Atmospheric Sciences 441/503  
Homework #3  
Due 10:30am Monday 2 November 2015

Instructions: Answer all questions assigned to you (see below), show all work, and please write clearly. Hand in all worksheets with your name on all of them. Total possible points are shown at the end of each question.

1. Prove the identity below. (3)

\[- \frac{1}{\rho} \frac{\partial p}{\partial z} \approx - \frac{1}{\rho_0} \frac{\partial p}{\partial z} - \frac{1}{\rho_0} \frac{\partial p'}{\partial z} - \frac{\rho'}{\rho_0} g\]

where the total pressure and density fields are decomposed into the basic state and perturbation components \( p(x, y, z, t) = p_0(z) + p'(x, y, z, t), \rho(x, y, z, t) = \rho_0(z) + \rho'(x, y, z, t) \). Assume that the magnitude of perturbation part is much smaller than that of the basic state part \(|p_0| \gg |p'|, |\rho_0| \gg |\rho'|\). Also recall that the hydrostatic balance holds in the basic state.

2. Prove the identity below. (7)

\[- \frac{\rho'}{\rho_0} \approx \frac{\theta'}{\theta_0}\]

where \( \theta_0(z) \) and \( \rho_0(z) \) are the basic state temperature and density, and \( \theta'(x, y, z, t) \) and \( \rho'(x, y, z, t) \) are perturbation (deviation from the basic state) potential temperature and density, respectively. Assume that \(|\theta_0| \gg |\theta'|, |\rho_0| \gg |\rho'|\), and \( p = p_0 \) (pressure perturbation is negligible). (*Tip: Use \( \ln(1 + \varepsilon) \approx \varepsilon \), when \( \varepsilon \ll 1 \))