3.1 TROPOPAUSE DYNAMICS BEYOND QUASIGEOSTROPHY
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1. INTRODUCTION

The tropopause represents an abrupt transition zone separating the well-mixed troposphere from the more-stable stratosphere. As a result of the relatively strong (weak) mixing in the troposphere (stratosphere), the stratification and potential vorticity (PV) are small (large). We are interested here with the dynamics of balanced wave motions supported by undulations in the tropopause. Motivation for understanding this problem derives from the recognition that tropopause undulations are important for extratropical weather, since these features produce organized patterns of vertical motion in the troposphere (e.g., Bluestein 1992). Here we explore the steadily propagating nonlinear wave solutions supported by the tropopause under the assumptions of a uniform-PV jump at the tropopause, constant wind shear on either side of the tropopause, and small Rossby-number dynamics.

Some primary attributes of tropopause disturbances are illustrated in Figures 1 and 2, which result from a composite average of the strongest quartile of maxima (minima) in the vertical component of cyclonic (anticyclonic) vorticity maxima (minima) over North America during December 1988-February 1989; there are 1681 cyclonic events and 1533 anticyclonic events. Further details on the composing method and an analysis of the cyclonic disturbances can be found in Hakim (1999). Plan views of pressure on the dynamic tropopause (defined here as the 1.5 × 10^{-6} m^2 K kg^{-1} s^{-1}, hereafter PVU, Ertel PV surface) illustrates an asymmetry between cyclones and anticyclones (Fig. 1). Mean tropopause pressure for the cyclone case reaches 480 hPa, an 168 hPa anomaly, whereas tropopause pressure for the anticyclone case reaches 230 hPa, an -67 hPa anomaly (Fig. 1). Zonal cross sections illustrate further the asymmetry between cyclones and anticyclones (Fig. 2). Cyclonic disturbances exhibit a localized depression of the tropopause with stratospheric values of potential vorticity extending to lower altitude. The PV depression is flanked by a dipole of vertical motion that extends deeper into the troposphere than the stratosphere. In contrast, anticyclonic disturbances exhibit a comparatively weaker upward deflection of the tropopause and a weaker dipole of vertical motion.

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2. METHOD

The tropopause is modeled here as a fluid system comprising two regions of constant PV which are separated by a free internal boundary—a material interface across which the discontinuity in PV is supported. The stratospheric and tropospheric regions are distinguished by differences in their mean background states which are defined by exact linear-shear solutions of the Boussinesq primitive equations (PE). Tropopause waves are disturbance solutions from this basic Eady (1949) state.

In addition to lateral periodicity, wave disturbances are also assumed to decay in the vertical away from the tropopause $z \to \pm \infty$. At the tropopause, the physical conditions to be satisfied are continuity of pressure and temperature, and a dynamical constraint on the interface. The disturbed tropopause is described as a surface which is displaced normal to the reference tropopause, $z_i = \epsilon h(x, y, t)$; where $\epsilon$ is the Rossby number. As noted in Rivest et al. (1992) and Juckes (1994), this weak $O(\epsilon)$ scaling of the tropopause displacement is a requirement for the temperature disturbances to be consistent with quasigeostrophic (QG) balance. Dimensionally, this means that this analysis for tropopause waves will be strictly valid only when the height of the tropopause disturbances is smaller than the Rossby height as defined by the ambient stratification.

Our analysis begins with a reformulation of the PE, similar to that in Muraki et al. (1999), which facilitates QG approximations to the PE for small $\epsilon$ asymptotics. The leading-order nonlinear wave solutions recover the QG results of Rivest et al. (1992), and at next order (QG+1) realistic cyclone-anticyclone asymmetries are discovered. A brief discussion of these results follows.

3. RESULTS

The tropopause height perturbation fields for the QG and QG+1 flat- (sloped-) tropopause solutions are shown in Figure 3 (Figure 4). Cyclonic and anticyclonic cells are symmetric for the QG wave, whereas the cyclonic (anticyclonic) cell is notably stronger (weaker) for the QG+1 wave. Furthermore, for the sloped solution, the largest height gradient is located on the southern portion of the cyclone, where the disturbance and ambient tropopause slope are in phase (cf. Fig. 1). A zonal cross section through the flat solutions highlights the profile of the disturbed interface, with an enhanced cyclone.
and a flattened anticyclone for the QG$^{+1}$ solution relative to the QG solution (Fig. 5); note the qualitatively similar structure in the observations (Fig. 2). Vertical motions are damped above the interface, and reach their largest magnitude below the interface for both QG and QG$^{+1}$ solutions. The QG$^{+1}$ solution exhibits slightly stronger vertical motions that are concentrated closer to the cyclone center when compared to the QG solution; the QG$^{+1}$ solution structure is qualitatively similar to the dipole of vertical motion noted in observations (cf. Fig. 4 with Fig. 2). This behavior may be explained by the constraints imposed by the dynamical interface condition, which requires greater vertical motions to elevate and depress the interface near the QG$^{+1}$ cyclonic interface deflection. The potential temperature lines in the troposphere bulge upward below the cyclone and downward below the anticyclone in a manner similar to observations (cf. Figs. 5 and 2), and as one would expect from PV reasoning based on local cyclonic and anticyclonic PV anomalies.

4. SUMMARY

Through an asymptotic approximation to the PE, tropopause dynamics one order beyond quasigeostrophy are recovered. Here we study the analytical nonlinear wave solutions supported by the next-order theory and find that these solutions compare favorably with observations. Specifically, the cyclone-anticyclone asymmetry noted in observations, and lacking in QG dynamics, is captured by the next-order solutions. The next-order theory for tropopause dynamics discussed here offers opportunities for further understanding of tropopause behavior; two examples concern time-dependent modeling of the interface in isolation, and baroclinic instability with a dynamically evolving tropopause.

REFERENCES

Figure 1. Plan views of tropopause pressure (contours every 25 hPa) for upper-tropospheric (top) cyclones and (bottom) anticyclones. Latitude and longitude are given by dotted lines every 10°.

Figure 2. Zonal cross section of Ertel potential vorticity (bold solid lines, every 0.25 PVU from 0.75 to 2 PVU; every 1 PVU thereafter; 1.5 PVU is highlighted by a heavy solid line) potential temperature (light dashed lines, every 5 K), and vertical motion (medium lines, every 0.25 × 10$^{-1}$ Pa s$^{-1}$; negative values dashed) through upper tropospheric (top) cyclones, and (bottom) anticyclones. The cross sections are approximately 3900 km wide and extend from 100 to 1000 hPa.
Figure 3. Plan views of tropopause height $h$ comparing (left) QG and (right) QG$^{+1}$ solutions for the flat tropopause basic state. Anticyclones (cyclones) are located in the upper (lower) portion of the figure. All units are nondimensional.

Figure 5. Zonal cross section of potential temperature and vertical motion comparing (top) QG and (bottom) QG$^{+1}$ solutions; the heavy solid line gives the location of the tropopause. Note that positive values for $w$ denote upward vertical motion, whereas the reverse is true in Fig. 2. All units are nondimensional.

Figure 4. Plan views of tropopause height $h$ comparing (left) QG and (right) QG$^{+1}$ solutions for the sloped tropopause basic state. Anticyclones (cyclones) are located in the upper (lower) portion of the figure. All units are nondimensional.