1. Implementation of the Lorenz model.

The mathematical formulation of the Lorenz turbulence model follows that described previously (Lorenz 1969; Rotunno and Snyder 2008). The curves in Fig. 1 are produced using Lorenz’s original dynamical formulation based on linearized barotropic vorticity dynamics with the saturation background kinetic energy spectrum given by (52) of Lorenz (1969). In contrast to previous studies (Lorenz 1969; Rotunno and Snyder 2008), a smooth approach to nonlinear saturation is enforced by solving the system of ordinary differential equations (Durran and Gingrich 2014)

$$\frac{dY_k}{dt} = \sum_{l=1}^{n} C_{kl} Z_l, \quad \frac{dZ_k}{dt} = \left( \frac{X_k - Z_k}{X_k} \right) Y_k.$$

Here \( n \) is the total number of spectral bands, and \( Z_k \) is the ensemble mean of the kinetic energy of the perturbation velocities about the ensemble mean, integrated with respect to \( \ln(k) \) over the spectral band at two-dimensional horizontal wavenumber \( k \). (The wavenumber is \( 2\pi \) divided by the wavelength.) \( X_k \) is the background kinetic energy spectrum again integrated with respect to \( \ln(k) \). \( Y_k \) is a dummy variable used to write a second order system of differential equations as a larger first-order system. Adjacent wave-numbers in our truncation differ by a factor of \( \sqrt{2} \), which is twice the spectral resolution used by Lorenz (1969) and Rotunno and Snyder (2008); we retain 40 wave-numbers. Following Lorenz (1969), the shortest retained wavelength is 38 m, the longest length scale is \( L = 40,000 \) km; the total integrated background kinetic energy is set to \( E = 148 \) m$^2$/s$^{-2}$, and the dimensional scale for time is \( LE^{-1/2} \). The MATLAB code used to run the Lorenz model is available from the authors.

2. The squall line simulations.

The numerical model solves the nonlinear, nonhydrostatic, compressible equations of motion together with a warm-rain microphysics parameterization (Durran and Klemp 1983); Coriolis forces are neglected. The numerical domain is periodic along both horizontal coordinates (thereby facilitating the spectral analysis) and spans a distance of 512 km in both the east-west coordinate \( x \) and the north-south coordinate \( y \). The upper boundary permits gravity-wave radiation (Klemp and Durran 1983; Bougeault 1983). The grid spacing is \( \Delta x = \Delta y = 1 \) km with \( \Delta z \) gradually increasing from 200 m near the surface to 500 m above 5 km. The model is integrated using a two-time-step algorithm in which the large time step is 3 s and the acoustic time step is 1.5 s.

The idealized atmospheric environment in which the squall line grows is similar to that in Weisman and Klemp (1982). A unidirectional horizontally uniform background wind profile is specified in which winds from the west increase from zero at the surface to 20 m s$^{-1}$ at a height of 5 km and remain 20 m s$^{-1}$ at higher levels. The background north-south wind component is zero. The vertical profile of potential temperature and relatively humidity (RH) for the horizontally homogeneous initial state is taken from (1) and (2) of Weisman and Klemp (1982), with a constant mixing ratio of 14 g kg$^{-1}$ between the surface and 1 km, except that the RH is capped at 90% for \( 1 \leq z \leq 1.4 \) km and at 75% at any level above 1.4 km where (2) would yield a higher value (Wandishin et al. 2008).

Three localized warm bubbles with positive temperature perturbations 2 K warmer than their environment trigger the initial updrafts, which owing to the wind shear, ultimately evolve into a line of thunderstorms (Skamarock et al. 1994). These bubbles are spheroidal with 2 K perturbations in their center, 10 km horizontal radius and 1.4 km vertical radius. Their centers are 1.4 km above the surface, and their \((x, y)\) coordinates in km are (100, 250), (150, 300) and (125, 175). Variations in the position and
strength of the squall line within each 25-member ensemble are produced by small-amplitude background perturbations in the surface temperature field that decay exponentially with height over an e-folding scale of 1 km. In terms of potential temperature $\theta'$, these perturbations have the form

$$\theta' = ae^{-z} \sin \left[ 2\pi \left( \frac{x}{L} - \phi_x \right) \right] \sin \left[ 2\pi \left( \frac{y}{L} - \phi_y \right) \right].$$

Here the phase of the perturbation in each ensemble member is determined by the random numbers $(\phi_x, \phi_y)$ in the interval $[0,1]$; $L$ is either 8 km in the small-scale ensemble or 128 km in the large-scale ensemble, and $x$, $y$ and $z$ are distances in km along each coordinate. The amplitude $a$ is set to 0.5 K for the 8-km-scale perturbations and is reduced by a factor of 1/4 to 0.125 K for the 128-km perturbations. The synthetic radar data displayed in Fig. 2 are computed from the simulated rain fields (Koch et al. 2005).

3. The KE spectrum.

The ensemble mean $KE$ spectrum for the squall line simulations was computed as horizontal. At a given vertical level, let $u_{i,j,m}$ and $v_{i,j,m}$ denote the zonal and meridional velocities at horizontal mesh point $(i,j)$ for ensemble member $m$. The discrete Fourier transform was applied along the $x$-coordinate to each $u_{i,j,m}$ and $v_{i,j,m}$. Denoting the transform of a function $\phi$ by $\hat{\phi}$, and the complex conjugate by $\phi^*$, the kinetic energy spectral density for each $j$ and $m$ was computed as

$$\hat{KE}_{j,m}(\hat{k}) = \frac{\Delta x}{2N_x} \left[ \hat{u}_{j,m}(\hat{k}) \hat{u}_{j,m}^*(\hat{k}) + \hat{v}_{j,m}(\hat{k}) \hat{v}_{j,m}^*(\hat{k}) \right].$$

Here $\hat{k}$ is the (one-dimensional) wavenumber along the $x$-coordinate ($\hat{k} = 2\pi/\lambda_x$, where $\lambda_x$ is the wavelength parallel to the $x$-axis), and $N_x = 512$ is the total number of grid points. Then, $\hat{KE}_{j,m}(\hat{k})$ was averaged over both $j$ and $m$ to give the ensemble- and meridional-averaged one-dimensional total kinetic energy spectrum, $KE$. The perturbation kinetic energy spectral density $KE'$ is calculated in the same manner as $KE$, except that $u_{i,j,m}$ and $v_{i,j,m}$ are replaced with $\hat{u}_{i,j,m} = u_{i,j,m} - \langle u_{i,j} \rangle$ and $\hat{v}_{i,j,m} = v_{i,j,m} - \langle v_{i,j} \rangle$, where $\langle \phi \rangle$ indicates the average of $\phi$ over all 25 ensemble members. Only those wavelengths greater than $7\Delta x$ are shown in Fig. 3; $7\Delta x$ is the (slightly arbitrary) cutoff scale beyond which Skamarock (2004) found numerical dissipation excessively reduced the $KE$ in mesoscale numerical models. The normalized perturbation kinetic energy plotted in Fig. 5 is computed as

$$KE'_n(\hat{k}) = Ak^{-5/3} \frac{KE'(\hat{k})}{KE(\hat{k})}$$

where $A$ is a constant mapping the result to the observed mesoscale background $KE$ spectrum.

References


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1 When 0.5 K amplitude perturbations were specified for the 128-km case, their influence was so strong that the comparison with the 8-km case became difficult. The factor of 1/4 reduction in amplitude for the 128-km case was adopted, without testing other possible values, because it gave similar ensemble error evolutions at 8 and 128 km. The sensitivity of our results to different values of $a$ is currently under investigation.