1. INTRODUCTION

An important element in the current paradigm of extratropical cyclogenesis is the upper-level disturbance. Synopticians have long referred to these features as “short-wave troughs” and “vort maxes,” usually with regard to the height and vorticity fields near these features as they appear on the 500 hPa chart. Alternatively, on maps of the dynamic tropopause, these features appear as regions of relatively cold potential temperature and relatively high pressure (low height). This perspective illustrates the often large-amplitude mesoscale distortion of the tropopause and the accompanying localized region of large values of potential vorticity (PV). These properties suggest that these features may be better understood in terms of mesoscale nonlinear coherent structures (such as vortices), as compared to quasilinear Rossby waves. This hypothesis is addressed in the present work for a cyclogenesis event over northeastern North America by extracting the disturbance from the background flow using wavelet transforms (WT) and by assessing the degree to which this disturbance traps fluid parcels using trajectory calculations.

Considerable attention has been given to the wave interpretation of upper disturbances, both in theory (e.g., normal-mode life cycles) and in diagnosis of observations (e.g., time series analysis). While the vortex interpretation has been explored through idealized modeling (Takayabu 1991) and analytical (Mitsudera 1994) approaches, observational verification of these ideas is lacking, due in part to difficulty in extracting localized disturbances from the background flow (Takayabu 1991). This difficulty arises primarily because the traditional methods of extraction are based on thresholding (i.e., selectively partitioning) Fourier expansions of observed fields. Since the basis functions (sin, cos) are global, Fourier transforms of fields characterized by localized features “smear” these features in the transform domain. In contrast, the wavelet transforms use functions that are oscillatory but that decay rapidly to zero away from a local extremum. Localized disturbances therefore are not smeared by the wavelet transform and can be readily isolated by inversion.

Issues that can be addressed once the localized cyclogenesis precursor has been extracted from the background flow include: 1) scale and structure of the disturbance, 2) an estimate of the importance of nonlinearity, 3) the contribution of this disturbance to the induction of a surface cyclone, and 4) the interaction of this disturbance with other flow structures. In the present work, the following method is applied to a cyclogenesis event that occurred over eastern North America in order to address the first two issues: 1) extraction of the disturbance by WT of the quasigeostrophic potential vorticity (QGPV) field, 2) recovery of the three-dimensional structure by inversion of the QGPV field, and 3) tracking of fluid parcels near the tropopause to assess the degree to which they are trapped by the disturbance.

2. METHOD

The method by which the precursor disturbance is isolated is based upon the QGPV:

\[ q = \frac{1}{f_0} \nabla^2 \phi + f + \int \frac{\partial}{\partial p} \left( \frac{1}{\sigma_r} \frac{\partial \phi}{\partial p} \right). \]

Here, \( \phi \) is the geopotential, \( f \) the Coriolis parameter \( (f_0 = 10^{-4} \text{ s}^{-1}) \), and \( \sigma_r = -\frac{\partial q}{\partial p} \) is the static stability of the reference atmosphere. Primed quantities denote perturbations from the reference atmosphere.

To extract a disturbance, a discrete WT is applied to the \( q \) field. For simplicity, the subsequent discussion refers to a one-dimensional WT, whereas in the applications that follow a two-dimensional WT is used. Wavelet transforms expand a function in terms of two parameters: a “scale” or dilation parameter, \( a \), and a “position” or translation parameter, \( b \). The discrete WT in one dimension is given by (e.g., Daubechies 1992):

\[ W_q(m,n) = \langle q, \Psi_{m,n} \rangle. \]

Here, \( \Psi_{m,n}(x) = a_0^{-m} \Psi \left( \frac{x-nb_0a_0^m}{a_0} \right) \) is a scaled and dilated version of the real analyzing wavelet, \( \Psi \), for discrete values of \( m \) and \( n \) indexed by \( m \) and \( n \), respectively, and the brackets denote the standard real inner product. In this study, the analyzing wavelet is the second derivative of the Gaussian distribution (the “Mexican hat”). The wavelet coefficients, \( W_q(m,n) \), are large in the vicinity of the upper disturbance. This subset of large coefficients, denoted by \( W^*_{q}(m,n) \), is inverted to recover the disturbance portion of the \( q \) field.
Figure 1. Analyses at 500 hPa for 1200 UTC 6 January 1989: (a) perturbation geopotential height [solid, contour interval (CI) 60 m] and QGPV (dashed, CI 0.5 PVU), (b) disturbance QGPV (contour interval 0.5 PVU), (c) background (solid, CI 60 m) and disturbance (dashed, CI 15 m) perturbation geopotential height, and (d) q–ψ scatter plot and cubic-polynomial fit for normalized disturbance fields.

\[ q_d = C \sum_m \sum_n W_q(m,n) \Psi_{m,n} \]  

The constant \( C \) is related to the constants \( a_0 \) and \( b_0 \). The disturbance geopotential field is then recovered by an inversion of \( q_d \) subject to homogeneous Dirichlet boundary conditions.

Three-dimensional parcel trajectories are computed by the method of successive approximation described by Petterssen (1956, §2.4). Data for these calculations are taken from the four-dimensional data assimilation system of the European Centre for Medium-Range Weather Forecasts. These mandatory-level grids are interpolated to a 1° × 1° latitude-longitude horizontal grid and have 6 h temporal resolution.

3. RESULTS

The 500 hPa perturbation geopotential height and \( q \) fields for 1200 UTC 6 January 1989 are shown in Fig 1a. The disturbance of interest is located over the Ohio Valley within a predominantly westerly current that is confluent over the western North Atlantic Ocean. The disturbance QGPV field (Fig. 1b), obtained by the WT method, is clearly separated from the background flow (cf. Figs. 1a,b) and when inverted yields a nearly symmetric monopolar vortex (Fig. 1c). In the vicinity of the disturbance, the resultant background flow field is westerly at roughly 20 m s\(^{-1}\).

It is of interest to determine if the disturbance structure approximates any of the localized and steady solutions of the QG equations [e.g., solitary waves (weakly nonlinear) (e.g., Malanotte-Rizzoli 1982) and modons (strongly nonlinear) (e.g., Flierl et al. 1980)]. The \( q–\psi \) relationship for the disturbance in this case exhibits scatter about a cubic-polynomial fit, reflecting the fact that it is not steady in isolation (Fig. 1d). Most of the curvature (and scatter) comes from the exterior field \( (q < 0.25) \), while the interior field \( (q > 0.25) \) has less scatter and exhibits a nearly linear \( q–\psi \) relationship. The interior field approximates the linear \( q–\psi \) relationship for the analytical class of modon solutions. Specifically, the monopolar structure of the disturbance suggests that it may be a modon with a strong rider (i.e., an axisymmetric solution superposed on the dipolar vortex of the modon).

Significant nonlinearity is a characteristic of coherent structures that distinguishes them from linear Rossby waves. One measure of nonlinearity
is given by the ratio of advection to dispersion in the nondimensional QGPV equation. This nonlinearity parameter is \( \delta = \frac{U}{L} \), where \( U \) and \( L \) are the characteristic (maximum) eddy wind speed, length scale, and the background (planetary) QGPV gradient, respectively (Flierl 1977; Davey and Killworth 1984). From the disturbance fields, \( L \) approximated by the mean distance from the point of maximum geostrophic vorticity to the first zero crossing in this field, is 360 km, and \( U \), the maximum disturbance geostrophic wind, is 19 m s\(^{-1}\). These values, along with \( \beta = 1.7 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \), give \( \delta = 8.6 \). Approximating the total background QGPV gradient by averaging over the area of the eddy gives \( \delta \sim 4 \)—still very robust nonlinearity.

Another test of nonlinearity is given by tracking fluid parcels that reside within the eddy. Parcels escape from linear eddies and are trapped in nonlinear eddies. A swarm of parcel trajectories is determined for parcels originating on the dynamical tropopause (1.5 Ertel PV\( U \) surface) within the disturbance at 0000 UTC 7 January and tracked backwards for 48 h. This swarm is found to move with the eddy during this time period (not shown). A sample parcel in the swarm, plotted relative to the center of the eddy (defined as the mean position of parcels constituting the swarm), shows the parcel orbiting the eddy in a cyclonic manner and thus trapped by the eddy (Fig. 2).

![Figure 2. Parcel trajectory (48 h) relative to the center of the eddy, denoted by the "+" symbol. The indicated grid spacing is 50 km, and the large dot denotes the origin of the back trajectory.](image)

The swarm of parcel trajectories also permits determination of the speed of the disturbance. At 1200 UTC 6 January, the eddy is traveling eastward at 12 m s\(^{-1}\), or westward at 8 m s\(^{-1}\) relative to the 20 m s\(^{-1}\) westerly geostrophic background current. Since this speed falls in the range of linear Rossby wave speeds for midlatitudes, it suggests that the disturbance is likely not a modon or solitary wave, which have phase speeds that are complementary to those of linear Rossby waves.

4. CONCLUSIONS

Using wavelet transform and QGPV inversion techniques, the structure of an upper-level precursor disturbance to cyclogenesis is found to be that of a nearly symmetric monopolar vortex having a radius of 360 km. Although structural considerations (i.e., the \( q-\psi \) relationship) are suggestive of a modon-with-strong-rider analytical solution, the phase speed of the eddy is found to be in the range of linear Rossby waves. An estimate of the importance of nonlinearity based on the extracted disturbance fields indicates that it is quite significant. This result is confirmed by trajectory calculations, which show trapped parcels orbiting the core of the eddy. In sum, the results suggest an interpretation of this disturbance as a strong monopolar vortex traveling westward relative to the background current.

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REFERENCES


