The Variability and Predictability of Axisymmetric Hurricanes in Statistical Equilibrium

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ABSTRACT

The variability and predictability of axisymmetric hurricanes are determined from a 500-day numerical simulation of a tropical cyclone in statistical equilibrium. By design, the solution is independent of the initial conditions and environmental variability, which isolates the “intrinsic” axisymmetric hurricane variability.

Variability near the radius of maximum wind is dominated by two patterns: one associated primarily with radial shifts of the maximum wind, and one primarily with intensity change at the time-mean radius of maximum wind. These patterns are linked to convective bands that originate more than 100 km from the storm center and propagate inward. Bands approaching the storm produce eyewall replacement cycles, with an increase in storm intensity as the secondary eyewall contracts radially inward. A dominant time period of 4–8 days is found for the convective bands, which is hypothesized to be determined by the time scale over which subsidence from previous bands suppresses convection; a leading-order estimate based on the ratio of the Rossby radius to band speed fits the hypothesis.

Predictability limits for the idealized axisymmetric solution are estimated from linear inverse modeling and analog forecasts, which yield consistent results showing a limit for the azimuthal wind of approximately 3 days. The optimal initial structure that excites the leading pattern of 24-h forecast-error variance has largest azimuthal wind in the midtroposphere outside the storm and a secondary maximum just outside the radius of maximum wind. Forecast errors grow by a factor of 24 near the radius of maximum wind.

1. Introduction

Forecasts of tropical cyclone position have steadily improved in the past decade (e.g., Elsberry et al. 2007) along with the global reduction in forecast error for operational models. If tropical cyclone intensity were mainly determined by the storm environment (e.g., Frank and Ritchie 1999; Emanuel et al. 2004) such as through variability in sea surface temperature and large-scale vertical wind shear, then one might expect improvements in tropical cyclone intensity forecasts similar to those for track. The fact that tropical cyclone intensity forecasts from numerical models have been resistant to these environmental improvements suggests that the challenge to improve these forecasts lies with the dynamics of the tropical cyclone itself. While increasing observations and novel data assimilation strategies may reduce forecast errors, it is unclear what improvements one should expect, even in the limit of vanishing initial error, since the fundamental predictability of tropical cyclone structure and intensity is essentially unknown.

We take a step in the direction of addressing this problem here by examining the variability and predictability of idealized axisymmetric numerical simulations of a tropical cyclone in statistical equilibrium. By eliminating variability due to the environment, asymmetries, and storm motion we isolate what we define as the “intrinsic” variability. A 500-day equilibrium solution allows this variability to be estimated with sample sizes much larger than can be obtained from observed data or from the simulation of individual storms.

Since the structure of tropical cyclones is dominated by features on the scale of the vortex, primarily the azimuthal mean, a starting hypothesis for predictability and variability in storm structure and intensity is that it is controlled by the axisymmetric circulation. However, it is clear that although asymmetries have much smaller amplitude compared to the azimuthal mean, they may affect storm intensity through, for example,
wave–mean flow interaction (e.g., Montgomery and Kallenbach 1997; Wang 2002; Chen et al. 2003). A prominent example of structure and intensity variability involves eyewall replacement cycles, where the eyewall is replaced by an inward-propagating ring of convection (e.g., Willoughby et al. 1982; Willoughby 1990). While this process may be initiated by asymmetric disturbances, it is unclear whether such variability requires asymmetries. Here we explore variability associated with only the symmetric circulation, which provides a basis for comparison to three-dimensional studies that include asymmetries (Brown and Hakim 2013).

In terms of predictability, most research studies focus on forecasts of the location and intensity of observed storms (e.g., DeMaria 1996; Goerss 2000). Since there are a wide variety of environmental influences on observed storms, it is very difficult to determine the intrinsic predictability independent of these influences. Given the aforementioned resistance of hurricane intensity forecasts to improvements in the larger-scale environment, the intrinsic variability provides a starting point for understanding how the environment adds, or removes, predictive capacity. Ultimately, we wish to know the limit of tropical cyclone structure and intensity predictability, which may be approached by forecasts as a result of improvements to models, observations, and data assimilation systems. Again, the present study provides a starting point for the limiting case of axisymmetric storms in statistical equilibrium.

One dynamical perspective on tropical cyclone predictability views the vortex as analogous to the extratropical planetary circulation, where growth of the leading Lyapunov vectors is determined mostly by balanced perturbations developing on horizontal and vertical shear (e.g., Snyder and Hamill 2003). Some initial conditions are more sensitive than others, and the upscale influence of moist convection may accelerate large-scale error growth in some cases (e.g., Zhang et al. 2002), but these appear to be exceptional. An alternative perspective, given the central role of convection in tropical cyclones, is that upscale error growth from convective scales may be relatively more important for tropical cyclones as compared to extratropical weather predictability. For example, Van Sang et al. (2008) randomly perturb the low-level moisture distribution of a 10-member ensemble for a three-dimensional numerical simulation of tropical cyclogenesis, and find maximum azimuthal-mean wind speed differences among the members as large as 15 m s\(^{-1}\). Based on these simulations, Van Sang et al. (2008) conclude that inner-core asymmetries are “random and intrinsically unpredictable” (p. 580). If hurricane predictability is indeed more closely linked to the convective decorrelation time, then intrinsic predictability may be limited to a few hours or less. While we cannot completely answer this question here, we take the first step of acknowledging its fundamental importance to understanding tropical cyclone structure and intensity predictability, and make estimates of predictive time scales and structures that control forecast errors in an idealized axisymmetric model.

Using a radiative–convective equilibrium (RCE) modeling approach to generate samples for statistical analysis has a long history in tropical atmospheric dynamics. For example, Held et al. (1993) employ this approach to examine the statistics of simulated two-dimensional tropical convection for a uniform sea surface temperature, and show that in the absence of shear the convection aggregates into a subset of the domain. A similar, three-dimensional, study by Bretherton et al. (2005) finds upscale aggregation of convection and, when ambient rotation is included, tropical cyclogenesis. Nolan et al. (2007) use RCE to examine the sensitivity of tropical cyclogenesis to environmental parameters, and also find the spontaneous development of tropical cyclones. The current paper uses RCE of a specific state, a mature tropical cyclone, to examine the variability and predictability of these storms in an axisymmetric framework: results for three-dimensional simulations are presented in Brown and Hakim (2013).

The remainder of the paper is organized as follows. Section 2 describes the axisymmetric numerical model used to produce a 500-day solution in statistical equilibrium. The intrinsic axisymmetric variability of this solution is explored in section 3, and predictability is discussed in section 4. Section 5 provides a concluding summary.

2. Experiment setup

The axisymmetric numerical model used here is a modified version of the one used by Bryan and Rotunno (2009) as described in Hakim (2011). The primary modification involves the use of the interactive Rapid Radiative Transfer Model, GCM version (RRTM-G), radiation scheme (Mlawer et al. 1997; Iacono et al. 2003) as a replacement for the Rayleigh damping scheme used by Bryan and Rotunno (2009). Interactive radiation allows for the generation of convection by reducing the static stability as the atmosphere cools by emitting infrared radiation to space. Radiation allows the storm to achieve statistical equilibrium, maintain active convection, and produce robust variability in storm intensity. In contrast, the only mechanism to destabilize the atmosphere with the Rayleigh damping scheme is surface fluxes and, as a result, the storm does not achieve equilibrium, and there is little convection or variability.
in storm intensity (Hakim 2011). Solar radiation is excluded in the interest of eliminating the complicating effects of the diurnal and seasonal cycles.

The model uses the Thompson et al. (2008) microphysics scheme; aerodynamic formulas for surface fluxes of entropy and momentum, with equal values for the exchange coefficients for these quantities; and a constant sea surface temperature of 26.3°C. The computational grid is 1500 km in radius and 25 km in height, with 2-km horizontal resolution and 250-m vertical resolution from 0 to 10 km, gradually increasing to 1 km at the model top. A 500-day numerical solution from this model is obtained from a state of rest with full gridded data sampled every 3 h, and maximum azimuthal wind speed sampled every 15 min. The mean state of the equilibrium solution is described in Hakim (2011).

3. Intrinsic axisymmetric variability

A radius–time plot of the azimuthal wind reveals substantial fluctuation of the lowest model-level wind (125-m elevation) about the time-mean value of 59 m s\(^{-1}\) (Fig. 1, left panel). Initial storm development occurs at small radii, and the radius of maximum wind increases with time as the storm size increases [see Hakim (2011) for further details]. After approximately 25 days, a statistically steady storm is evident, with a time-mean radius of maximum wind around 30 km. Storm intensity fluctuates between about 40 and 70 m s\(^{-1}\), with bursts of stronger wind lasting for a few days. This behavior is more apparent in Fig. 1 (right panel), which also shows that, in many cases, these bursts of stronger wind originate more than 100 km from the storm center, and propagate inward. Moreover, these patterns of stronger wind move radially inward more slowly as they reach the radius of maximum wind. Finally, although not periodic, we note that the bursts of stronger wind appear on average roughly every 5 days.

The temporal Fourier spectrum of the maximum wind speed at the lowest model level (Fig. 2) exhibits two peaks: one near periods of 4–8 days, close to the range evident for the convective bands noted in Fig. 1, and the second near periods of 3 h, which may correspond to bursts of convection near the eyewall. Spectra for two single-lag autoregressive [AR(1)] models are also shown by gray lines in Fig. 2, one for the single-step (15 min) autocorrelation and the other for the single-step autocorrelation consistent with the autocorrelation e-folding
time (24.75 h); the dashed lines give the upper range of the spectra at the 95% confidence interval based on a chi-squared test (Wilks 2005, p. 397). The single-step model provides an approximate fit of the observed spectrum for periods shorter than about 12 h, except for the peak near 3 h, which suggests that the peak is distinct from persistence forced by white noise. The \( e \)-folding-time AR(1) model provides an approximate fit of the observed spectrum for periods longer than about 1 day, except for the peak near 4–8 days, which suggests this peak is also distinct from persistence forced by white noise.

The leading empirical orthogonal functions (EOFs) of the azimuthal wind field at the lowest model level reveal two patterns that dominate the variance in this field (Fig. 3). The leading pattern, which accounts for 40% of the variance, crosses zero near the radius of maximum wind; and therefore suggests a radial shift of the radius of maximum wind. The dipole structure of this EOF is asymmetric in radius, indicating that the response in the azimuthal wind is largest inside the radius of maximum wind; and therefore suggests a radial shift of the radius of maximum wind. The second pattern, which explains 27% of the variance, is largest near the radius of maximum wind, which suggests this peak is also distinct from persistence forced by white noise.

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Sample-mean structure of the azimuthal wind for times corresponding to the upper and lower tercile for each PC reveal the storm structure associated with these dominant patterns of variability for both large positive and negative EOF amplitude. For the first EOF, results show that the positive composite is associated with an inward shift of the radius of maximum wind and an increase in the radial gradient of azimuthal wind inside the eyewall, whereas the negative composite is associated

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### Table 1. Correlation between the first two EOF principal component time series and the maximum wind speed (max speed) and radius of maximum wind (RMW).

<table>
<thead>
<tr>
<th>Field</th>
<th>Max speed</th>
<th>RMW</th>
</tr>
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<tbody>
<tr>
<td>PC1</td>
<td>-0.02</td>
<td>-0.59</td>
</tr>
<tr>
<td>PC2</td>
<td>0.82</td>
<td>-0.37</td>
</tr>
<tr>
<td>PC1 upper tercile</td>
<td>0.46</td>
<td>-0.44</td>
</tr>
<tr>
<td>PC1 lower tercile</td>
<td>-0.19</td>
<td>-0.07</td>
</tr>
<tr>
<td>PC2 upper tercile</td>
<td>0.84</td>
<td>0.18</td>
</tr>
<tr>
<td>PC2 lower tercile</td>
<td>0.17</td>
<td>-0.40</td>
</tr>
</tbody>
</table>
with an outward shift and a decrease in the gradient (Fig. 4, top panel). In both cases, the composite storm is stronger than the mean, which suggests that weaker storms are associated with smaller PC values. Results for composites of the second EOF show that for positive (negative) events the storm is stronger (weaker) than the mean, consistent with the interpretation suggested previously (Fig. 4, bottom panel). The radius of maximum wind is diffuse, but displaced toward larger radius in the negative sample, consistent with the weak negative correlation described for the entire PC time series; the positive example exhibits little shift (Table 1).

The temporal evolution of variability associated with the leading two EOFs is described through lag regression of the azimuthal wind onto the PC time series. Results for PC-1 indicate that this structure is linked to inward propagating bands of weaker and stronger wind that originate more than 100 km from the storm center (Fig. 5); results for PC-2 are qualitatively similar (not shown). These bands travel radially inward at approximately 1–2 m s\(^{-1}\) until they reach about 50 km from the storm center, at which point they slow substantially to less than 0.5 m s\(^{-1}\). This pattern also appears for regression of the azimuthal wind onto the wind speed at the radius of maximum wind, and is also evident in Fig. 1. Note that the dominant time scale of \(\sim 6\) days is also apparent in the regression field.

Lag regression of the azimuthal wind at locations outside the storm provides a link between variability in the environment and near the storm. At a radius of 100 km, bands of stronger wind\(^2\) clearly propagate inward all the way to the eyewall (Fig. 6, top panel) similar to Fig. 5. In the outer environment, 600 km from the storm center, the bands propagate faster, at a rate of 5–10 m s\(^{-1}\), but vanish by a radius of 400 km (Fig. 6, bottom panel).

The vertical structure of the bands at these locations is determined by regressing the radial and vertical wind, water vapor mixing ratio, and azimuthal wind onto the 100- and 600-km azimuthal wind time series at the lowest model level (Fig. 7). At a radius of 100 km,

\(^2\) Although by linearity this discussion applies to either sign, we discuss only the positive case.
results reveal that stronger azimuthal winds at this location are associated with a deep layer of anomalously large mixing ratio and an overturning vertical circulation (Fig. 7a). Specifically, the results suggest a convective band located near 75–100-km radius, with rising air in the band and sinking air behind. Positive anomalous azimuthal wind is coincident with the band over a deep layer, with a maximum near 2–3-km altitude (Fig. 7b). Radial inflow at low levels near the region of positive azimuthal wind is expected to accelerate the azimuthal wind as the band moves radially inward. It appears that the band is interacting with the eyewall, enhancing the mixing ratio near a radius of about 25 km. Sinking motion is found at smaller radii, and also above an altitude of 2 km outside the band. Near the eyewall, subsidence is coincident with a narrow region of weaker azimuthal wind located radially inward from the convective band. The vertical circulation is qualitatively in accord with a balanced Eliassen secondary circulation, with a clockwise (counterclockwise) circulation where the radial gradient of anomalous mixing ratio, and presumably latent heating, is negative (positive); one exception to this agreement is the radial inflow region near 6–10-km altitude outside the band. Results for the regression onto lowest-model-level azimuthal wind at a radius of 600 km reveal a similar vertical structure (Figs. 7c,d), except that the convective band and associated vertical circulation are shallower and wider. The deeper layer of anomalous mixing ratio ahead of the band and shallower layer behind the band is suggestive of a convective band with trailing stratiform precipitation (Fig. 7c). A further distinction from the results at a radius of 100 km is that the positive anomalous azimuthal wind is more vertically aligned with two distinct maxima: one at the surface and another at 6-km altitude (Fig. 7d).

Fig. 6. Time-lag regression of the azimuthal wind onto the azimuthal wind at the lowest model level and a radius of (top) 100 and (bottom) 600 km. Positive (negative) values are given by black (gray) lines, with contours every (top) 1 and (bottom) 0.2 m s\(^{-1}\) starting at 0.4 m s\(^{-1}\); the 0 contour is omitted. Black dots denote the location of the base point for the regression.

Another perspective on storm variability is provided by the time series of the radius of maximum wind (Fig. 8). This field shows frequent jumps from around 20–30 km to much larger values before returning again to smaller values. These jumps are objectively identified after day 20 by finding times when the radius of maximum wind increase by at least 30 km over a 15-min period. A second requirement is that any jump must be separated by at least 6 h from other events, which leaves a total of 131 events. Composite averages of these events show that after the initial outward jump in the radius of maximum wind by about 45 km, it becomes smaller at a decreasing rate, returning close to the original value after about 2 days (Fig. 9, top panel). For 2 days prior to the time of the jump in the radius of maximum wind, the storm slowly weakens, reaching minimum intensity shortly after the radius of maximum wind jumps to larger radius (Fig. 9, bottom panel). The maximum wind rapidly increases after reaching the minimum, returning after about 24 h to values observed nearly 2 days before the jump in the radius of maximum wind. This asymmetric response suggests slow weakening of a primary eyewall, followed by a rapid intensification of its replacement, qualitatively similar to what is observed in eyewall replacement cycles in real storms, although somewhat slower (e.g., Willoughby et al. 1982).

4. Predictability

Although there are many approaches to investigating the predictability of the axisymmetric storm, we continue with the statistical theme of this paper by sampling from the long equilibrium simulation to
estimate predictability time scales and structures that control predictability. One simple measure of predictability is the autocorrelation time scale. Near the radius of maximum wind, the autocorrelation drops to 0 around 42 h, reaching a minimum around 4 days before becoming weakly positive again (Fig. 10, top panel). This behavior is consistent with the average eyewall replacement cycle period of around 4–8 days. The

Fig. 7. Regression of (a) radial and vertical wind (arrows) and water vapor mixing ratio (colors; g kg⁻¹), and (b) the azimuthal wind, onto the azimuthal wind at the lowest model level at a radius of 100 km. (c),(d) As in (a),(b), but for regression onto the azimuthal wind at the lowest model level at a radius of 600 km. Vectors are shown only at points where the regression of both components of the vector are significant at the 95% confidence level. Azimuthal wind contour interval is 0.25 m s⁻¹ (thick, ±0.5 m s⁻¹) in (b) and 1 m s⁻¹ (thick, ±3 m s⁻¹) in (d); negative values shown in gray.

Fig. 8. Time series of radius of maximum wind (km).
autocorrelation e-folding time reaches a maximum inside the eye around 33 h, with a relative minimum of 15 h in the range 30–50-km radius, and an increase to a secondary maximum around 21 h centered near a radius of 100 km (Fig. 10, bottom panel). A similar behavior is obtained for the time when the lower bound on the autocorrelation 95% confidence range reaches 0, except that outside the eyewall this time scale increases from about 33 h near the radius of maximum wind to 90–100 h beyond a radius of about 140 km.

A more direct estimate of predictability time scales involves solving for error growth over a large number of initial conditions. Here we construct a linear model for this purpose, derived from the statistics of the long simulation (e.g., Penland and Magorian 1993). In general, the linear model can be written

\[
\frac{dx}{dt} = Lx, \tag{1}
\]

where \(x\) is the state vector of model variables and \(L\) is a taken here as a constant matrix that defines the system dynamics. The solution of (1) over the time interval \(t: t + \tau\) is

\[
x(t + \tau) = M(t, t + \tau)x(t), \tag{2}
\]

where \(M(t, t + \tau) = e^{L\tau}\) is the propagator matrix that maps the initial condition \(x(t)\) into the solution vector.
\( M(t, t + \tau) = \{ x(t + \tau)x(t)^T \} \{ x(t)x(t)^T \}^{-1}. \) (3)

Superscript \( T \) denotes a transpose, and braces denote an expectation, which is estimated by an average over a “training” sample consisting of 500 randomly chosen times out of a total of 3760 during the last 470 days of the simulation. Once determined, the propagator matrix is then applied to an independent sample consisting of 100 randomly chosen times that do not overlap with the training sample. This process is repeated over 100 trials to provide a bootstrap error estimate for the calculation.

Four prognostic variables are included in the linear model: azimuthal wind \( v \); radial wind \( u \); potential temperature \( \theta \); and cloud water mixing ratio \( q \). To reduce dimensionality, the gridpoint values of these variables are projected onto the leading 20 EOFs in each variable over all vertical levels from the origin to a radius of 200 km. In terms of the fraction of the total variance accounted for by these EOFs, the leading 20 EOFs capture 90% for \( v \), 84% for \( \theta \), 73% for \( u \), and 52% for \( q \). This approach reduces \( M \) to an 80 by 80 matrix.

Forecast error is summarized for each forecast lead time \( \tau \) by computing the error variance in gridpoint space

\[ e = x(t + \tau) - x_\tau(t + \tau). \] (4)

where \( x_\tau(t + \tau) \) is the projection of the true state onto the EOF basis. For each variable, at each time, the spatial variance in \( e \) is computed, leaving a time series of error variance. This error variance is normalized by the variance in \( x_\tau \), which means that skill is lost when the normalized error variance reaches a value of 1. The mean and standard deviation over the 100 trials as a function of \( \tau \) are summarized in Fig. 11. Results show fastest error growth in cloud water mixing ratio, with error saturation at 15 h (Fig. 11d). Error growth in radial wind is similar for the first 6 h, and then becomes much slower, reaching saturation around 36 h (Fig. 11a). The slowest error growth is observed in the azimuthal wind.
field, which grows linearly for the first 18 h, and then slowly approaches the saturation value around 48 h (Fig. 11b). A similar behavior is observed in the potential temperature field, but the initial growth is larger in $u$ (Fig. 11c). The summary interpretation of these results is that convective motions are most strongly evident in the cloud water and radial wind fields, and these fields lose predictability the fastest, whereas the slow response is in the azimuthal wind field, which responds through the secondary circulation. The azimuthal wind and potential temperature fields should couple strongly through the thermal wind equation except near convective and other unbalanced motions, so we hypothesize that the relatively larger errors for $\theta$ compared to $\psi$ at smaller $t$ are related to convection.

To test the effect of the linear assumption in these calculations, an independent predictability estimate is derived from solution divergence rates estimated by analogs in the 500-day simulation. The metric for comparing states is the summed squared difference in azimuthal wind over all vertical levels from the storm center to a radius of 200 km. The closest analog state is defined for each time by the state with minimum metric that is also separated in time by at least 4 days. Taking one state to be the “truth” and the other to be the “forecast” allows error statistics to be determined as defined above for the linear inverse model. Results show that the initial errors are larger than for the linear inverse model, reflecting the finite resource of available analogs, but the growth rate at this amplitude is similar (Fig. 12). The analog results lack an error estimate, but it appears that the predictability limit may be longer, by perhaps 12 h, than in the linear inverse model.

To put these results in context, operational forecast errors from the National Hurricane Center (NHC) are provided for comparison (Fig. 13), normalized by the errors at 84 h. Compared to the linear inverse modeling (LIM) results, the NHC errors follow a similar growth curve with somewhat slower growth, and saturate approximately 12–24 h later, similar to the analog results. Although the close comparison between the LIM results and NHC forecast errors suggests that intrinsic storm predictability may be an important factor in operational intensity prediction, further research is needed to make this link more than circumstantial.

Another aspect of interest in predictability concerns the initial-condition errors that evolve into structures that dominate forecast-error variance. Here we objectively identify initial conditions that optimally excite the leading EOF of the forecast-error variance in the linear inverse model, subject to the constraint that the initial structure is drawn from a distribution of equally likely disturbances that are consistent with short-term error statistics. The theory for this calculation, described completely in Houtekamer (1995), Ehrendorfer and Tribbia (1997), and Snyder and Hakim (2005), begins with a measure for the size of forecast errors relative to initial-condition errors:

$$\lambda = \frac{\langle \mathbf{x}(t + \tau), \mathbf{x}(t + \tau) \rangle_{t+\tau}}{\langle \mathbf{x}(t), \mathbf{x}(t) \rangle_t}.$$  (5)

The initial-time inner product, $\langle \cdot, \cdot \rangle_t$, is defined by the probability density function for the initial errors $p[x(t)]$.
which we take to be Gaussian with mean zero and covariance matrix $B$ so that

$$\langle x(t), x(t) \rangle_t = x(t)B^{-1}x(t) = -\ln p[x(t)].$$

This relationship implies that the “size” of initial disturbances is determined by the covariance matrix for the initial errors, so that disturbances of equal size have equal probability. Making the substitution

$$x(t) = E\hat{x}(t),$$

where $E$ is a symmetric matrix that is a square root of $B$ (i.e., $B = EE$), (5) becomes

$$\lambda = \langle EM^TME\hat{x} \rangle_{t+\tau}.$$

Here we have assumed that the initial disturbances have equal probability. The eigenvector of $EM^TME$ having the largest eigenvalue gives the initial disturbance that evolves into the leading eigenvector of the forecast-error covariance matrix; that is, the structure that dominates the forecast error (Ehrendorfer and Tribbia 1997). For these calculations, the leading forecast-error EOF accounts for 20% of forecast-error variance. We calculate the leading eigenvector for a forecast lead time of 24 h, and use as a proxy for $B$ the 3-h forecast-error covariance; the “true” analysis-error covariance matrix requires an assimilation system, and depends on observations, which is beyond the scope of the current paper.

For brevity, results are summarized for only the azimuthal wind, which shows an initial structure with largest amplitude outside the storm, near 125-km radius and 5-km altitude (Fig. 14a). A second region of

![Figure 14](image-url)
enhanced azimuthal wind is found just outside the radius of maximum wind above the surface. This second region grows rapidly in time from about 1 m s⁻¹ to more than 24 m s⁻¹ by 24 h (Figs. 14b–d) for the (arbitrary) amplitude assigned to the initial condition. Further experimentation shows that the solution at 24 h is due mostly to the initial structure of \( \nu \) and \( \theta \) (not shown).

5. Summary and conclusions

We have defined intrinsic variability of tropical cyclones as the structure and intensity change independent of the environment in which the storm resides. Insight into intrinsic variability is useful for determining the internal dynamics of the storm, and also for predictability research, since the intrinsic variability controls forecast errors in the absence of environmental forcing. Here, leading-order estimates of intrinsic variability are estimated from a 500-day numerical simulation of an axisymmetric tropical cyclone begun from a state of rest. This framework is useful because, unlike simulations of real or idealized three-dimensional tropical cyclones, the results are independent of the initial disturbance, and large sample sizes are available to document variability and predictability.

Results for the axisymmetric solution show that variability in storm intensity, as measured by the maximum azimuthal wind speed, varies over 30 m s⁻¹. This variability is controlled by convective bands in the distant environment (\( r > 100 \text{ km} \)) that propagate inward and lead to eyewall replacement cycles. Bands in the distant environment are relatively shallow compared to bands closer to the storm center. Moreover, the radial motion of the bands is fastest in the distant environment, and slowest near the eyewall, consistent with expectations based on the role of inertial stability on the strength of the Eliassen circulation forced by radial gradients in latent heating. A composite of 131 eyewall replacement cycles indicates that the secondary wind maximum exceeds the primary nearly 50 km from the primary radius of maximum wind, and reaches the location of the primary maximum after approximately 24–36 h. The sample-mean maximum wind speed increases by about 13 m s⁻¹ during this period where the secondary eyewall contracts.

A dominant period of 4–8 days is evident for the convective bands and eyewall replacement cycles. We speculate that this time period is related to suppression of convection outside the bands because of subsidence from the balanced circulation forced by the band (see Fig. 7). In this case, a scaling for the time period where convection is suppressed is given by the ratio of a band-circulation length scale to the band speed,

\[
P_{\text{band}} \sim \frac{L}{c_{\text{band}}}.
\]

Based upon the regression calculation shown in Fig. 7, subsidence extends about 400 km from the band, while \( c_{\text{band}} \) is approximately 2 m s⁻¹, giving \( P_{\text{band}} \approx 55 \text{ h} \). Beyond this time, convection may begin to deepen and moisten the troposphere. Band formation in the simulation likely involves an upscale aggregation of convection, the study of which is beyond the scope of this paper.

Predictability time scales for the axisymmetric model are estimated by autocorrelation times and by linear inverse modeling, with similar results. Autocorrelation e-folding times for the azimuthal wind at the lowest model level are largest just inside the eyewall, at around 33 h, and smallest near and just outside the eyewall, at around 18 h; a tail of statistically significant, but small, autocorrelation hints at predictability time scales from around 24 h near the eyewall to 100 h in the far field. Linear inverse modeling provides a more quantitative measure of predictability. The azimuthal wind is predictable, on average, for at least 36–48 h, with marginal predictability to about 3 days. The radial wind and cloud water field are less predictable, with errors saturating around 24 h. The optimal initial condition that evolves into the leading EOF of the forecast-error covariance matrix has a weak convective band just outside the radius of maximum wind that rapidly develops, increasing azimuthal-wind forecast errors by a factor of roughly 24 near the radius of maximum wind. The optimal structure is also characterized by a maximum in azimuthal wind near 125-km radius and 5-km altitude, which grows more slowly than wind errors near the radius of maximum wind.

Although this work provides only a starting point from an axisymmetric framework, results for idealized three-dimensional storms yield similar patterns of variability and limits of predictability (Brown and Hakim 2013). An important limitation of this research involves the imposition of boundary conditions at finite radius. While the location of the boundary is important for storm size (D. R. Chavas and K. A. Emanuel 2012, unpublished manuscript), the sensitivity of variability and predictability is unknown and the subject of future research.

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