Variability and Predictability of a Three-Dimensional Hurricane in Statistical Equilibrium

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ABSTRACT

The internal variability and predictability of idealized three-dimensional hurricanes is investigated using 100-day-long, statistically steady simulations in a compressible, nonhydrostatic, cloud-resolving model. The equilibrium solution is free of the confounding effects of initial conditions and environmental variability in order to isolate the “intrinsic” characteristics of the hurricane.

The variance of the axisymmetric tangential velocity is dominated by two patterns: one characterized by a radial shift of the maximum wind, and the other by intensity modulation at the radius of maximum wind. These patterns are associated with convectively coupled bands of anomalous wind speed that propagate inward from large radii with a period of roughly 5 days, the strongest of which is associated with an eyewall replacement cycle. The asymmetric tangential wind is strongest radially inward of the radius of maximum wind. On average, asymmetries decelerate the azimuthal-mean tangential wind at the radius of maximum wind and accelerate it along the inner edge of eyewall.

Predictability of axisymmetric storm structure is measured through the autocorrelation e-folding time and linear inverse modeling. Results from both methods reveal an intrinsic predictability time scale of about 2 days. The predictability and variability of the axisymmetric storm structure are consistent with recently obtained results from idealized axisymmetric hurricane modeling.

1. Introduction

Official National Hurricane Center (NHC) forecasts and operational forecast models exhibit significantly more skill when predicting the future track of a tropical cyclone (TC) than when forecasting the future intensity (Elsberry et al. 2007). While the track of a TC is mainly determined by the synoptic-scale steering pattern, the representation and prediction of which has shown steady improvement, forecasting TC intensity is a multiscale problem in which the ambient environment is just one of several factors. The disparity in skill between forecasts of the large-scale environment and TC intensity suggests that a better understanding of the finer-scale, internal dynamics of the storm itself is required to accelerate improvements in TC intensity prediction.

Because of the highly axisymmetric nature of mature hurricanes, axisymmetric models have often been employed to investigate the dominant structure and dynamics while reducing the dimensionality of the system. This simplified framework represents well the distinctive primary and secondary circulations that define TCs, and enables theories for the mechanism of hurricane formation and predictions of maximum steady intensity of TCs (e.g., Emanuel 1986, 1988; Holland 1997). However, by design, an axisymmetric framework cannot simulate asymmetric motions, and steady-state models yield little insight on variability and predictability.

Asymmetric storm structure is relatively small in magnitude compared to the axisymmetric component but is inherent to all hurricanes in three dimensions (e.g., Houze 2010). Some studies of the effects of asymmetries in observed storms find that they can weaken storms (e.g., Black et al. 2002; Zehr 2003); however, these results can be difficult to generalize given that each storm is embedded in a unique environment and the effects of environmentally induced asymmetries are virtually impossible to separate from the effects of internally induced asymmetries that are intrinsic to the structure of three-dimensional tropical cyclones. Furthermore, the sample of observed mature storms is relatively small and, in the case of real
observations, the storm is sampled inhomogenously. Many of these issues may be addressed through the use of idealized, three-dimensional modeling: fields may be sampled uniformly on a grid, the surrounding environment may be set to quiescent conditions, external forcing may be damped or eliminated, and as many samples as desired may be simulated.

A hierarchy of idealized models has been used to investigate the role of asymmetries in hurricanes, with differing results. Montgomery and Kallenbach (1997) suggest that the axisymmetrization of asymmetries accelerates the tangential wind and propose this wave–mean-flow interaction as a spinup mechanism for developing storms. Möller and Montgomery (1999) also find that asymmetries transfer energy to the basic (axisymmetric) state and strengthen the storm in a barotropic model. Möller and Montgomery (2000) use a three-dimensional balance model to study the effect of convective bursts using double-cluster, pulsed potential vorticity (PV) perturbations and find that the resulting asymmetries strengthen the storm. Nolan and Montgomery (2002) and Nolan and Grasso (2003) apply three-dimensional, nonhydrostatic perturbations to TC vortices of varying strength and, in contrast to earlier work, find that the purely asymmetric disturbances weaken the symmetric vortex. While Chen and Yau (2001), Chen et al. (2003), and Rozoff et al. (2009) conclude that vorticity mixing due to asymmetries, through a variety of mechanisms, does intensify hurricanes, Yang et al. (2007) and Wang (2007) conclude that asymmetric motions cause three-dimensional simulated storms to be weaker than those simulated in two dimensions. Idealized investigations of environmentally induced asymmetries support the conclusion that asymmetries generally weaken hurricanes (e.g., Frank and Ritchie 2001; Riemer et al. 2010).

With accelerated improvements in data assimilation systems and increasing types and numbers of observations in TCs, it is important to investigate the role of the environment and internal dynamics in storm predictability in order to gauge the utility of new information from observations. Existing studies of hurricane predictability overwhelmingly focus on forecasts of the location and intensity of observed storms (e.g., Zhang et al. 2009; Reynolds et al. 2009). Environmental factors such as large-scale vertical shear, sea surface temperature variations, and synoptic weather patterns have significant effects on the intensity and structure of tropical cyclones (e.g., Emanuel et al. 2004; Wang and Wu 2004). Numerous environmental factors affect observed storms, making it difficult to determine TC intrinsic predictability, which we define as the predictability independent of external influences. This issue has been considered for idealized developing storms by Van Sang et al. (2008), who create an ensemble of idealized TCs free of environmental forcing by randomly perturbing the moisture field of an intensifying storm. Differences in the azimuthal-mean wind speed of up to 15 m s⁻¹ at day 4 lead the authors to conclude that convective processes dominate the variability and that intrinsic predictability is virtually non-existent. The simulations of Van Sang et al. (2008) end at 96 h, with the assumption that storms are in “quasi-steady state” after 72 h—which, their conclusions apply only to the development stage of tropical cyclones. The question of intrinsic predictable time scales for mature, steady-state hurricanes is taken up in Hakim (2013), where an analysis of a long axisymmetric numerical solution reveals intrinsic predictable times scales of 2–3 days. In this paper, we use a fully three-dimensional model to test the generality of the predictability results obtained by Hakim (2013).

The remainder of the paper is organized as follows: Section 2 describes the numerical model and analysis methods used to produce two- and three-dimensional steady-state TC simulations. The intrinsic variability of the axisymmetric and asymmetric components is discussed in section 3, and the intrinsic predictability of the axisymmetric component is discussed in section 4. Finally, a concluding summary is provided in section 5.

2. Method

The model used here is release 15 of the compressible, nonhydrostatic cloud model of Bryan and Fritsch (2002) (CM1). This model has been designed in particular to explicitly simulate moist convective processes, making it well suited for the study of tropical cyclones. The axisymmetric setup is described in Bryan and Rotunno (2009), and although both three-dimensional (3D) and axisymmetric (AXI) simulations are considered here, the main emphasis is on the 3D simulation.

The domain for the 3D simulation consists of a single grid with a square region of 256 × 256 points with uniform grid spacing of 4 km (i.e., 1024 km × 1024 km) bordered on all sides by a region in which the horizontal resolution gradually increases to 36 km at the lateral boundaries to give a total domain grid size of 352 × 352 points covering a domain that is 2944 km × 2944 km. The vertical resolution is 250 m from a height of 0 to 1.25 km, and then gradually increases to a resolution of 1 km from a height of 10 to 25 km. To calculate the axisymmetric component of the 3D simulation, the center of the storm is found by locating the point of minimum wind at the first model level within a window centered on the minimum central pressure. In practice, the center of the storm is virtually stationary. The axisymmetric simulation is configured identically to the three-dimensional
simulation, except that the axisymmetric simulation does not have the lateral grid-stretching portion of the domain.

To reach and maintain statistical equilibrium, these long simulations require radiational cooling to offset condensational heating. As demonstrated in Hakim (2011), Newtonian cooling schemes fail to achieve equilibrium for assumed soundings that have neutral stability and, moreover, it is preferable to allow the solution to select the equilibrium thermodynamic profile through radiative–convective adjustment. Here we use the National Aeronautics and Space Administration (NASA) Goddard longwave scheme (Chou and Suarez 1994), and neglect shortwave radiation to avoid the complicating effects of diurnal and seasonal cycles. All simulations use the NASA Goddard version of the Lin et al. (1983) ice microphysics scheme. This scheme simulates the bulk effects of six water forms: water vapor, cloud water, cloud ice, rain, snow, and a “large ice” category that we take to be hail. The effects of heat dissipated by turbulent kinetic energy are included in all of the model simulations, in a manner similar to Bister and Emanuel (1998).

The CM1 surface layer scheme chosen here calculates the surface exchange coefficient for momentum from Deacon’s formula (e.g., Moss and Rosenthal 1975):

$$C_D = 1.1 \times 10^{-3} + 4 \times 10^{-5} (u^2 + v^2)^{1/2} \frac{1}{(1/2) \Delta z},$$

where $\Delta z$ is the vertical grid spacing. The surface exchange coefficient for enthalpy is set equal to the coefficient for momentum as in Rotunno and Emanuel (1987). This configuration is chosen for simplicity and for comparison with Hakim (2013) even though current studies suggest that $C_D$ should be larger and the ratio of the exchange coefficients should be smaller than 1 (e.g., Davis et al. 2008). The additional axisymmetric simulation described below suggests that the results presented here are not highly sensitive to the ratio of the exchange coefficients as long as the turbulent mixing lengths are chosen appropriately.

Unresolved motions are parameterized as a Smagorinsky scheme following Rotunno and Emanuel (1987), with horizontal and vertical eddy length scales of 1500 and 200 m, respectively. These values, recommended by Bryan and Rotunno (2009), are used in both 3D and AXI simulations, which yield a 3D solution with a relatively large mean radius of maximum wind compared to the AXI simulation. Though recent work indicates that the mixing lengths in observed storms may be smaller than these values, there is as yet no clear consensus on specific values for use in idealized modeling studies (Rotunno and Bryan 2012; Zhang and Montgomery 2012). Furthermore, larger mixing lengths are consistent with a larger ratio of surface exchange coefficients such as we use here (Bryan 2012).

In any event, the variability reported below appears insensitive to these details. An additional axisymmetric simulation with horizontal and vertical eddy length scales of 700 and 50 m, respectively, and the surface exchange coefficients following Donelan et al. (2004) (drag coefficient) and Drennan et al. (2007) (enthalpy exchange coefficient) yields intensity and variability (determined using empirical orthogonal function (EOF) analysis as described in section 3b) similar to the control AXI simulation (not shown).

All model simulations apply to an $f$ plane at 20°N in a quiescent environment, which is designed to be homogeneous in order to eliminate environmentally induced intensity changes. The surface boundary condition consists of a constant, horizontally uniform sea surface temperature of 26.3°C. The environment is initialized with a sounding free of CAPE as in Rotunno and Emanuel (1987) and there is no environmental flow. A quiescent environment is maintained by damping perturbations to zero in the $u$, $v$, and $w$ fields in a zone near the lateral boundaries. For the AXI simulations, this zone is located within 100 km of the domain boundary. For the 3D simulation, the inner edge of this zone is defined by a circle circumscribed inside a square located 100 km from the domain boundary. This quiescent environment provides a constant source of angular momentum, which is needed to maintain equilibrium, since air parcels lose angular momentum on inflow and conserve angular momentum on outflow as they circulate the closed domain.

In equilibrium, small CAPE (less than 100 J kg$^{-1}$) is found near the core of the storm, and CAPE increases with radius to between 800 and 900 J kg$^{-1}$ at radii greater than 200 km, for an undiluted air parcel raised from the lowest model level, which is qualitatively in accord with observations (e.g., Molinari et al. 2012).

A large and statistically steady dataset is required to make meaningful statements on the statistical significance of the findings. To this end, a 100-day integration is sampled every 3 h, giving a total of 800 samples for each simulation. All analyses are performed on the “quasi-steady” period of about 81 days (discussed in the next section), which excludes spinup, and leaves 649 samples.

### 3. TC variability and structure

#### a. TC intensity

After spinup and a transient high-intensity period, the simulations exhibit statistically steady behavior in the maximum tangential wind speed (Fig. 1). Note that the axisymmetric wind component is shown for the 3D simulation in the interest of comparison with the AXI
Simulation. For brevity, unless otherwise specified, “tangential wind” is used to refer to the axisymmetric tangential wind speed. From hour 450 (about 19 days) onward, which we define to be the steady period, the mean tangential wind speed is 53 (AXI) and 52 m s$^{-1}$ (3D), and the mean minimum central pressure is 940 (AXI) and 944 hPa (3D). The transient period early in the simulation with winds near 70 m s$^{-1}$ occurs while the storm circulation expands and comes into equilibrium with the environment [see Hakim (2013) for a more detailed diagnosis of this transient period].

Interestingly, the mean intensity during the steady period does not differ appreciably between the AXI and 3D simulations. These comparable intensities are in contrast to previous work comparing three-dimensional and axisymmetric numerically modeled hurricanes such as Yang et al. (2007), who found that the azimuthal-mean tangential wind of a three-dimensional simulation was 15% less than for a similarly configured axisymmetric simulation. One possibility is that the relatively strong horizontal turbulent diffusion used here may damp asymmetries that would otherwise weaken the three-dimensional circulation.

b. Axisymmetric variability

The time-mean, azimuthal-mean radial structure of the primary circulation at the lowest model level (125 m) of the 3D simulation shows a canonical hurricane structure, with weak tangential wind at small radii, a sharp increase to the maximum tangential wind in the eyewall, and then a more gradual decrease of the tangential wind toward larger radii (Fig. 2). The area of highest temporal variability (defined by the standard deviation of the azimuthal-mean tangential wind) is located just inside the time-mean radius of maximum wind (RMW), where small shifts of the large radial gradient dominate the variability. The asymmetric variability is defined by the standard deviation in azimuth of the asymmetric tangential wind—that is, $(\overline{r'^2})^{1/2}$, where the overbar indicates the azimuthal mean and a prime denotes the deviation from the azimuthal mean. In this manner, we obtain a measure of the asymmetric structure at the lowest model level that varies only in radius and time (Fig. 2, solid line). The amplitude of the asymmetries reaches a maximum at a radius of about 40 km, on the inside edge of the RMW, and drops sharply at the RMW.

About 80% of the variance in the lowest-model-level tangential wind field is accounted for by the leading two EOFs of the azimuthal-mean wind for both 3D and AXI simulations (Table 1). The EOFs have largest amplitude in the region of the eyewall where the variability in the axisymmetric component is strongest (Fig. 3). To account for different storm sizes, the EOFs of 3D and AXI

| Simulation | EOF1 (%) | EOF2 (%) | $V_{\text{max}}$ (m s$^{-1}$) | $P_{\text{min}}$ (hPa) |
|------------|----------|----------|-----------------|----------------|}
| AXI        | 59       | 24       | 53              | 940            |
| 3D         | 63       | 16       | 52              | 944            |
are plotted against radius normalized by the time-mean axisymmetric radius of maximum wind (54 and 34 km, respectively). The leading EOF (EOF1) peaks inside the RMW, crosses zero near the RMW, and is negative outside the RMW, suggesting that this pattern represents a radial shift of the RMW. The second EOF (EOF2) peaks at the radius of maximum wind and crosses zero on either side of the RMW, suggesting that this pattern represents intensity modulation at the RMW. The leading two EOFs of the AXI simulation are qualitatively similar to those of the 3D simulations, although the shifting mode and the intensity modulation mode are not as distinctly separated; in particular, EOF1 is does not quite reach zero at the RMW (Fig. 3b). Nevertheless, by the test of North et al. (1982), the first five EOFs of the AXI simulation and the first four EOFs of the 3D simulation are statistically distinct from each other and from the remainder of the modes. Though these EOFs are only shown at the lowest model level, they exhibit the same basic structure and amount of variance explained throughout the boundary layer and into the midtroposphere. In previous axisymmetric modeling using release 14 of the CM1, Hakim (2013) found leading EOFs that closely resemble those presented here. It appears that these variability patterns are an inherent property of internal storm dynamics since there is no environmental or other external forcing in these simulations.

Since EOF analysis is an objective method designed to optimally partition variance, the structures it produces are not bound to have a physical interpretation. To confirm the interpretations assumed above for the leading axisymmetric EOFs, a composite analysis is performed for the 3D and AXI simulations. The principal component (PC) time series for each EOF (i.e., projection coefficients) are ordered into terciles by the value of the projection coefficients. The original data (tangential wind at the lowest model level) associated with the upper and lower terciles are then averaged to create a composite of the radial structure associated with the upper and lower terciles for each pattern. For EOF1, positive values (the upper tercile) are associated with a RMW that is located radially inward of the mean-state RMW, and negative values (the lower tercile) exhibit a RMW that is outside of the mean-state RMW (Figs. 4a,c). Results for EOF2 reveal maximum wind speeds that are greater (less) than the full-sample mean for the upper (lower) tercile, while both have a RMW located at roughly the same radius as the mean (Figs. 4b,d). These composites are consistent with the interpretations described above; EOF1 (EOF2) does indeed represent radial shifting (intensity modulation), with the upper and lower terciles of the data leading to a 10% deviation from the time-mean value. The composite analysis also reveals that, although the leading EOFs are nonzero at large radii, there is very little difference in the radial profiles of the composite terciles at large radii. This is explained by the fact that the simulations exhibit very little total variability at these radii (cf. Fig. 2).

We consider now the temporal evolution of the variability associated with the leading EOFs through lag correlation. By definition, the PC time series are orthogonal (i.e., have zero correlation) for zero lag, but for nonzero lag times, the relationship between the EOFs appears to be one of quadrature, with the second EOF leading the first by about 24 h (Fig. 5). This result implies that an increase (decrease) in intensity precedes an inward (outward) shift of the RMW. Regressing the axisymmetric tangential wind at each radius onto the PC time series reveals that the EOFs are linked to inward-propagating bands of anomalous azimuthal wind that originate at large radii (Fig. 6). This behavior is particularly apparent in the regression of PC2 where bands of enhanced wind form at radii of 120–140 km and move inward at 1–1.5 m s$^{-1}$ until nearing the RMW, at which point they slow considerably (Fig. 6b). The dominant time scale of these bands, as well as the pulsing at the eyewall associated with PC1 (Fig. 6a) is roughly 5 days. Propagating bands of weaker and stronger wind are also apparent in the analysis of Hakim (2013), who found a comparable time scale and similar structure. A similar regression analysis on the azimuthal-mean vertical motion
reveals similar propagating band features, indicating that bands of stronger tangential wind are associated with enhanced vertical motion (Figs. 6c,d).

To assess the possibility that propagating band structures are convectively coupled, the azimuthal-mean vertical velocity, the azimuthal-mean radial velocity $u$, and the azimuthal-mean water vapor mixing ratio $q$ are regressed onto PC2 as a function of lag time ($2^12$, $2^24$ and $2^36$ h). Results show that the band is collocated with an area of convergence in the boundary layer, rising air in the troposphere, and enhanced water vapor (Fig. 7). This band is flanked by regions of anomalous subsidence and dry air. In particular, subsidence and anomalously dry air extend more than 100 km radially outward from the band, suggesting that these circulations are forced by the secondary circulation induced by the radial heating gradient in the band (e.g., Shapiro and Willoughby 1982; Schubert and Hack 1982). We conclude that the band features associated with the leading modes of axisymmetric variability represent convective bands that propagate slowly radially inward because of the secondary circulation.

The radially propagating bands are apparent in a radius–time depiction of the tangential wind (Fig. 8a), and even more clearly when the data are limited to the projection onto the first two EOFs (Fig. 8b). At 1875 h, an eyewall replacement cycle (ERC; e.g., Willoughby et al. 1982) occurs concurrently with an enhanced band.
of tangential wind in the EOF projection field. Although this is the only case of an ERC identified in the 3D simulation, numerous other similar events (e.g., around 2000 and 2130 h) are apparent, which differ only in that they are not strong enough to form a clear secondary wind maximum. The ERC in the full tangential wind field slightly leads the associated feature in the truncated EOF space, which shows that not all of the variance is captured by the leading two modes. As with observed eyewall replacement cycles, the mechanism by which the propagating band features in the 3D simulation are excited is uncertain, but in this simulation the process is apparently linked to convection that organizes several hundred kilometers from the eyewall.

c. Aggregate role of asymmetries

As was seen in Fig. 2, the asymmetries have a maximum amplitude at a radius of about 40 km, just inside the
axisymmetric RMW. To quantify the effects of the asymmetries on the azimuthal-mean flow, the equations of motion can be transformed into storm-centered cylindrical coordinates (e.g., Bryan and Rotunno 2009). Partitioning the full momentum budget into contributions from the azimuthal mean and asymmetries gives

\[
\frac{\partial \mathbf{u}}{\partial t}_{\text{mean}} = - \left( \frac{\partial \mathbf{u}}{\partial r} + \frac{\partial \mathbf{v}}{\partial z} \right) - \left( f \mathbf{u} + \frac{\mathbf{u} u}{r} \right) + \mathcal{F} \quad \text{and} \quad (2)
\]

\[
\frac{\partial \mathbf{u}}{\partial t}_{\text{eddy}} = - \left( \frac{\partial \mathbf{u}}{\partial r} + \frac{\partial \mathbf{v}}{\partial z} + \frac{\mathbf{u} \mathbf{v}}{r} \right), \quad (3)
\]

respectively. Here, \( r \) is radius, \( z \) is height, and \( \mathcal{F} \) represents the combined effects of friction, diffusion, and subgrid turbulence. The time-mean contributions of the mean flow [right-hand side of (2), excluding \( \mathcal{F} \)] and the eddies [right-hand side of (3)] are shown in Figs. 9a and b. Since \( \mathcal{F} \) is not easily calculated in cylindrical coordinates directly from the model output, it is inferred from the time mean as a residual (Fig. 9c).

The mean-flow contributions involve an acceleration of the tangential flow in the eyewall due to boundary layer inflow and deceleration above (Fig. 9a), consistent with previous numerical modeling results (e.g., Wu and Braun 2004) and observations (e.g., Bell and Montgomery 2008). These tendency patterns are opposed by eddies and the contributions from the parameterized effects of diffusion, subgrid turbulence, and friction (Figs. 9b,c). The contribution from the eddies is an order of magnitude smaller than the dissipation terms in the boundary layer. Above the boundary layer, the magnitude of the acceleration by the eddies is comparable to the dissipation terms, the sum of which balances the deceleration by the mean flow along the sloping inner edge of the eyewall. Again, these results are consistent with previous hurricane modeling studies of linear momentum and related relative angular momentum diagnostics (e.g., Yang et al. 2007; Wang 2002; Kepert and Wang 2001).

For the AXI simulation, the effect of mean flow is similar to that in the 3D simulation, with acceleration in the boundary layer and weak deceleration above (Fig. 10). Differences in the location of local maxima in tendency between 3D and AXI simulations are due to storm structure; that is, the AXI simulation has a smaller RMW than the 3D simulation, although eyewall tilt is similar. Even though there are no explicitly resolved asymmetric motions in the AXI simulation, the parameterized diffusion
and subgrid turbulence have the same effect as in three dimensions; that is, their contributions balance the acceleration by the mean flow.

4. Predictability

Here we explore the predictability of the axisymmetric component of the flow. Results from this research provide a baseline estimate of the intrinsic predictability to which the environment may add or subtract.

The autocorrelation $e$-folding time provides a simple measure of predictability in terms of the average persistence of information. For the azimuthal-mean tangential wind at the lowest model level, the maximum autocorrelation $e$-folding time for the 3D and AXI simulations is found in the eye, with values over 100 h in the AXI simulation and 45 h in the 3D simulation (Fig. 11); all values are significant at the 95% confidence level. The eye region has both the lowest wind speeds and lowest wind speed variance, and the lower $e$-folding times in the eye may be due to asymmetric motion present in the three-dimensional configuration of the 3D simulation that is absent from the AXI simulation. Of greater interest are the much smaller $e$-folding times found in the...
eyewall and outer rainband regions. Minimum e-folding times of 22 and 25 h are found near the RMW in the 3D and AXI simulations, respectively; slightly longer times are found at larger radii. The pattern of long e-folding time in the eye, minimum e-folding time at the RMW, and recovery of e-folding time at large radii is similar to the radial structure of tangential wind autocorrelation e-folding time found by Hakim (2013). Similar results apply at a height of 1.4 km, where the time-mean, azimuthal-mean tangential wind is strongest (Fig. 11, dashed lines).

A more sophisticated method of estimating the axisymmetric predictability involves evaluating many forecasts using a model based on the equilibrium statistics. Here we approximate the evolution of the axisymmetric fields linearly through an inverse method (e.g., Penland and Magorian 1993). To reduce the dimensionality of the problem, the axisymmetric fields are projected onto a truncated EOF basis consisting of the leading 10 modes on the radius–height plane for all vertical levels and radii from the center to 200 km (variables used for the EOF basis are described below). The model may be expressed by

$$\frac{dx}{dt} = Lx,$$

where x represents the EOF basis and L a constant matrix that defines the dynamics in the EOF basis linearized around the time-mean state. Solving (4) over time $t$ to time $t + \tau$ gives

$$x(t + \tau) = M(t, t + \tau)x(t).$$

The matrix $M(t, t + \tau) = e^{L\tau}$ is calculated empirically from the equilibrium statistics of the full simulation. We use the least squares approach described in, for example, Hakim (2013):

$$M(t, t + \tau) = [x(t + \tau)x(t)^T][x(t)x(t)^T]^{-1},$$

where superscript T denotes a transpose. Braces indicate an expected value, which is estimated using an average over a “training” subset of data that consists of 500 randomly selected samples. Once matrix $M(t, t + \tau)$ has been calculated, the linear model is used to compute forecasts of the remaining independent sample of 149 initial conditions.
Variables used in the axisymmetric linear model consist of the radial ($u$) and azimuthal ($y$) wind, potential temperature ($u$), and cloud water mixing ratio ($qc$) of the 3D simulation. The variance explained by the first 10 EOFs in $u$, $y$, $u$, and $qc$ is 90%, 93%, 94%, and 88%, respectively. Forecast error is determined by taking the difference of the forecast, $x(t + \tau)$, and the projection of the true state onto the EOF basis at the forecast time; predictability is measured by the normalized spatial variance of this forecast error. Normalization applies to the error at long lead time, when the forecast-error variance reaches that of a randomly drawn sample of states from the equilibrium solution. Results discussed below represent an average over 149 forecasts. To ensure statistical robustness, the linear inverse process is repeated 50 times in order to obtain a bootstrap estimate of the error in the calculation.

The most rapid error growth is apparent in the cloud water and radial wind fields, reaching half of their predictability limit at about 10 h and a predictability limit at 30–38 h (Figs. 12a,d). These fields may be more strongly influenced by convection as compared to the azimuthal wind and potential temperature fields, which exhibit slower error growth, losing half of their predictability by 15–20-h forecast lead time, and a predictability limit around 2–3 days (Figs. 12b,c). These results are qualitatively in agreement with similar axisymmetric solutions presented by Hakim (2013).

The predictability of specific, point values in the 3D simulation can be determined from this linear inverse modeling method by projecting the forecast and “truth” truncated EOF bases back on to the original data. Here we focus on the maximum tangential wind and the radius at which it is located, which are quantities often associated with operational forecasts as representative metrics of storm intensity and size. Figure 13 shows that the error variance in this idealized simulation reaches saturation at 48 h for the maximum tangential wind and 42 h for the RMW. These predictability limits are similar to the predictability limit of the tangential wind field as a whole.

5. Summary and conclusions

We have investigated the variability of and interaction between the axisymmetric and asymmetric components of simulated idealized three-dimensional hurricanes. Intrinsic variability is defined here as the component of storm variability that is independent of the environment. Intrinsic variability provides a baseline for understanding how environmental forcing affects internal storm dynamics. We estimate the intrinsic variability with 100-day numerical simulations of a storm in a homogeneous environment. The primary simulation is a fully three-dimensional, stationary hurricane, with an axisymmetric simulation for comparison.

Results show a dominant axisymmetric flow in the three-dimensional simulations, which varies by about 20 m s$^{-1}$ at the RMW during the equilibrium period. Two patterns account for about 80% of the variance in the azimuthal-mean tangential wind: 1) a radial shift of
the RMW and 2) intensity modulation at the RMW. These interpretations are consistent with a composite analysis of the corresponding principal component time series, which also shows that the intensity modulation pattern leads the radial shifting pattern in time. The EOF structures are associated with convective bands that originate in the distant environment and propagate toward the center of the storm at roughly 1.25 m s$^{-1}$, slowing as they reach the eyewall region. These propagating bands have a period of about 5 days and are shown to be convectively coupled. The projection of the two leading EOFs onto the axisymmetric tangential wind at the lowest model level shows that these bands are coincident with secondary wind maxima and one case of a full eyewall replacement cycle. The axisymmetric modeling study of Hakim (2013) revealed numerous, regular eyewall replacement cycles that were closely linked to propagating band features similar to those found here. We hypothesize that in three dimensions the axisymmetric variability of an idealized hurricane is manifested in anomalous banded features that are not as strong as the full eyewall replacement cycles in Hakim (2013) yet represent a similar mechanism.

These results imply that the leading modes of axisymmetric variability are excited by a process outside of the RMW that initiates an inward-propagating disturbance. However, it would be premature to conclude that the intrinsic dynamics simply depend on variability outside of the RMW considering that the spatial and temporal resolution of the model used here prevents an analysis of inner-core phenomena such as vortex Rossby waves or convective bursts.

Asymmetric flow in the 3D simulation reaches a maximum just inside the RMW. In the time mean, the eddies decelerate the azimuthal-mean tangential wind at the RMW, and accelerate the flow along the inside of the eyewall. A budget for the azimuthal-mean tangential wind in the 3D simulation shows that statistically steady state is achieved in the hurricane boundary layer through a balance between the mean-flow and dissipation contributions.
with the eddies playing a relatively small role below a height of 2 km. Above the boundary layer, the eddy and dissipation terms contribute roughly equally to balancing deceleration by the mean flow. Idealized hurricane simulations are known to be sensitive to the mixing lengths that control turbulence closure schemes (e.g., Rotunno and Bryan 2012), but sensitivity to mixing lengths has been investigated mainly using axisymmetric models. Though computationally intensive, the sensitivity of the momentum tendency and resolved eddies to parameterized diffusion in statistically steady three-dimensional hurricanes is needed to test the generality of the results presented here.

Predictability is estimated using the autocorrelation $e$-folding time of the tangential wind and linear inverse modeling. The autocorrelation $e$-folding time is maximized in the eye region and minimized at or near the RMW. In the convectively active region of the eyewall, $e$-folding times are between 20 and 40 h. Linear inverse modeling for the 3D simulation gives a predictability limit of about 48 h, which is in rough agreement with the autocorrelation $e$-folding results (note that the inverse model results apply to a complete loss of predictive skill). Predictability of specific values that are of interest to operational forecasters (i.e., the maximum azimuthal-mean tangential wind and the radius of maximum wind) is found to be limited to around 42–48 h. It should be kept in mind that this limit applies to an idealized simulation in which, by design, the ambient environment provides no additional information to the forecast.

Though we find here, as in Hakim (2013), that intrinsic predictability is lost after about 2 days, operational forecasts for observed TCs are routinely made at lead times of 120 h. So-called statistical–dynamical models that aid these operational forecasts have demonstrated skill against climatology and persistence when using environmental predictors such as vertical shear and upper-level divergence (Kaplan and DeMaria 2003). These models have also recently included rough measures of the storm structure derived from satellite data. However, the inclusion of these measurements, which might be expected to project on the intrinsic aspect of predictability because they represent the inner core of the storm, is shown only to improve the statistical–dynamical forecasts to 48-h lead time (DeMaria et al. 2005). This suggests that, because of the limitations of intrinsic predictability, observations of the inner core of mature hurricanes may only be expected to improve forecasts at short (i.e., 24–48 h) lead times, whereas other types of observations farther from the core of the storm may supply additional skill at longer lead times. This speculation remains to be tested in future research.

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