GFD II: Balance Dynamics
ATM S 542

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The QG system
- Started with 5 equations for 5 unknowns
- Reduced to 1 equation for 1 unknown (streamfunction)
- It’s nonlinear though (so has very rich behavior)
- Balanced in that there are no gravity waves
Response to a ball of PV
Response to a surface temperature anomaly

Just like top half of the previous picture!
PV Thinking

- Boussinesq, f-plane, adiabatic, frictionless QG PV equation:

\[ \frac{D_g q}{Dt} = 0 \]

- Advection by geostrophic velocities

- Inversion:

\[ q = \nabla^2 \psi + \frac{f_0^2}{N^2} \frac{\partial^2 \psi}{\partial z^2} \]

- Can invert PV to get all the geostrophic quantities:

\[ \psi \ (\text{and } u_g, v_g, \zeta_g, b) \]
Ageostrophic velocities

- Any change in the individual quantities happens due to ageostrophic motions though:

\[
\frac{D u_g}{D t} = f_0 v_a \\
\frac{D v_g}{D t} = -f_0 u_a \\
\frac{D \zeta_g}{D t} = f_0 \frac{\partial w}{\partial z} \\
\frac{D b}{D t} = -N^2 w
\]
Can reconstruct \( w \) from geostrophic quantities with a diagnostic relation: the omega equation

- Derive this by eliminating partial derivatives with respect to time in the buoyancy and vorticity equation
W-thinking

- Thinking about factors that change the components of **thermal wind balance** leads to a completely different way of interpreting the ageostrophic velocities

- Geostrophic velocities are constantly trying to break apart thermal wind balance
  - Changing each the buoyancy gradient and shear in equal magnitudes, in the wrong direction to cancel
Ageostrophic velocities are quietly working in the background to keep the flow in thermal wind balance.

This derivation also suggests a better version of the “omega equation”
- “Q-vectors”: best way to depict vertical motion
  
\[
Q = - \left( \frac{\partial u_g}{\partial x} \cdot \nabla b, \frac{\partial u_g}{\partial y} \cdot \nabla b \right)
\]

\[
\nabla \cdot Q = \frac{N^2}{2} \left( \nabla^2 w + \frac{f_0^2}{N^2} \frac{\partial^2 w}{\partial z^2} \right)
\]

Q-vectors point towards ascent.