Adiabatic lapse rate and static stability
(Ch. 2.7.2, 2.7.3)

Adiabatic lapse rate

Start with the definition of potential temperature, take logarithm and total differential w.r.t. height

\[
\begin{align*}
    c_p \frac{\partial \ln \theta}{\partial z} &= c_p \frac{\partial \ln T}{\partial z} - R \frac{\partial \ln p}{\partial z} \\
    \frac{T}{\theta} \frac{\partial \theta}{\partial z} &= \_ = \_ \\
\end{align*}
\]

If potential temperature is constant in vertical

\[
\Gamma_d \equiv -\frac{\partial T}{\partial z} = \_ \\
\]

Static stability

If potential temperature varies in vertical

\[
\begin{align*}
    \Gamma &\equiv -\frac{\partial T}{\partial z} \\
    \frac{T}{\theta} \frac{\partial \theta}{\partial z} &= \_ = \_ \\
\end{align*}
\]

stratification?
Adiabatic lapse rate and static stability
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What would be the vertical acceleration that the air parcel would get when it is adiabatically moved up by delta $z$?

$$z + \delta z$$

$$\rho_0(z + \delta z) = \rho_0(z) + \theta_0(z + \delta z) = \theta_0(z) + \rho_0(z)$$

(unscaled) vertical momentum equation after decomposing pressure and density into resting atmosphere and perturbation

$$\frac{Dw}{Dt} = \frac{u^2 + v^2}{a}$$

the parcel has no motion, assume pressure adjust to environmental value quickly

$$\frac{Dw}{Dt} =$$

thermodynamic energy equation
Adiabatic lapse rate and static stability (Ch. 2.7.2, 2.7.3)

The acceleration of the air parcel

\[
\frac{Dw}{Dt} = \frac{D^2 \delta z}{Dt^2} = \delta z
\]

Buoyancy frequency

\[
N^2 = \frac{D^2 \delta z}{Dt^2} = -N^2 \delta z
\]

general solution

\[
\delta z = A \exp(iNt)
\]

What would be the trajectory of the air parcel depending on the sign of \(N^2\)?

What’s the relationship between the buoyancy frequency and the static stability?