ATMS 321: Science of Climate  Mid Term Exam  7 May 2014
Science of Climate

Atmospheric Sciences 321
Mid-Term Examination: Closed Book 50 minutes (110 pts total)
Show all your work on these sheets, use back if necessary. Include units.
Put a box around your final answer where numerical answers are required.
Data and formulas on page 6.

Your Name: Dennis Hartmann (10pts)

Multiple Choice: (5 points each)

1. What is the equilibrium Bowen ratio in the Tropics, where the surface temperature is about 300K?
   a) 6.0, b) 3.0, c) 1.0, d) 0.3, e) 0.05
   Answer: d

2. If the ocean temperature rises by 1°C, by how much does the saturation vapor pressure increase?
   a) 20%, b) 10%, c) 7%, d) 3%, e) 1/273.12
   Answer: c

3. Which is the most important greenhouse gas?
   a) CO₂, b) N₂O, c) O₃, d) CH₄, e) H₂O
   Answer: e

4. The global annual average downward longwave radiation the surface is?
   a) 396 Wm⁻², b) 345 Wm⁻², c) 240 Wm⁻², d) 160 Wm⁻², e) 80 Wm⁻²
   Answer: b

Problems: Points as marked

1. The net radiative heat balance at the top of the atmosphere over the region north of 60N is a loss to space of 100 Wm⁻². Calculate the northward heat flux across 60N in Joules per second that is necessary to balance this radiative heat loss and maintain a steady equilibrium. Hint: use area formula on last page, convert Wm⁻² to Watts. \( \sin(60°) = 0.866 \) (15pts)

\[
W = \text{Wm}^{-2} \times \text{Area} = 100 \text{Wm}^{-2} \times 2\pi a^2 \left(1 - 0.866\right) = 160 \text{Wm}^{-2} \times 3.42 \times 10^{15} \text{m}^2 = 3.42 \times 10^{15} \text{W}
\]

\[ F_{\text{northward}} = 3.42 \text{ PetaWatts} \]
2. A cylindrical planet rotates around a sun with the long axis of the cylinder always perpendicular to the radius of its orbit. The cylinder rotates about its long axis once every 24 hours. The cylinder has a diameter of \( d \) and a length of \( L \). It has an albedo of \( \alpha \). The planet is a very poor heat conductor, but is covered with ocean with a deep mixed layer and has a high heat capacity so its surface temperature is uniform around it circumference. (20 pts)

\[ \text{Direction of movement} \]

\[ \text{Sun} \rightarrow \text{Cylindrical Planet} \]

a) What is the shadow area of the planet?

\[ dL \]

b) What is the emission area of the planet? Ignore the ends because they would be very cold, eh?

\[ \pi dL \]

c) Write down the energy balance for the planet, if it emits like a black body at temperature \( T_e \). And the total solar irradiance at the planet is \( S \).

\[ S (1-\alpha) dL = \sigma T_e^4 \pi dL \]

d) Solve for the emission temperature of the planet if \( S=1360 \text{Wm}^{-2} \) and \( \alpha=0.30 \).

\[ T_e = \sqrt[4]{\frac{S (1-\alpha)}{\pi \sigma}} = 276.3 K \]
3. Lake Chad is near the Sahara Desert. If 70% of the surface solar absorption of 280 Wm\(^{-2}\) is used to evaporate water, what depth of water will be evaporated in one year? Give your answer in millimeters of liquid water. (20 pts)

\[
\frac{dh_w}{dt} = \frac{Q}{P_w L} \\
Q = 0.7 \times 280 \text{ Wm}^{-2} \\
dt = 1 \text{ year}
\]

\[
dh_w = dt \cdot \frac{Q}{P_w L}
\]

\[
= 365.25 \times 86,400 \text{ sec/day} \cdot \frac{0.7 \times 280 \text{ Wm}^{-2}}{10^3 \text{ kgm}^{-3} \cdot 2.5 \times 10^7 \text{ J/kg}^7}
\]

\[
= 3.156 \times 10^{-6} \text{ sec} \cdot 7.84 \times 10^{-8} \text{ m/s}^{-1}
\]

\[
h_w = 2.47 \text{ m} = 2470 \text{ mm}
\]
4. A meteor hits Earth and kicks up a global cloud of dust and soot aerosols. The radiative properties of the atmosphere change such that the planetary albedo is changed to 0.25, and the disposition of the absorbed solar radiation is changed so that 60% of the radiation is absorbed into the atmosphere and 40% is absorbed at the surface. The solar radiation that is absorbed by the atmosphere is evenly distributed 30%/30% between the two atmospheric layers. Assume that the atmosphere can be represented with two layers at 2.5 and 5 km that are black bodies for infrared. Assume that the atmosphere fluxes heat vertically via convection to maintain a lapse rate of 6.5 K/km. Specify unknown fluxes from the surface to level 2 ($F_{s-2}$) and from level 2 to level 1($F_{2-1}$) that are necessary to satisfy energy balance. Remember to compute a new $T_e$. (25pts total)

a) Draw a diagram showing all of the radiative and convective energy fluxes (be neat): (5pts)

\[ \begin{align*}
0.3 \sigma T_e^4 \downarrow & \quad \downarrow 0.3 \sigma T_e^4 \downarrow \\
\uparrow \sigma T_1^4 & \quad \uparrow \sigma T_2^4 \\
0.4 \sigma T_e^4 \downarrow & \quad \downarrow \sigma T_3^4 \\
F_{s-2} & \quad F_{2-1} \\
\downarrow & \\
T_{surface}
\end{align*} \]

b) Write down the energy balance equations for the top of the atmosphere, the two atmospheric layers, and the surface. (5pts)

TOA \quad \sigma T_e^4 = \sigma T_1^4

Layer 1 \quad 0.3 \sigma T_e^4 + \sigma T_2^4 + F_{2-1} = 2 \sigma T_1^4

Layer 2 \quad 0.3 \sigma T_e^4 + \sigma T_1^4 + \sigma T_3^4 + F_{s-2} = 2 \sigma T_2^4 + F_{2-1}

Surface. \quad 0.4 \sigma T_e^4 + \sigma T_2^4 = \sigma T_3^4 + F_{s-2}

\[ m = m' \]
c) Manipulate the equations for the energy balance to provide formulas for the temperature of the top layer and the two unknown convective fluxes. (5pts)

\[ T_1 = T_e \]

\[ F_{s-2} = \frac{2\sigma T_1^4 - 0.3 \sigma T_e^4 - 0.12}{2} \cdot 0.4T_1^4 + 0.4T_2^4 - 0.4T_5^4 \]

\[ F_{2-1} = 2\sigma T_1^4 - 0.3 \sigma T_e^4 - 0.4T_2^4 \]

d) Solve for emission temperature and the two fluxes. The solar constant is 1360 Wm\(^{-2}\). (5pts)

\[ T_e = \sqrt[4]{\frac{1360}{\sigma}} = 259K \]

\[ T_2 = \frac{259 + 6.5 \times 2.5}{2} = 278.25K \]

\[ T_5 = \frac{259 + 6.5 \times 5}{2} = 291.5K \]

\[ F_{s-2} = 102.06 \text{ Wm}^{-2} + 325 \text{ Wm}^{-2} - 409.4 = 17.67 \text{ Wm}^{-2} \]

\[ F_{2-1} = 510.3 \text{ Wm}^{-2} - 76.54 - 325 \text{ Wm}^{-2} = 108.3 \text{ Wm}^{-2} \]

e) How do these fluxes compare with the fluxes obtained in the standard case where the atmosphere does not absorb solar radiation and the albedo is 0.3? In the case done for homework remember that \( F_{s-2} = 159 \text{ Wm}^{-2} \) and \( F_{2-1} = 172 \text{ Wm}^{-2} \). How do you think the rainfall would change? Explain (5pts)

A bit less, so convection needs to move less energy, so less rainfall, even though surface nominally warmer virtually NO deep convection.
Data and Formulas:

\[ \sigma = 5.68 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4} \quad \pi = 3.1415926 \quad \frac{\partial T}{\partial t} = Q \]

\[ G = R_s \quad LE = SH - \Delta F_{\phi0} \quad I = \sigma T^4 \quad \sigma \theta_e^4 = \frac{S_0}{4} (1 - \alpha_p) \]

Surface area of Earth poleward of latitude \( \phi \) \n
\[ \text{Area} = 2\pi a^2 \cos \phi \, d\phi \, d\lambda = 2\pi a^2 (1 - \sin \phi) \]

Area of circle = \( \pi r^2 \)

Area of Sphere = \( 4\pi r^2 \)

Density of water = 1000 kg m\(^{-3}\)

\[ LE = R_s = \rho_w L \frac{dh_w}{dt} \Rightarrow \frac{dh_w}{dt} = \frac{R_s}{\rho_w L} \]

Area of cylinder of diameter \( d \) and length \( L \) = \( \pi dL + \frac{1}{2} \pi d^2 \) - including ends. Without ends Area = \( \pi dL \)

Latent heat of vaporization of water = \( L = 2.5 \times 10^6 \text{ J kg}^{-1} \)

Specific heat of water = 4218 J K\(^{-1}\)kg\(^{-1}\)

Average radius of Earth = \( a = 6.37 \times 10^6 \text{ m} \)

\[ S_o = 1367 \text{ Wm}^{-2} \]

\[ E = M c^2 \quad \epsilon = 0.622 \quad q^* = \frac{611 \text{ Pa}}{p} \exp \left\{ \frac{L}{R_v} \left( \frac{1}{273} - \frac{1}{T} \right) \right\} \]

\[ R = 287 \text{ JK}^{-1} \text{kg}^{-1} \quad Q = \left( \frac{d^2}{d} \right) \cos \theta_s \]

\[ \cos \theta_s = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \]

At Noon \( \theta_s = \phi - \delta \)

\[ C_o = \rho_w c_w d_w = d_w \times 4.2 \times 10^6 \text{ JK}^{-1} \text{m}^{-2} \text{m}^{-1} \quad 1 \text{ day} = 86,400 \text{ seconds} \]