the optical depth $\tau_1$ is equal to 0.02, 2.5, and 5, the accuracy of the $\delta$-two- and four-stream combination approximation based on the quadrature scheme is similar to that of the $\delta$-four-stream approximation. In other cases, the $\delta$-four-stream approximation performs better. Hence, the $\delta$-two- and four-stream combination approximation based on the quadrature scheme is better than those based on the Eddington and hemispheric constant schemes.

Both cloud and haze affect the radiation field. For overcast and hazy atmospheres, the error in photolysis rates incurred by the use of the two-stream approach becomes larger than that of clear-sky situations (Kylling et al. 1995). The diffuse actinic fluxes of two test cases, one from the Haze L scattering model (case 1 in Table 21 of Garcia and Siewert 1985) and the other from the Cloud C1 scattering model (case 4 in Table 21 of Garcia and Siewert 1985), are shown in Fig. 5. The phase function

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**Fig. 3.** As in Fig. 2 but for the $\delta$-two- and four-stream combination and $\delta$-two-stream approximations based on the quadrature scheme, denoted SFQ4 and Quadrature, respectively.
of Cloud C1 and Haze L are specified by Garcia and Siewert (1985). The two cases are summarized in our Table 6. Figure 5 shows that the $d$-two- and four-stream combination approximation based on the quadrature scheme is much better than $d$-two-stream approximations based on the quadrature scheme. Since the $d$-two-stream approximation based on the quadrature scheme is the same as the $d$-two-stream method used by Kylling et al. (1995), the accuracy of the $d$-two- and four-stream combination approximations based on the quadrature scheme is better than the method reported by Kylling et al. (1995). For both Haze L and Cloud C1, the accuracy of the $d$-two- and four-stream combination approximations based on the Eddington and hemispheric constant schemes is slightly better than or similar to that of the $d$-two-stream approximation based on the Eddington and hemispheric constant schemes. Hence, the $d$-two-and four-stream combination approximation based on the
FIG. 5. Diffuse actinic flux $F$ for the haze and cloud vs the optical depth $\tau$: $\tau_1 = (\text{left})$ 1.00 and (right) 64.0; (top) quadrature and SFQ4, (middle) Eddington and SFQ4, and (bottom) Hemispheric constant and SFH4. Four and Exact are for comparison references to the left and right. Here, the left of the figure is about Haze L, and the right of it is about Cloud C. The $\delta$-four-stream approximation is labeled Four; the $\delta$-two-and four-stream combination approximations based on the Eddington, quadrature, and hemispheric constant schemes are labeled SFE4, SFQ4, and SFH4, respectively. In addition, the $\delta$-two-stream methods based on Eddington, quadrature, and hemispheric constant schemes are labeled Eddington, Quadrature, and Hemispheric constant, respectively.
quadrature scheme is better than those based on the Eddington and hemispheric constant schemes.

In conclusion, the δ-two- and four-stream combination approximation based on the quadrature scheme may be used for the calculation of diffuse actinic fluxes when accuracy greater than that of the δ-two-stream approximations is required. In some cases, such as Rayleigh scattering, its accuracy is similar to that of the δ-four-stream approximation.

For applications to three-dimensional atmospheric chemistry modeling, radiative computations of diffuse actinic flux are required. Thus, it is important to examine both the accuracy and efficiency of the radiative transfer parameterization. Table 7 shows the calculation times using various approximations. For these comparisons, the atmosphere was divided into 1000 layers, and the computing time was normalized by that of the δ-two-stream approximations.

The δ-two-stream approximations are the most computationally efficient. However, as demonstrated in Tables 3–5 and Figs. 1–5, they produce significant errors in the diffuse actinic flux calculations. High accuracy in diffuse actinic fluxes can be obtained using the δ-four-stream approximation. However, its computation time is 6.49 times more than that of the δ-two-stream approximations (see Table 7).

As shown in Tables 3–5 and Figs. 1–5, the accuracy of the δ-two- and four-stream combination approximations based on the quadrature scheme is close to that of the δ-four-stream approximation, but their computational time is less than half that of the δ-four-stream approximation (see Table 7).

4. Summary and conclusions

We formulated δ-two- and four-stream combination approximations for the calculation of diffuse actinic fluxes. We investigated the accuracy and efficiency of these methods and compared them to the δ-two-stream and δ-four-stream approximations. A wide range of solar zenith angles, optical depths, and surface albedos were considered for considering molecular, aerosol, haze, and cloud scattering.

We found that for the calculation of diffuse actinic fluxes, the errors in the δ-two-stream methods were largest. For the δ-four-stream approximation, reliable results were obtained under all conditions. The accuracy of the δ-two- and four-stream combination approximations based on the quadrature scheme was close to that of the δ-four-stream approximation.

In view of their accuracy and computational efficiency, the δ-two- and four-stream combination method based on the quadrature scheme is well suited to diffuse actinic flux calculations.

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