Supplementary Materials for

Slow climate mode reconciles historical and model-based estimates of climate sensitivity

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Other Supplementary Material for this manuscript includes the following:
(available at advances.sciencemag.org/cgi/content/full/3/7/e1602821/DC1)

- data file S1 (.zip format). Ensemble of posterior draws.
Supplementary Materials

Supplementary Text

S1: Spatial Projection of Eigenmodes

Generalizing equation (1) in the main manuscript, we can write the evolution of local temperatures in the abrupt quadrupling experiment as

\[ T(x, y, t) = 2 T_{2\times}(x, y) \sum_n \alpha_n(x, y)(1 - \exp(-t / \tau_n)) \]  

(1.1)

where \( T_{2\times} \) is the local equilibrium sensitivity to a doubling of CO2 and \( \alpha_n(x, y) \) is the local relative contribution of each eigenmode, with \( \sum_n \alpha_n(x, y) = 1 \). The global analysis of eigenmodes was predicated on the assumption that equilibrium is reached when \( H \) asymptotes to zero. Due to lateral heat fluxes in the system, however, equilibrium does not generally imply that \( H(x, y) \to 0 \). Furthermore, it has been shown that linearity in the temperature response to radiative forcing holds much better globally than locally (49). A more complete eigenmode decomposition approach that is applicable at the local level and computationally tractable awaits development.

Here we simply examine how local temperatures evolve along the timescales defined at the global mean. We fix \( \tau_n \) to the posterior median values obtained from the global analysis, such that we can rewrite equation (1) as

\[ T(x, y, t) = \sum_n T_n(x, y) \phi_n(t) \]  

(1.2)

where \( \phi_n(t) = 1 - \exp(-t / \tau_n) \), and \( T_n(x, y) = 2 T_{2\times}(x, y) \alpha_n(x, y) \) is the equilibrium warming associated with each eigenmode for a quadrupling of CO2. \( T_n(x, y) \) is obtained by performing a multiple linear regression of \( T(x, y, t) \) on \( \phi_n(t) \). The amplitudes of these modes are illustrated in fig. S2.

Although the accuracy of this local projection is not quantified in the way that our global projections are and our main results do not depend upon these calculations, these local results are nonetheless useful for purposes of comparison against other simulations and studies.
Convolving the local eigenmodes with the AR5 historical forcing gives an approximation of the historical pattern of warming. The local warming between a reference period of 1961-1990 to 2000-2011 is similar to that found in historical GCM runs and the HadCRUT4 observational dataset (50), both in pattern and magnitude (fig. S3).

S2: Generalized Linear Model

We provide solutions based on linear response theory for two simple models used in the literature and cited in the main text to describe time dependency of the net radiative feedback.

**Linear Model Response:** Let $y$ be the state vector the climate system. If we assume that $y$ describes small anomalies around a state of equilibrium, the evolution of the climate system in response to an external forcing $F$ is described by

$$\frac{d}{dt} y = Jy + F(t)$$

(1.3)

where $J$ is the Jacobian of the system. We apply classic linear system methods and diagonalize the Jacobian as

$$J = P^{-1}DP$$

(1.4)

where $P$ is the modal matrix containing the eigenvectors of the Jacobian, and $D$ is the diagonal spectral matrix containing the eigenvalues. The exponential decay time scales of the eigenmodes are the negative inverses of the eigenvalues, such that $\tau_n = -1/D_{nn}$. The system has the well-known solution (51)

$$y = y(0) + \int_0^t e^{P \tau} DPF(t-\tau) d\tau$$

(1.5)

Global mean temperature is a weighted sum of members of the state vector corresponding to surface temperature
Further, we assume that the system starts from a state of equilibrium, \( y(0) = 0 \), and that there is a single forcing time series, \( F(t) \), imposed on the system that is projected onto the different dimensions of the system according to

\[
F = w_F F(t)
\] (1.7)

We can now write the global temperature response, \( T \), as

\[
T = \int_0^t w_T P^{-1} e^{D_p} P w_T F \left( t - \tau \right) d\tau
\] (1.8)

\[
T = \sum_n \sum_k \sum_l w_{Tl} P^{-1} \ln P_{ln} w_{Fk} \exp \left( -t / \tau_n \right) * F(t)
\] (1.9)

If we divide and multiply by \( \tau_k \), and by the equilibrium sensitivity parameter

\[
\frac{T}{F} = -w_T J^{-1} w_F = -w_T P^{-1} D^{-1} P w_F
\] (1.10)

we recover Eqn. 1 in the main manuscript

\[
T = \frac{T}{F} \frac{\alpha}{\tau} \sum_n \exp \left( -t / \tau_n \right) * F(t)
\] (1.11)

where

\[
\alpha_n = \sum_m \sum_k w_{Tm} P^{-1} \ln P_{nk} w_{Fk}
\]

\[
\frac{\alpha_n}{w_T P^{-1} D^{-1} P w_F}
\] (1.12)

with \( \sum_n \alpha_n = 1 \).

The net global radiative response, \( R \), will likewise be a linear function of the state vector.
\[ R_y = w_R y \]  

which has to satisfy the condition that in equilibrium \( R_{eq} = F_{eq} \), and thus

\[ w_R P^{-1} D^{-1} P w_F = 1 \]  

We can now write

\[ R = \sum_m \sum_k \sum_n w_{Rm} P^{-1} P_{mn} \tau_{nk} w_{Fk} \exp \left( -t / \tau_n \right) \ast F(t) \]  

\[ R = \sum_n \beta_n \frac{\exp \left( -t / \tau_n \right)}{\tau_n} \ast F(t) \]  

with

\[ \beta_n = \sum_m \sum_k w_{Rm} P^{-1} \tau_{nk} w_{Fk} \]  

The radiative feedbacks associated with each eigenmode can be computed (as per Eqn. 5 in the main text) as \( \lambda_n = \beta_n / \alpha_n F_{eq} / T_{eq} \)

\[ \lambda_n = \frac{\sum_m \sum_k w_{Rm} P^{-1} \tau_{nk} w_{Fk}}{\sum_m \sum_k w_{Tm} P^{-1} \tau_{nk} w_{Fk}} \]  

**Regional feedbacks model:** In order to explain the time dependency of the net feedback, a three-box model has been proposed \((16)\). The model has three independent regions, each evolving with their own characteristic time scale and radiative feedback

\[ C_j \frac{dT_j}{dt} = -\lambda_j T_j + F \]
The matrices associated with this model are

\[
\begin{align*}
\mathbf{w}_f &= \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \\
\mathbf{w}_r &= \begin{bmatrix} 1/C_1 \\ 1/C_2 \\ 1/C_3 \end{bmatrix}, \\
\mathbf{w}_r &= \begin{bmatrix} \lambda_1/3 \\ \lambda_2/3 \\ \lambda_3/3 \end{bmatrix}
\end{align*}
\]

\[J = \begin{bmatrix}
-\frac{\lambda}{C_1} & 0 & 0 \\
0 & -\frac{\lambda_2}{C_2} & 0 \\
0 & 0 & -\frac{\lambda_3}{C_3}
\end{bmatrix}
\]

Since the Jacobian matrix is already diagonal, each region represents an eigenmode, such that the solution is trivial

\[
\tau_j = C_j / \lambda_j
\]

\[
\alpha_j = \frac{1}{3} \sum_{j=1}^{n} \lambda_j^{-1}
\]

and the radiative feedbacks associated with each eigenmode are equal to the regional feedbacks. Energy fluxes between regions can be modeled by adding symmetrical off-diagonal terms in the Jacobian matrix, however in that case the radiative feedbacks associated with each eigenmode would no longer be equal to the regional feedbacks.

Two-layer ocean model: Another standard conceptualization of the evolution of the system is one where a fast-responding upper ocean is coupled to a slow-responding deep ocean,
\[ C_s \frac{dT_s}{dt} = -\lambda_{eq} T_s - \varepsilon \gamma (T_s - T_d) + F \]  
(1.25)

\[ C_d \frac{dT_d}{dt} = \gamma (T_s - T_d) \]  
(1.26)

\( C_{s,d} \) are the heat capacities of the surface and deep ocean, and \( T_{s,d} \) are the temperatures. Thermal coupling between the surface and deep is represented by \( \gamma \), the equilibrium feedback parameter is \( \lambda_{eq} \) and radiative forcing is \( F \). Also included is a term for the efficacy of ocean heat uptake, \( \varepsilon \).

This term can be understood in the context of an effective forcing whereby the same global radiative forcing can illicit different global temperature responses (52), but where it is the transfer of heat between surface and deeper layers that forces surface temperature (13, 14). The radiative response of this two-layer ocean model can be written as

\[ R = \lambda_{eq} T - (1 - \varepsilon)(T - T_d) \]  
(1.27)

and the matrices associated with this model are

\[
\begin{bmatrix}
\frac{1}{C_s} \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
1 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
\lambda_{eq} + (\varepsilon - 1) \\
1 - \varepsilon
\end{bmatrix}
\]

(1.28)

\[
J = \begin{bmatrix}
-\frac{\lambda_{eq} + \varepsilon \gamma}{C_s} & \frac{\varepsilon \gamma}{C_s} \\
\frac{\gamma}{C_d} & -\frac{\gamma}{C_d}
\end{bmatrix}
\]

(1.29)

They can be used to solve for

\[ \tau_{1,2} = \frac{2C_d C_s}{(\varepsilon \gamma + \lambda_{eq})C_d + \gamma C_s \pm \Delta} \]  
(1.30)

\[ \alpha_{1,2} = \frac{\Delta \pm (\varepsilon \gamma - \lambda_{eq})C_d + \gamma C_s}{2\Delta} \]  
(1.31)
\[ \lambda_{1,2} = \lambda_{eq} \frac{1 + \varepsilon}{2} + (\varepsilon - 1) \frac{\gamma C_s + \varepsilon \gamma C_d \pm \Delta}{2 \varepsilon C_d} \]  

(1.32)

where

\[ \Delta = \sqrt{\left((\varepsilon \gamma + \lambda_{eq} C_d + \gamma C_s\right)^2 - 4 \gamma \lambda_{eq} C_d C_s} \]  

(1.33)

As expected, for the case of unit efficacy the feedbacks along the two eigenmodes are the same.

Another formalism proposed to account for time-variability in feedback strengths is that of a virtual forcing \((22)\), but because this representation is mathematically equivalent to the two-layer ocean model described here, the same form of eigenmode solution holds.
**fig. S1. Structure of residuals.** Residual temperature and top-of-atmosphere energy flux between GCM simulations and Bayesian fits. Residuals are shown for two-eigenmode (a, c) and three-eigenmode (b, d) fits for each GCM (colored lines). The median residual across models (thick black line) shows systematic residuals for the two-eigenmode fits during initial decades after abrupt quadrupling of CO2 concentrations of model adjustment that are not present in the three-eigenmode fit. All eigenmode fits use posterior maximum likelihood value.
fig. S2. Figure 1 continued.
fig. S3. Figure 1 continued.
fig. S4. Figure 1 continued.
**fig. S5.** Figure 1 continued.
Spatial projection of the eigenmodes. Local equilibrium amplitude, \(a_n(x,y)T_{2n}(x,y)\), for the fast (0.7 years, a), intermediate (9 years, b) and slow (350 years, c) eigenmodes, under a uniform global radiative forcing value of 3.6 W/m².
**fig. S7. Historical temperature anomalies** averaged over 2000-2011, relative to a reference period of 1961-1990, obtained from the ensemble average of the spatial projection of eigenmodes (a), the ensemble average of CMIP5 historical model runs (b), and the HadCRUT4 dataset (50) (c).
**fig. S8. Autocovariance of temperature residuals.** Autocovariance of the residuals between the GCM simulations and the Bayesian fit for each model (red), and that inferred for residual noise by the Bayesian fit (blue, from values of $\sigma_T$ and $\rho_T$). Maximum likelihood parameters associated with each Bayesian fit are used in these calculations. The p-value of a Kolmogorov-Smirnov normality test for the residuals that accounts for autocorrelation (46) is also displayed for each GCM. Only MIROC 5 shows significant deviations from normality ($p < 0.05$) in the temperature residuals.
fig. S9. Autocovariance of energy flux residuals. Same as Fig. S8, but for the top-of-atmosphere energy flux, H. Statistically significant deviations from a normal distribution are found for MIROC 5, CCSM4 and IPSL-CM5B-LR. Notwithstanding the significance of the p-value for MIROC 5, four rejections out of 48 trials across the results for temperature and energy flux is consistent with that expected when the null hypothesis holds and a test is performed at the 5% significance level.
### Table S1. Posterior parameter values for each GCM.

Median values for parameters in the Bayesian fit to each GCM. Ensemble values at bottom are computed assuming each GCM is equally likely.