

Financial Toolbox

For Use with MATLAB®

Computation

Visualization

Programming



User's Guide

Version 2

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Financial Toolbox User's Guide

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Preface

Introducing the Financial Toolbox	x
Using This Guide	xi
Expected Background	xi
Organization of the Document	xii
Examples	xii
Related Products	xiii
Prerequisites	xiv
Compatibility	xv
Configuration Information	xvi
Additional Resources	xvii
Financial Demonstration Programs	xvii
Finding Additional Information	xvii
Typographical Conventions	xviii

Getting Started

1

Using Matrix Functions for Finance	1-3
Key Definitions	1-3
Referencing Matrix Elements	1-3
Transposing Matrices	1-5
Matrix Algebra Refresher	1-6
Adding and Subtracting Matrices	1-6
Multiplying Matrices	1-7

Multiplying Vectors	1-8
Computing Dot Products of Vectors	1-8
Multiplying Vectors and Matrices	1-9
Multiplying Two Matrices	1-10
Multiplying a Matrix by a Scalar	1-11
Dividing Matrices	1-12
Solving Simultaneous Linear Equations	1-12
Operating Element-by-Element	1-15
Function Input/Output Arguments	1-17
Input Arguments	1-17
Matrix Input	1-17
Matrices of String Input	1-18
Function Output Arguments	1-19
Interest Rate Arguments	1-20

Tutorial

2

Handling and Converting Dates	2-4
Date Formats	2-4
Date Conversions	2-5
Input Conversions	2-5
Output Conversions	2-7
Current Date and Time	2-8
Determining Dates	2-8
Formatting Currency	2-12
Charting Financial Data	2-13
High-Low-Close Chart Example	2-13
Bollinger Chart Example	2-14
Analyzing and Computing Cash Flows	2-16
Interest Rates / Rates of Return	2-16
Present or Future Values	2-17
Depreciation	2-17

Annuities	2-18
Pricing and Computing Yields for	
Fixed-Income Securities	2-20
Terminology	2-20
SIA Framework	2-22
SIA Default Parameter Values	2-23
Single Bond Example	2-24
Bond Portfolio Example	2-25
SIA Coupon Date Calculations	2-26
SIA Semi-Annual Yield Conventions	2-26
Pricing Functions	2-27
Yield Functions	2-27
Fixed-Income Sensitivities	2-28
Term Structure of Interest Rates	2-29
Pricing and Analyzing Equity Derivatives	2-32
Sensitivity Measures	2-32
Delta	2-32
Gamma	2-32
Lambda	2-32
Rho	2-32
Theta	2-33
Vega	2-33
Implied Volatility	2-33
Analysis Models	2-33
Black-Scholes Model	2-33
Binomial Model	2-34
Analyzing Portfolios	2-37
Portfolio Optimization Functions	2-37
Portfolio Construction Examples	2-39
Efficient Frontier Example	2-39
Portfolio Selection and Risk Aversion	2-41
Optimal Risky Portfolio Example	2-42
Constraint Specification Example	2-45
Linear Constraint Equations	2-47
Specifying Additional Constraints	2-50

Solving Sample Problems

3

Common Problems in Finance	3-3
Sensitivity of Bond Prices to Changes in Interest Rates	3-3
Constructing a Bond Portfolio to Hedge Against Duration and Convexity	3-6
Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve	3-8
Constructing Greek-Neutral Portfolios of European Stock Options	3-12
Term Structure Analysis and Interest Rate Swap Pricing ...	3-15
 Producing Graphics with the Toolbox	3-19
Plotting an Efficient Frontier	3-19
Plotting Sensitivities of an Option	3-21
Plotting Sensitivities of a Portfolio of Options	3-23

Function Reference

4

Functions by Category	4-2
Handling and Converting Dates	4-2
Formatting Currency and Charting Financial Data	4-4
Analyzing and Computing Cash Flows	4-5
Fixed-Income Securities	4-7
Analyzing Portfolios	4-9
Pricing and Analyzing Derivatives	4-10
GARCH Processes	4-10
Obsolete Bond Price and Yield Functions	4-11
 Alphabetical List of Functions	4-12
accrfrac	4-16
acrubond	4-19
acrudisc	4-20
amortize	4-21
annurate	4-23
annuterm	4-24

bdtbond	4-25
bdttrans	4-33
beytbill	4-36
binprice	4-37
blkprice	4-39
blsdelta	4-41
blsgamma	4-42
blsimpv	4-43
blslambda	4-44
blsprice	4-45
blsrho	4-47
blstheta	4-48
blsvega	4-49
bndconvp	4-50
bndconvy	4-53
bnddurp	4-56
bnddury	4-59
bndprice	4-62
bndyield	4-65
bolling	4-68
busdate	4-69
candle	4-70
cfamounts	4-71
cfconv	4-76
cfdates	4-77
cfdur	4-80
cfport	4-81
cftimes	4-84
corr2cov	4-86
cov2corr	4-87
cpncount	4-88
cpndaten	4-91
cpndatenq	4-94
cpndatep	4-98
cpndatepq	4-101
cpndaysn	4-105
cpndaysp	4-108
cpnpersz	4-111
cur2frac	4-114
cur2str	4-115

dateaxis	4-116
datedisp	4-118
datefind	4-119
datemnth	4-120
datenum	4-122
datestr	4-125
datevec	4-128
datewrkdy	4-130
day	4-131
days360	4-132
days365	4-133
daysact	4-134
daysdif	4-135
depfixdb	4-136
depgendb	4-137
deprdv	4-138
depsoyd	4-139
depstln	4-140
disc2zero	4-141
discrate	4-144
effrr	4-145
eomdate	4-146
eomday	4-147
ewstats	4-148
fbusdate	4-150
frac2cur	4-151
frontcon	4-152
fvdisc	4-155
fvfix	4-156
fvvar	4-157
fwd2zero	4-159
highlow	4-163
holidays	4-164
hour	4-165
irr	4-166
isbusday	4-167
lbusdate	4-168
lweekdate	4-170
m2xdate	4-172
minute	4-174

mirr	4-175
month	4-176
months	4-177
movavg	4-178
nomrr	4-179
now	4-180
nweekdate	4-181
opprofit	4-183
payadv	4-184
payodd	4-185
payper	4-186
payuni	4-187
pcalims	4-188
pcgcomp	4-191
pcglims	4-193
pcpval	4-196
pointfig	4-198
portalloc	4-199
portcons	4-202
portopt	4-206
portrand	4-209
portsim	4-210
portstats	4-212
portvrisk	4-214
prbyzero	4-216
prdisc	4-220
prmat	4-221
prtbill	4-223
pvfix	4-224
pvvar	4-225
pyld2zero	4-227
ret2tick	4-231
second	4-233
taxedrr	4-234
tbl2bond	4-235
tick2ret	4-237
today	4-239
tr2bonds	4-240
ugarch	4-243
ugarchllf	4-245

ugarchpred	4-247
ugarchsim	4-250
weekday	4-255
wrkdydif	4-257
x2mdate	4-258
xirr	4-260
year	4-262
yeardays	4-263
yearfrac	4-264
ylddisc	4-265
yldmat	4-266
yldtbill	4-268
zbtprice	4-269
zbtyield	4-274
zero2disc	4-279
zero2fwd	4-282
zero2pyld	4-286

Glossary

A

Bibliography

B

Bond Pricing and Yields	B-2
Term Structure of Interest Rates	B-2
Derivatives Pricing and Yields	B-2
Portfolio Analysis	B-3
Other References	B-3

Preface

Introducing the Financial Toolbox	x
Using This Guide	xi
Expected Background	xi
Organization of the Document	.xii
Examples	.xii
Related Products	xiii
Prerequisites	xiv
Compatibility	.xv
Configuration Information	xvi
Additional Resources	xvii
Financial Demonstration Programs	xvii
Finding Additional Information	xvii
Typographical Conventions	xviii

Introducing the Financial Toolbox

MATLAB® and the Financial Toolbox provide a complete integrated computing environment for financial analysis and engineering. The toolbox has everything you need to perform mathematical and statistical analysis of financial data and display the results with presentation-quality graphics. You can quickly ask, visualize, and answer complicated questions.

In traditional or spreadsheet programming you must deal with all sorts of housekeeping details: declaring, data typing, sizing, etc. MATLAB does all that for you. You just write expressions the way you think of problems. And there's no need to switch tools, convert files, or rewrite applications.

With MATLAB and the Financial Toolbox, you can:

- Compute and analyze prices, yields, and sensitivities for derivatives and other securities, and for portfolios of securities.
- Perform Securities Industry Association (SIA) compatible fixed-income pricing, yield, and sensitivity analysis.
- Analyze or manage portfolios.
- Design and evaluate hedging strategies.
- Identify, measure, and control risk.
- Analyze and compute cash flows, including rates of return and depreciation streams.
- Analyze and predict economic activity.
- Create structured financial instruments, including foreign-exchange instruments.
- Teach or conduct academic research.

Using This Guide

This guide helps you learn to use MATLAB and the Financial Toolbox for financial analysis and engineering applications. After reading this manual, you will understand Financial Toolbox concepts, content, functions, and uses. You will have successfully executed several examples, and you will be able to use the functions of choice.

Expected Background

In designing the Financial Toolbox and this manual, we assume your title is similar to one of these:

- Analyst, quantitative analyst
- Risk manager
- Portfolio manager
- Fund manager, asset manager
- Economist
- Financial engineer
- Trader
- Student, professor, or other academic

We also assume your background, education, training, and responsibilities match some aspects of this profile:

- Finance, economics, perhaps accounting
- Engineering, mathematics, physics, other quantitative sciences
- Bachelor's degree minimum; MS or MBA likely; Ph.D. perhaps; CFA
- Comfortable with probability, statistics, and algebra
- May understand linear or matrix algebra, calculus, and differential equations
- Previously resigned to doing traditional programming (C, Fortran, etc.)
- May be responsible for instruments or analyses involving large sums of money
- Perhaps new to MATLAB

Organization of the Document

Chapter 1, “Getting Started” reviews key definitions and elementary matrix operations and discusses the application of matrix algebra fundamentals in a financial context.

Chapter 2, “Tutorial” describes the use of the Financial Toolbox to perform a wide range of common financial tasks.

Chapter 3, “Solving Sample Problems” shows how the toolbox solves real-world financial problems and how it produces presentation-quality graphics. If you are already familiar with MATLAB, matrix algebra, and financial computations, you can turn immediately to these examples.

Chapter 4, “Function Reference” groups the Financial Toolbox functions by task, then describes each toolbox function in alphabetical order. Purpose, syntax, arguments, description, examples, and related functions are given for each function.

Appendix A “Glossary” defines the financial terms used in this manual.

Appendix B “Bibliography” provides references for the toolbox formulas and concepts.

Examples

We encourage you to try the examples throughout the text. In addition, we provide M-files of the longer common financial problems and graphics examples in Chapter 3, “Solving Sample Problems”. The five common financial problems are `ftspex1.m` through `ftspex5.m`; the three graphics examples are `ftgex1.m` through `ftgex3.m`.

Related Products

The MathWorks provides several products that are especially relevant to the kinds of tasks you can perform with the Financial Toolbox.

For more information about any of these products, see either:

- The online documentation for that product if it is installed or if you are reading the documentation from the CD
- The MathWorks Web site, at <http://www.mathworks.com>; see the “products” section

Note The toolboxes listed below all include functions that extend MATLAB’s capabilities.

Product	Description
Database Toolbox	Tool for connecting to, and interacting with, most ODBC/JDBC databases from within MATLAB
Datafeed Toolbox	MATLAB functions for integrating the numerical, computational, and graphical capabilities of MATLAB with financial data providers
Excel Link	Tool that integrates MATLAB capabilities with Microsoft Excel for Windows
Financial Derivatives Toolbox	Tool that extends the Financial Toolbox in the areas of fixed income derivatives and of securities contingent to interest rates, with functions for analyzing individual financial derivative instruments and portfolios composed of them

Product	Description
Financial Time Series Toolbox	Tool for analyzing time series data in the financial markets
GARCH Toolbox	MATLAB functions for univariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) volatility modeling
MATLAB Compiler	Compiler for automatically converting MATLAB M-files to C and C++ code
MATLAB C/C++ Math Library	Library for automatically converting MATLAB applications that contain math and graphics to C and C++ code for stand-alone applications
MATLAB Web Server	Tool for the development and distribution of Web-based MATLAB applications
Optimization Toolbox	Tool for general and large-scale optimization of nonlinear problems, as well as for linear programming, quadratic programming, nonlinear least squares, and solving nonlinear equations
Simulink Report Generator	Tool for documenting information in Simulink and Stateflow in multiple output formats
Statistics Toolbox	Tool for analyzing historical data, modeling systems, developing statistical algorithms, and learning and teaching statistics

Prerequisites

The Financial Toolbox requires the Statistics and Optimization Toolboxes, but you need not read those manuals before reading this one. Some examples use functions in the Spline Toolbox, but that toolbox is not a prerequisite for the Financial Toolbox.

Compatibility

The Financial Toolbox is compatible with Release 11 (MATLAB Version 5.3) and later.

Configuration Information

To determine whether the Financial Toolbox is installed on your system, type this command at the MATLAB prompt:

```
ver
```

When you enter this command, MATLAB displays information about the version of MATLAB you are running, including a list of all toolboxes installed on your system and their version numbers.

To install the Financial Toolbox see the *MATLAB Installation Guide for PC* or the *MATLAB Installation Guide for UNIX*. Check the *Release Notes* for any last minute information concerning installation of this release.

Note For the most up-to-date information about system requirements, see the system requirements page, available in the products area at the MathWorks Web site (<http://www.mathworks.com>).

Additional Resources

Financial Demonstration Programs

The MATLAB Financial Toolbox Exposition ships with a large number of programs that illustrate many of the features of the toolbox, including charting, options pricing, portfolio analysis, and others. Type `help findemos` for a complete list of available financial demonstration programs.

Finding Additional Information

For additional information about MathWorks financial products and the Financial Toolbox, visit our Web site

<http://www.mathworks.com/products/finprod>

or send e-mail to

finance@mathworks.com

Typographical Conventions

This manual uses some or all of these conventions.

Item	Convention to Use	Example
Example code	Monospace font	To assign the value 5 to A, enter <code>A = 5</code>
Function names/syntax	Monospace font	The <code>cos</code> function finds the cosine of each array element. Syntax line example is <code>MLGetVar ML_var_name</code>
Keys	Boldface with an initial capital letter	Press the R eturn key.
Literal strings (in syntax descriptions in Reference chapters)	Monospace bold for literals	<code>f = freqspace(n, 'whole')</code>
Mathematical expressions	<i>Italics</i> for variables Standard text font for functions, operators, and constants	This vector represents the polynomial $p = x^2 + 2x + 3$
MATLAB output	Monospace font	MATLAB responds with <code>A =</code> <code>5</code>
Menu names, menu items, and controls	Boldface with an initial capital letter	Choose the F ile menu.
New terms	<i>Italics</i>	An <i>array</i> is an ordered collection of information.
String variables (from a finite list)	<i>Monospace italics</i>	<code>sysc = d2c(sysd, 'method')</code>

Getting Started

Using Matrix Functions for Finance	1-3
Key Definitions	1-3
Referencing Matrix Elements	1-3
Transposing Matrices	1-5
 Matrix Algebra Refresher	 1-6
Adding and Subtracting Matrices	1-6
Multiplying Matrices	1-7
Dividing Matrices	1-12
Solving Simultaneous Linear Equations	1-12
Operating Element-by-Element	1-15
 Function Input/Output Arguments	 1-17
Input Arguments	1-17
Function Output Arguments	1-19
Interest Rate Arguments	1-20

This chapter uses MATLAB to review the fundamentals of matrix algebra you need for financial analysis and engineering applications. It contains these sections:

- **Using Matrix Functions for Finance**

Reviews Key Definitions and some matrix algebra fundamentals, such as Referencing Matrix Elements and Transposing Matrices.

- **Matrix Algebra Refresher**

Provides a brief refresher on using matrix functions in financial analysis and engineering

This material explains some MATLAB concepts and operations using financial examples to help get you started.

Using Matrix Functions for Finance

Many financial analysis procedures involve *sets* of numbers; for example, a portfolio of securities at various prices and yields. Matrices, matrix functions, and matrix algebra are the most efficient ways to analyze sets of numbers and their relationships. Spreadsheets focus on individual cells and the relationships between cells. While you can think of a set of spreadsheet cells (a range of rows and columns) as a matrix, a matrix-oriented tool like MATLAB manipulates sets of numbers more quickly, easily, and naturally.

Key Definitions

Matrix. A rectangular array of numeric or algebraic quantities subject to mathematical operations; the regular formation of elements into rows and columns. Described as an “m-by-n” matrix, with m the number of rows and n the number of columns. The description is always “row-by-column.” For example, here is a 2-by-3 matrix of two bonds (the rows) with different par values, coupon rates, and coupon payment frequencies per year (the columns) entered using MATLAB notation.

```
Bonds = [1000    0.06    2
          500    0.055   4]
```

Vector. A matrix with only one row or column. Described as a “1-by-n” or “m-by-1” matrix. The description is always “row-by-column.” Here is a 1-by-4 vector of cash flows in MATLAB notation.

```
Cash = [1500    4470    5280    -1299]
```

Scalar. A 1-by-1 matrix; i.e., a single number.

Referencing Matrix Elements

To reference specific matrix elements use (row, column) notation. For example,

```
Bonds(1,2)
```

```
ans =
```

```
0.06
```

```
Cash(3)
```

```
ans =
```

```
5280.00
```

You can enlarge matrices using small matrices or vectors as elements. For example,

```
AddBond = [1000    0.065    2];  
Bonds = [Bonds; AddBond]
```

adds another row to the matrix and creates

```
Bonds =
```

```
1000    0.06    2  
500     0.055   4  
1000    0.065   2
```

Likewise,

```
Prices = [987.50  
475.00  
995.00]
```

```
Bonds = [Prices, Bonds]
```

adds another column and creates

```
Bonds =
```

```
987.50    1000    0.06    2  
475.00     500    0.055   4  
995.00    1000    0.065   2
```

Finally, the colon (:) is important in generating and referencing matrix elements. For example, to reference the par value, coupon rate, and coupon frequency of the second bond.


```
BondItems = Bonds(2, 2:4)
```

```
BondItems =
```

```
500.00    0.055    4
```

Transposing Matrices

Sometimes matrices are in the wrong configuration for an operation. In MATLAB, the apostrophe or prime character (') transposes a matrix: columns become rows, rows become columns. For example,

```
Cash = [1500    4470    5280   -1299]'
```

produces

```
Cash =
```

```
1500
4470
5280
-1299
```

Matrix Algebra Refresher

Matrix algebra and matrix operations are fundamental to using MATLAB in financial analysis and engineering. The topics discussed in this section include:

- “Adding and Subtracting Matrices” on page 1-6
- “Multiplying Matrices” on page 1-7
- “Dividing Matrices” on page 1-12
- “Solving Simultaneous Linear Equations” on page 1-12
- “Operating Element-by-Element” on page 1-15

These explanations should help refresh your skills.

William Sharpe’s “electronic work-in-progress,” *Macro-Investment Analysis*, also provides an excellent explanation of matrix algebra operations using MATLAB. It is available on the Web at

<http://www.stanford.edu/~wfsharpe/mia/mia.htm>

Hint When you are setting up a problem, it helps to “talk through” the units and dimensions associated with each input and output matrix. In the example under “Multiplying Matrices” below, one input matrix has “five days’ closing prices for three stocks,” the other input matrix has “shares of three stocks in two portfolios,” and the output matrix therefore has “five days’ closing values for two portfolios.” It also helps to name variables using descriptive terms.

Adding and Subtracting Matrices

Matrix addition and subtraction operate element-by-element. The two input matrices must have the same dimensions. The result is a new matrix of the same dimensions where each element is the sum or difference of each corresponding input element. For example, consider combining portfolios of different quantities of the same stocks (“shares of stocks A, B, and C [the rows] in portfolios P and Q [the columns] plus shares of A, B, and C in portfolios R and S”).

```
Portfolios_PQ = [100  200
                  500  400
                  300  150];
```

```
Portfolios_RS = [175  125
                  200  200
                  100  500];
```

```
NewPortfolios = Portfolios_PQ + Portfolios_RS
```

```
NewPortfolios =

    275.00    325.00
    700.00    600.00
    400.00    650.00
```

Adding or subtracting a scalar and a matrix is allowed and also operates element-by-element.

```
SmallerPortf = NewPortfolios-10
```

```
SmallerPortf =

    265.00    315.00
    690.00    590.00
    390.00    640.00
```

Multiplying Matrices

Matrix multiplication does *not* operate element-by-element. It operates according to the rules of linear algebra. In multiplying matrices, it helps to remember this key rule: the inner dimensions must be the same. That is, if the first matrix is m -by-3, the second must be 3-by- n . The resulting matrix is m -by- n . It also helps to “talk through” the units of each matrix, as mentioned above.

Matrix multiplication also is *not* commutative; i.e., it is not independent of order. $A*B$ does *not* equal $B*A$. The dimension rule illustrates this property. If A is 1-by-3 and B is 3-by-1, $A*B$ yields a scalar (1-by-1) but $B*A$ yields a 3-by-3 matrix.

Multiplying Vectors

Vector multiplication follows the same rules and helps illustrate the principles. For example, a stock portfolio has three different stocks and their closing prices today are

```
ClosePrices = [42.5    15    78.875]
```

The portfolio contains these numbers of shares of each stock.

```
NumShares = [100
              500
              300]
```

To find the value of the portfolio, simply multiply the vectors

```
PortfValue = ClosePrices * NumShares
```

which yields

```
PortfValue =

          35412.50
```

The vectors are 1-by-3 and 3-by-1; the resulting vector is 1-by-1, a scalar. Multiplying these vectors thus means multiplying each closing price by its respective number of shares and summing the result.

To illustrate order dependence, switch the order of the vectors

```
Values = NumShares * ClosePrices
```

```
Values =

          4250.00          1500.00          7887.50
          21250.00          7500.00          39437.50
          12750.00          4500.00          23662.50
```

which shows the closing values of 100, 500, and 300 shares of each stock — not the portfolio value, and meaningless for this example.

Computing Dot Products of Vectors

In matrix algebra, if X and Y are vectors of the same length

$$Y = [y_1, y_2, \dots, y_n]$$

$$X = [x_1, x_2, \dots, x_n]$$

then the dot product

$$X \bullet Y = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

is the scalar product of the two vectors. It is an exception to the commutative rule. To compute the dot product in MATLAB, use `sum(X .* Y)` or `sum(Y .* X)`. Just be sure the two vectors have the same dimensions. To illustrate, use the previous vectors.

```
Value = sum(NumShares .* ClosePrices')
```

```
Value =
```

```
35412.50
```

```
Value = sum(ClosePrices .* NumShares')
```

```
Value =
```

```
35412.50
```

As expected, the value in these cases is exactly the same as the `PortfValue` computed previously.

Multiplying Vectors and Matrices

Multiplying vectors and matrices follows the matrix multiplication rules and process. For example, a portfolio matrix contains closing prices for a week. A second matrix (vector) contains the stock quantities in the portfolio.

```
WeekClosePr = [42.5      15      78.875
                42.125   15.5     78.75
                42.125   15.125    79
                42.625   15.25    78.875
                43       15.25    78.625];
```

```
PortQuan = [100
            500
            300];
```

To see the closing portfolio value for each day, simply multiply

```
WeekPortValue = WeekClosePr * PortQuan
```

```
WeekPortValue =
```

```
35412.50
35587.50
35475.00
35550.00
35512.50
```

The prices matrix is 5-by-3, the quantity matrix (vector) is 3-by-1, so the resulting matrix (vector) is 5-by-1.

Multiplying Two Matrices

Matrix multiplication also follows the rules of matrix algebra. In matrix algebra notation, if A is an m -by- n matrix and B is an n -by- p matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1p} \\ b_{21} & \cdots & b_{2j} & \cdots & b_{2p} \\ \vdots & & \vdots & & \vdots \\ b_{n1} & & b_{nj} & & b_{np} \end{bmatrix}$$

then $C = A*B$ is an m -by- p matrix; and the element c_{ij} in the i th row and j th column of C is

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

To illustrate, assume there are two portfolios of the same three stocks above but with different quantities.

```
Portfolios = [100  200
              500  400
              300  150];
```

Multiplying the 5-by-3 week's closing prices matrix by the 3-by-2 portfolios matrix yields a 5-by-2 matrix showing each day's closing value for both portfolios.

```
PortfolioValues = WeekClosePr * Portfolios
```

```
PortfolioValues =
```

```

35412.50    26331.25
35587.50    26437.50
35475.00    26325.00
35550.00    26456.25
35512.50    26493.75

```

Monday's values result from multiplying each Monday closing price by its respective number of shares and summing the result for the first portfolio, then doing the same for the second portfolio. Tuesday's values result from multiplying each Tuesday closing price by its respective number of shares and summing the result for the first portfolio, then doing the same for the second portfolio. And so on through the rest of the week. With one simple command, MATLAB quickly performs many calculations.

Multiplying a Matrix by a Scalar

Multiplying a matrix by a scalar is an exception to the dimension and commutative rules. It just operates element-by-element.

```
Portfolios = [100    200
              500    400
              300    150];
```

```
DoublePort = Portfolios * 2
```

```
DoublePort =
    200.00    400.00
   1000.00    800.00
    600.00    300.00

```

Dividing Matrices

Matrix division is useful primarily for solving equations, and especially for solving simultaneous linear equations (see the next section). For example, you want to solve for X in $A * X = B$.

In ordinary algebra, you would simply divide both sides of the equation by A , and X would equal B/A . However, since matrix algebra is not commutative ($A * X \neq X * A$), different processes apply. In formal matrix algebra, the solution involves matrix inversion. MATLAB, however, simplifies the process by providing two matrix division symbols, left and right ($\backslash$ and $/$). In general,

$X = A \backslash B$ solves for X in $A * X = B$

$X = B / A$ solves for X in $X * A = B$.

In general, matrix A must be a nonsingular square matrix; i.e., it must be invertible and it must have the same number of rows and columns. (Generally, a matrix is invertible if the matrix times its inverse equals the identity matrix. To understand the theory and proofs, please consult a textbook on linear algebra such as the one by Hill listed in the “*Bibliography*.”) MATLAB gives a warning message if the matrix is singular or nearly so.

Solving Simultaneous Linear Equations

Matrix division is especially useful in solving simultaneous linear equations. Consider this problem: given two portfolios of mortgage-based instruments, each with certain yields depending on the prime rate, how do you weight the portfolios to achieve certain annual cash flows? The answer involves solving two linear equations.

A linear equation is any equation of the form

$$a_1x + a_2y = b$$

where a_1 , a_2 , and b are constants (with a_1 and a_2 not both zero), and x and y are variables. (It’s a linear equation because it describes a line in the xy -plane. For example the equation $2x + y = 8$ describes a line such that if $x = 2$ then $y = 4$.)

A system of linear equations is a set of linear equations that we usually want to solve at the same time; i.e., simultaneously. A basic principle for exact answers in solving simultaneous linear equations requires that there be as many equations as there are unknowns. To get exact answers for x and y there

must be two equations. For example, to solve for x and y in the system of linear equations

$$2x + y = 13$$

$$x - 3y = -18$$

there must be two equations, which there are. Matrix algebra represents this system as an equation involving three matrices: A for the left-side constants, X for the variables, and B for the right-side constants

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 13 \\ -18 \end{bmatrix}$$

where $A * X = B$.

Solving the system simultaneously simply means solving for X . Using MATLAB,

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix};$$

$$B = \begin{bmatrix} 13 \\ -18 \end{bmatrix};$$

$$X = A \setminus B$$

solves for X in $A * X = B$.

$$X = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

So $x = 3$ and $y = 7$ in this example. In general, you can use matrix algebra to solve any system of linear equations such as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

by representing them as matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

and solving for X in $A \cdot X = B$.

To illustrate, consider this situation. There are two portfolios of mortgage-based instruments, M1 and M2. They have current annual cash payments of \$100 and \$70 per unit, respectively, based on today's prime rate. If the prime rate moves down one percentage point, their payments would be \$80 and \$40. An investor holds 10 units of M1 and 20 units of M2. The investor's receipts equal cash payments times units, or $R = C \cdot U$, for each prime-rate scenario. As word equations,

	M1	M2
Prime flat:	\$100 * 10 units +	\$70 * 20 units = \$2400 receipts
Prime down:	\$80 * 10 units +	\$40 * 20 units = \$1600 receipts

As MATLAB matrices

```
Cash = [100  70
        80  40];

Units = [10
        20];

Receipts = Cash * Units

Receipts =

    2400.00
    1600.00
```

Now the investor asks the question: given these two portfolios and their characteristics, how many units of each should I hold to receive \$7000 if the

prime rate stays flat and \$5000 if the prime drops one percentage point? Find the answer by solving two linear equations.

	M1	M2
Prime flat:	$\$100 * x \text{ units}$	$+ \$70 * y \text{ units} = \7000 receipts
Prime down:	$\$80 * x \text{ units}$	$+ \$40 * y \text{ units} = \5000 receipts

In other words, solve for U (units) in the equation $R \text{ (receipts)} = C \text{ (cash)} * U \text{ (units)}$. Using MATLAB left division

```
Cash = [100  70
        80  40];

Receipts = [7000
            5000];

Units = Cash \ Receipts
Units =

    43.75
    37.50
```

The investor should hold 43.75 units of portfolio M1 and 37.5 units of portfolio M2 to achieve the annual receipts desired.

Operating Element-by-Element

Finally, element-by-element arithmetic operations are called *array* operations. To indicate an array operation in MATLAB, precede the operator with a period (.). Addition and subtraction, and matrix multiplication and division by a scalar, are already array operations so no period is necessary. When using array operations on two matrices, the dimensions of the matrices must be the same. For example, given vectors of stock dividends and closing prices

```
Dividends = [1.90  0.40  1.56  4.50];
Prices = [25.625  17.75  26.125  60.50];

Yields = Dividends ./ Prices
```

Yields =

0.0741 0.0225 0.0597 0.0744

Function Input/Output Arguments

MATLAB was designed to be a large-scale array (vector or matrix) processor. In addition to its linear algebra applications, the general array-based processing facility has the capability to perform repeated operations on collections of data. When MATLAB code is written to operate simultaneously on collections of data stored in arrays, the code is said to be vectorized. Vectorized code is not only clean and concise, but is also efficiently processed by the underlying MATLAB engine.

Input Arguments

Matrix Input

Because MATLAB can process vectors and matrices easily, most functions in the Financial Toolbox allow vector or matrix input arguments, rather than just single (scalar) values.

For example, the `irr` function computes the internal rate of return of a cash flow stream. It accepts a vector of cash flows and returns a scalar-valued internal rate of return. However, it also accepts a matrix of cash flow streams, a column in the matrix representing a different cash flow stream. In this case, `irr` returns a vector of internal rates of return, each entry in the vector corresponding to a column of the input matrix. Many other toolbox functions work similarly.

As an example, suppose you make an initial investment of \$100, from which you then receive by a series of annual cash receipts of \$10, \$20, \$30, \$40, and \$50. This cash flow stream may be stored in a vector

```
CashFlows = [-100 10 20 30 40 50]'
```

which MATLAB displays as

```
CashFlows =  
-100  
  10  
  20  
  30  
  40  
  50
```

The `irr` function can compute the internal rate of return of this stream.

```
Rate = irr(CashFlows)
```

The internal rate of return of this investment is

```
Rate =  
  
0.1201
```

or 12.01%.

In this case, a single cash flow stream (written as an input vector) produces a scalar output – the internal rate of return of the investment.

Extending this example, if you process a matrix of identical cash flow streams

```
Rate = irr([CashFlows CashFlows CashFlows])
```

you should expect to see identical internal rates of return for each of the three investments.

```
Rate =  
  
0.1201    0.1201    0.1201
```

This simple example illustrates the power of vectorized programming. The example shows how to collect data into a matrix and then use a toolbox function to compute answers for the entire collection. This feature can be useful in portfolio management, for example, where you might want to organize multiple assets into a single collection. Place data for each asset in a different column or row of a matrix, then pass the matrix to a Financial Toolbox function. MATLAB performs the same computation on all of the assets at once.

Matrices of String Input

Enter strings in MATLAB surrounded by single quotes ('string').

Strings are stored as character arrays, one ASCII character per element. Thus the date string

```
DateString = '9/16/2001'
```

is actually a 1-by-9 vector. Strings making up the rows of a matrix or vector all must have the same length. To enter several date strings, therefore, use a column vector and be sure all strings are the same length. Fill in with spaces

or zeros. For example, to create a vector of dates corresponding to irregular cash flows

```
DateFields = ['01/12/2001'
               '02/14/2001'
               '03/03/2001'
               '06/14/2001'
               '12/01/2001'];
```

DateFields actually becomes a 5-by-10 character array.

Don't mix numbers and strings in a matrix. If you do, MATLAB treats all entries as characters. For example,

```
Item = [83  90  99  '14-Sep-1999']
```

becomes a 1-by-14 character array, not a 1-by-4 vector, and it contains

```
Item =
```

```
SZc14-Sep-1999
```

Function Output Arguments

Some functions return no arguments, some return just one, and some return multiple arguments. Functions that return multiple arguments use the syntax

```
[A, B, C] = function(variables...)
```

to return arguments A, B, and C. If you omit all but one, the function returns the first argument. Thus for this example if you use the syntax

```
X = function(variables...)
```

function returns a value for A, but not for B or C.

Some functions that return vectors accept only scalars as arguments. Why could such functions not accept vectors as arguments and return matrices, where each column in the output matrix corresponds to an entry in the input vector? The answer is that the output vectors can be variable length and thus will not fit in a matrix without some convention to indicate that the shorter columns are missing data.

Functions that require asset life as an input, and return values corresponding to different periods over that life, cannot generally handle vectors or matrices as input arguments. Those functions are

<code>amortize</code>	Amortization
<code>depxfixdb</code>	Fixed declining-balance depreciation
<code>depxgendb</code>	General declining-balance depreciation
<code>depxsoyd</code>	Sum of years' digits depreciation

For example, suppose you have a collection of assets such as automobiles and you want to compute the depreciation schedules for them. The function `depxfixdb` computes a stream of declining-balance depreciation values for an asset. You might want to set up a vector where each entry is the initial value of each asset. `depxfixdb` also needs the lifetime of an asset. If you were to set up such a collection of automobiles as an input vector, and the lifetimes of those automobiles varied, the resulting depreciation streams would differ in length according to the life of each automobile, and the output column lengths would vary. A matrix must have the same number of rows in each column.

Interest Rate Arguments

One common argument, both as input and output, is interest rate. All Financial Toolbox functions expect and return interest rates as decimal fractions. Thus an interest rate of 9.5% is indicated as 0.095.

Tutorial

Handling and Converting Dates	2-4
Formatting Currency	2-12
Charting Financial Data	2-13
Analyzing and Computing Cash Flows	2-16
Pricing and Computing Yields for Fixed-Income Securities	2-20
Pricing and Analyzing Equity Derivatives	2-32
Analyzing Portfolios	2-37

The Financial Toolbox contains functions that perform many common financial tasks, including:

- Handling and converting dates

Calendar functions convert dates among different formats (including Excel formats), determine future or past dates, find dates of holidays and business days, compute time differences between dates, find coupon dates and coupon periods for coupon bonds, and compute time periods based on 360-, 365-, or 366-day years.

- Formatting currency

The toolbox includes functions for handling decimal values in bank (currency) formats and as fractional prices.

- Charting financial data

Charting functions produce a variety of financial charts including Bollinger bands, high-low-close charts, candlestick plots, point and figure plots, and moving-average plots. The Financial Time Series Toolbox provides additional charting functions. See the *Financial Time Series Toolbox User's Guide* for a description of these functions.

- Analyzing and computing cash flows

Cash-flow evaluation and financial accounting functions compute interest rates, rates of return, payments associated with loans and annuities, future and present values, depreciation, and other standard accounting calculations associated with cash-flow streams.

- Pricing and computing yields for fixed-income securities; analyzing the term structure of interest rates

Securities Industry Association (SIA) compliant fixed-income functions compute prices, yields, accrued interest, and sensitivities for securities such as bonds, zero-coupon bonds, and Treasury bills. They handle odd first and last periods in price/yield calculations, compute accrued interest and discount rates, and calculate convexity and duration. Another set of functions analyzes term structure of interest rates, including pricing bonds from yield curves and bootstrapping yield curves from market prices.

- Analyzing portfolios

Portfolio analysis functions provide basic utilities to compute variances and covariance of portfolios, find combinations to minimize variance, compute Markowitz efficient frontiers, and calculate combined rates of return.

-
- Pricing and analyzing equity derivatives

Derivatives analysis functions compute prices, yields, and sensitivities for derivative securities. They deal with both European and American options.

Black-Scholes functions work with European options. They compute delta, gamma, lambda, rho, theta, and vega, as well as values of call and put options.

Binomial functions work with American options, computing put and call prices. The optional Simulink system provides powerful tools for constructing simulation models for pricing these kinds of options.

The toolbox also contains sets of functions for pricing and analyzing option-embedded bonds and for modeling volatility in time series.

- **Black-Derman-Toy** functions work with both American and European options to compute prices, discounts, and sensitivity measures for bonds with embedded call or put options. See the Reference for additional information.
- **Generalized Autoregressive Conditional Heteroskedasticity (GARCH)** functions model the volatility of univariate economic time series. (The GARCH Toolbox provides a more comprehensive and integrated computing environment. For information see the *GARCH Toolbox User's Guide* or the financial products Web page at <http://www.mathworks.com/products/finprod>.)

Handling and Converting Dates

Since virtually all financial data is dated or derives from a time series, financial functions must have extensive date-handling capabilities. This section discusses date handling in the Financial Toolbox, specifically the topics:

- “Date Formats” on page 2-4
- “Date Conversions” on page 2-5
- “Current Date and Time” on page 2-8
- “Determining Dates” on page 2-8

Note If you specify a two-digit year, MATLAB assumes that the year lies within the 100-year period centered about the current year. See the function `datenum` for specific information. MATLAB internal date handling and calculations generate no ambiguous values. However, whenever possible, programmers should use serial date numbers or date strings containing four-digit years.

Date Formats

You most often work with date strings (14-Sep-1999) when dealing with dates. The Financial Toolbox works internally with *serial date numbers* (e.g., 730377). A serial date number represents a calendar date as the number of days that has passed since a fixed base date. In MATLAB, serial date number 1 is January 1, 0000 A.D. MATLAB also uses serial time to represent fractions of days beginning at midnight; for example, 6 p.m. equals 0.75 serial days. So 6:00 pm on 14-Sep-1999, in MATLAB, is date number 730377.75.

Many toolbox functions that require dates accept either date strings or serial date numbers. If you are dealing with a few dates at the MATLAB command-line level, date strings are more convenient. If you are using toolbox functions on large numbers of dates, as in analyzing large portfolios or cash flows, performance improves if you use date numbers.

The toolbox provides functions that convert date strings to serial date numbers, and vice versa.

Date Conversions

Functions that convert between date formats are

<code>datedisp</code>	Displays a numeric matrix with date entries formatted as date strings
<code>datenum</code>	Converts a date string to a serial date number
<code>datestr</code>	Converts a serial date number to a date string
<code>m2xdate</code>	Converts MATLAB serial date number to Excel serial date number
<code>x2mdate</code>	Converts Excel serial date number to MATLAB serial date number

Another function, `datevec`, converts a date number or date string to a date vector whose elements are [Year Month Day Hour Minute Second]. Date vectors are mostly an internal format for some MATLAB functions; you would not often use them in financial calculations.

Input Conversions

The `datenum` function is important for using the Financial Toolbox efficiently. `datenum` takes an input string in any of several formats, with `'dd-mmm-yyyy'`, `'mm/dd/yyyy'` or `'dd-mmm-yyyy, hh:mm:ss.ss'` most common. The input string can have up to six fields formed by letters and numbers separated by any other characters.

- The day field is an integer from 1 to 31.
- The month field is either an integer from 1 to 12 or an alphabetic string with at least three characters.
- The year field is a nonnegative integer: if only two numbers are specified, then the year is assumed to lie within the 100-year period centered about the current year; if the year is omitted, the current year is used as the default.
- The hours, minutes, and seconds fields are optional. They are integers separated by colons or followed by `'am'` or `'pm'`.

For example, if the current year is 1999, then these are all equivalent

```
'17-May-1999'
'17-May-99'
```

```
'17-may'  
'May 17, 1999'  
'5/17/99'  
'5/17'
```

and both of these represent the same time.

```
'17-May-1999, 18:30'  
'5/17/99/6:30 pm'
```

Note that the default format for numbers-only input follows the American convention. Thus 3/6 is March 6, not June 3.

With `datenum` you can convert dates into serial date format, store them in a matrix variable, then later pass the variable to a function. Alternatively, you can use `datenum` directly in a function input argument list.

For example, consider the function `bndprice` that computes the price of a bond given the yield-to-maturity. First set up variables for the yield-to-maturity, coupon rate, and the necessary dates.

```
Yield      = 0.07;  
CouponRate = 0.08;  
Settle     = datenum('17-May-2000');  
Maturity   = datenum('01-Oct-2000');
```

Then call the function with the variables

```
bndprice(Yield, CouponRate, Settle, Maturity)
```

Alternatively, convert date strings to serial date numbers directly in the function input argument list.

```
bndprice(0.07, 0.08, datenum('17-May-2000'), ...  
         datenum('01-Oct-2000'))
```

`bndprice` is an example of a function designed to detect the presence of date strings and make the conversion automatically. For these functions date strings may be passed directly.

```
bndprice(0.07, 0.08, '17-May-2000', '01-Oct-2000')
```

The decision to represent dates as either date strings or serial date numbers is often a matter of convenience. For example, when formatting data for visual display or for debugging date-handling code, it is often much easier to view

dates as date strings because serial date numbers are difficult to interpret. Alternatively, serial date numbers are just another type of numeric data, and can be placed in a matrix along with any other numeric data for convenient manipulation.

Remember that if you create a vector of input date strings, use a column vector and be sure all strings are the same length. Fill with spaces or zeros. See “Matrices of String Input” on page 1-18.

Output Conversions

The function `datestr` converts a serial date number to one of 19 different date string output formats showing date, time, or both. The default output for dates is a day-month-year string, e.g., 24-Aug-2000. This function is quite useful for preparing output reports.

Format	Description
01-Mar-2000 15:45:17	day-month-year hour:minute:second
01-Mar-2000	day-month-year
03/01/00	month/day/year
Mar	month, three letters
M	month, single letter
3	month
03/01	month/day
1	day of month
Wed	day of week, three letters
W	day of week, single letter
2000	year, four numbers
99	year, two numbers
Mar01	month year

Format	Description
15:45:17	hour:minute:second
03:45:17 PM	hour:minute:second AM or PM
15:45	hour:minute
03:45 PM	hour:minute AM or PM
Q1-99	calendar quarter-year
Q1	calendar quarter

Current Date and Time

The functions `today` and `now` return serial date numbers for the current date, and the current date and time, respectively.

```
today

ans =
    730693

now

ans =
    730693.48
```

The MATLAB function `date` returns a string for today's date.

```
date

ans =
    26-Jul-2000
```

Determining Dates

The toolbox provides many functions for determining specific dates, including functions which account for holidays and other nontrading days.

For example, you schedule an accounting procedure for the last Friday of every month. The `lweekdate` function returns those dates for 2000; the 6 specifies Friday.

```
Fridates = lweekdate(6, 2000, 1:12);
```

```
Fridays = datestr(Fridates)
```

```
Fridays =
```

```
28-Jan-2000
25-Feb-2000
31-Mar-2000
28-Apr-2000
26-May-2000
30-Jun-2000
28-Jul-2000
25-Aug-2000
29-Sep-2000
27-Oct-2000
24-Nov-2000
29-Dec-2000
```

Or your company closes on Martin Luther King Jr. Day, which is the third Monday in January. The `nweekdate` function determines those dates for 2001 through 2004.

```
MLKDates = nweekdate(3, 2, 2001:2004, 1);
```

```
MLKDays = datestr(MLKDates)
```

```
MLKDays =
```

```
15-Jan-2001
21-Jan-2002
20-Jan-2003
19-Jan-2004
```

Accounting for holidays and other nontrading days is important when examining financial dates. The toolbox provides the `holidays` function, which contains holidays and special nontrading days for the New York Stock

Exchange between 1950 and 2030, inclusive. You can edit the `holidays.m` file to customize it with your own holidays and nontrading days. In this example, use it to determine the standard holidays in the last half of 2000.

```
LHHDates = holidays('1-Jul-2000', '31-Dec-2000');
```

```
LHHDays = datestr(LHHDates)
```

```
LHHDays =
```

```
04-Jul-2000
```

```
04-Sep-2000
```

```
23-Nov-2000
```

```
25-Dec-2000
```

Now use the toolbox `busdate` function to determine the next business day after these holidays.

```
LHNextDates = busdate(LHHDates);
```

```
LHNextDays = datestr(LHNextDates)
```

```
LHNextDays =
```

```
05-Jul-2000
```

```
05-Sep-2000
```

```
24-Nov-2000
```

```
26-Dec-2000
```

The toolbox also provides the `cfdates` function to determine cash-flow dates for securities with periodic payments. This function accounts for the coupons per year, the day-count basis, and the end-of-month rule. For example, to determine the cash-flow dates for a security that pays four coupons per year on the last day of the month, on an actual/365 day-count basis, just enter the settlement date, the maturity date, and the parameters.

```
PayDates = cfdates('14-Mar-2000', '30-Nov-2001', 4, 3, 1);
```

```
PayDays = datestr(PayDates)
```

PayDays =

31-May-2000

31-Aug-2000

30-Nov-2000

28-Feb-2001

31-May-2001

31-Aug-2001

30-Nov-2001

Formatting Currency

The Financial Toolbox provides several functions to format currency and chart financial data. The currency formatting functions are

<code>cur2frac</code>	Converts decimal currency values to fractional values
<code>cur2str</code>	Converts a value to Financial Toolbox bank format
<code>frac2cur</code>	Converts fractional currency values to decimal values

These examples show their use.

```
Dec = frac2cur('12.1', 8)
```

returns `Dec = 12.125`, which is the decimal equivalent of $12\text{-}1/8$. The second input variable is the denominator of the fraction.

```
Str = cur2str(-8264, 2)
```

returns the string `($8264.00)`. For this toolbox function, the output format is a numerical format with dollar sign prefix, two decimal places, and negative numbers in parentheses; e.g., `($123.45)` and `$6789.01`. The standard MATLAB bank format uses two decimal places, no dollar sign, and a minus sign for negative numbers; e.g., `-123.45` and `6789.01`.

Charting Financial Data

The toolbox financial charting functions

<code>bolling</code>	Bollinger band chart
<code>candle</code>	Candlestick chart
<code>pointfig</code>	Point and figure chart
<code>highlow</code>	High, low, open, close chart
<code>movavg</code>	Leading and lagging moving averages chart

plot financial data and produce presentation-quality figures quickly and easily. They work with standard MATLAB functions that draw axes, control appearance, and add labels and titles. For users having additional charting requirements, the Financial Time Series Toolbox provides a more comprehensive set of charting functions.

Here are two plotting examples: a high-low-close chart of sample IBM stock price data, and a Bollinger band chart of the same data. These examples load data from an external file (`ibm.dat`), then call the functions using subsets of the data. `ibm` is a six-column matrix where each row is a trading day's data and where columns 2, 3, and 4 contain the high, low, and closing prices, respectively.

Note The data in `ibm.dat` is fictional and for illustrative use only.

High-Low-Close Chart Example

First load the data and set up matrix dimensions. `load` and `size` are standard MATLAB functions.

```
load ibm.dat;  
[ro, co] = size(ibm);
```

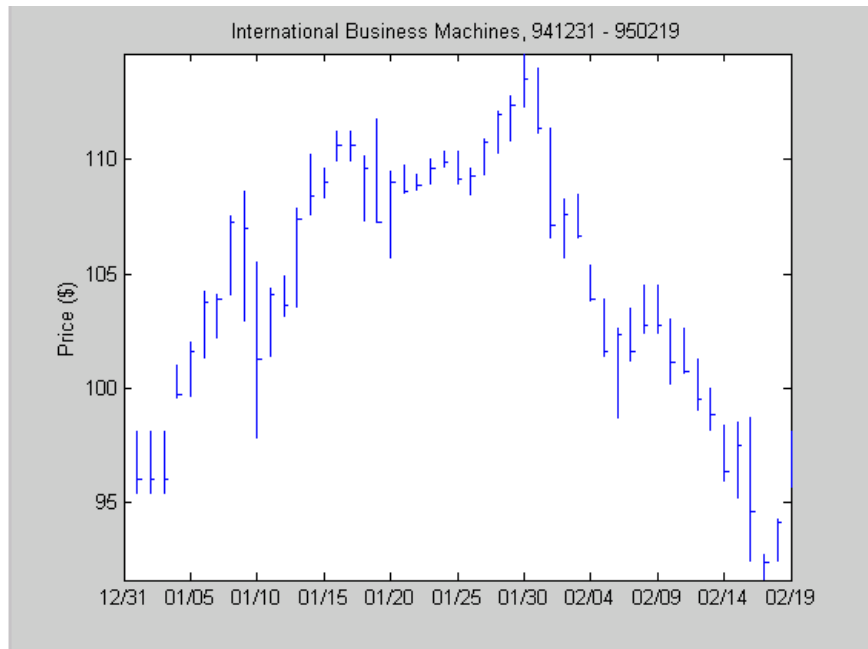
Open a figure window for the chart. Use the Financial Toolbox `highlow` function to plot high, low, and close prices for the last 50 trading days in the data file.

```
figure;
highlow(ibm(ro-50:ro,2),ibm(ro-50:ro,3),ibm(ro-50:ro,4),[],'b');
```

Add labels and title, and set axes with standard MATLAB functions. Use the Financial Toolbox `dateaxis` function to provide dates for the x-axis ticks.

```
xlabel('');
ylabel('Price ($)');
title('International Business Machines, 941231 - 950219');
axis([0 50 -inf inf]);
dateaxis('x',6,'31-Dec-1994')
```

MATLAB produces a figure similar to this. The plotted data and axes you see may differ. Viewed online, the high-low-close bars are blue.



Bollinger Chart Example

Next the Financial Toolbox `bollinger` function produces a Bollinger band chart using all the closing prices in the same IBM stock price matrix. A Bollinger band chart plots actual data along with three other bands of data. The upper

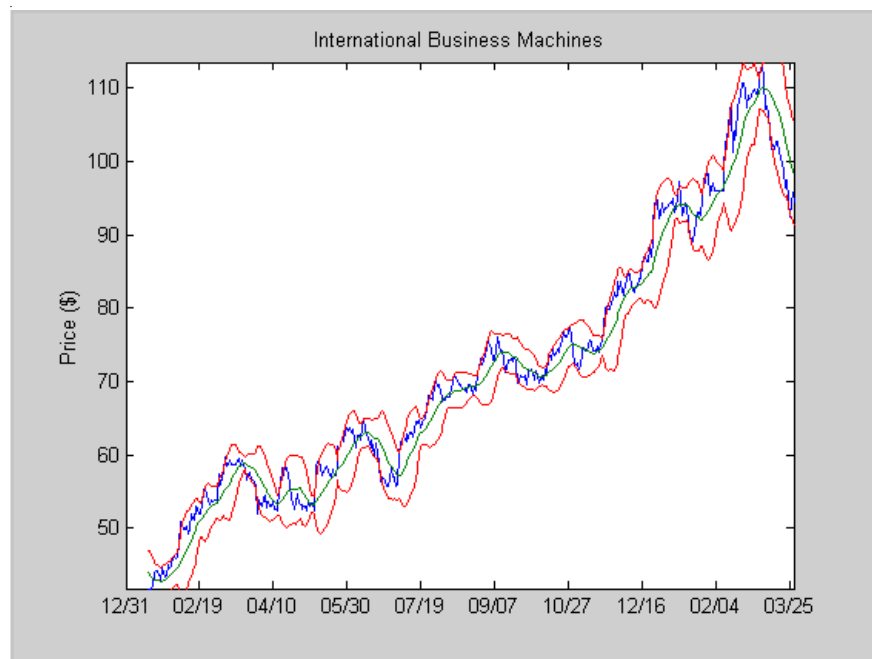
band is two standard deviations above a moving average; the lower band is two standard deviations below that moving average; and the middle band is the moving average itself. This example uses a 15-day moving average.

Assuming the previous IBM data is still loaded, simply execute the Financial Toolbox function.

```
bolling(ibm(:,4), 15, 0);
```

Specify the axes, labels, and titles. Again, use `dateaxis` to add the x-axis dates.

```
axis([0 ro min(ibm(:,4)) max(ibm(:,4))]);
ylabel('Price ($)');
title(['International Business Machines']);
dateaxis('x', 6, '31-Dec-1994')
```



For help using MATLAB plotting functions, see “Creating Plots” in the MATLAB documentation. See the MATLAB documentation for details on the `axis`, `title`, `xlabel`, and `ylabel` functions.

Analyzing and Computing Cash Flows

The Financial Toolbox cash-flow functions compute interest rates, rates of return, present or future values, depreciation streams, and annuities.

Some examples in this section use this income stream: an initial investment of \$20,000 followed by three annual return payments, a second investment of \$5,000, then four more returns. Investments are negative cash flows, return payments are positive cash flows.

```
Stream = [-20000, 2000, 2500, 3500, -5000, 6500, ...
          9500, 9500, 9500];
```

Interest Rates / Rates of Return

Several functions calculate interest rates involved with cash flows. To compute the internal rate of return of the cash stream, simply execute the toolbox function `irr`

```
ROR = irr(Stream)
```

which gives a rate of return of 11.72%.

Note that the internal rate of return of a cash flow may not have a unique value. Every time the sign changes in a cash flow, the equation defining `irr` can give up to two additional answers. An `irr` computation requires solving a polynomial equation, and the number of real roots of such an equation can depend on the number of sign changes in the coefficients. The equation for internal rate of return is

$$\frac{cf_1}{(1+r)} + \frac{cf_2}{(1+r)^2} + \dots + \frac{cf_n}{(1+r)^n} + Investment = 0$$

where *Investment* is a (negative) initial cash outlay at time 0, cf_n is the cash flow in the n th period, and n is the number of periods. Basically, `irr` finds the rate r such that the net present value of the cash flow equals the initial investment. If all of the cf_n s are positive there is only one solution. Every time there is a change of sign between coefficients, up to two additional real roots are possible. There is usually only one answer that makes sense, but it is possible to get returns of both 5% and 11% (for example) from one income stream.

Another toolbox rate function, `effrr`, calculates the effective rate of return given an annual interest rate (also known as nominal rate or annual percentage rate, APR) and number of compounding periods per year. To find the effective rate of a 9% APR compounded monthly, simply enter

```
Rate = effrr(0.09, 12)
```

The answer is 9.38%.

A companion function `nomrr` computes the nominal rate of return given the effective annual rate and the number of compounding periods.

Present or Future Values

The toolbox includes functions to compute the present or future value of cash flows at regular or irregular time intervals with equal or unequal payments: `fvfix`, `fvvar`, `pvinfos`, and `pvinfos`. The `-fix` functions assume equal cash flows at regular intervals, while the `-var` functions allow irregular cash flows at irregular periods.

Now compute the net present value of the sample income stream for which you computed the internal rate of return. This exercise also serves as a check on that calculation because the net present value of a cash stream at its internal rate of return should be zero. Enter

```
NPV = pvvar(Stream, ROR)
```

which returns an answer very close to zero. The answer usually is not *exactly* zero due to rounding errors and the computational precision of the computer.

Note Other toolbox functions behave similarly. The functions that compute a bond's yield, for example, often must solve a nonlinear equation. If you then use that yield to compute the net present value of the bond's income stream, it usually does not *exactly* equal the purchase price — but the difference is negligible for practical applications.

Depreciation

The toolbox includes functions to compute standard depreciation schedules: straight line, general declining-balance, fixed declining-balance, and sum of

years' digits. Functions also compute a complete amortization schedule for an asset, and return the remaining depreciable value after a depreciation schedule has been applied.

This example depreciates an automobile worth \$15,000 over five years with a salvage value of \$1,500. It computes the general declining balance using two different depreciation rates: 50% (or 1.5), and 100% (or 2.0, also known as double declining balance). Enter

```
Decline1 = depgendb(15000, 1500, 5, 1.5)
Decline2 = depgendb(15000, 1500, 5, 2.0)
```

which returns

Decline1 =					
	4500.00	3150.00	2205.00	1543.50	2101.50
Decline2 =					
	6000.00	3600.00	2160.00	1296.00	444.00

These functions return the actual depreciation amount for the first four years and the remaining depreciable value as the entry for the fifth year.

Annuities

Several toolbox functions deal with annuities. This first example shows how to compute the interest rate associated with a series of loan payments when only the payment amounts and principal are known. For a loan whose original value was \$5000.00 and which was paid back monthly over four years at \$130.00/month

```
Rate = annurate(4*12, 130, 5000, 0, 0)
```

The function returns a rate of 0.0094 monthly, or approximately 11.28% annually.

The next example uses a present-value function to show how to compute the initial principal when the payment and rate are known. For a loan paid at \$300.00/month over four years at 11% annual interest

```
Principal = pvfix(0.11/12, 4*12, 300, 0, 0)
```

The function returns the original principal value of \$11,607.43.

The final example computes an amortization schedule for a loan or annuity. The original value was \$5000.00 and was paid back over 12 months at an annual rate of 9%.

```
[Prpmt, Intpmt, Balance, Payment] = ...
    amortize(0.09/12, 12, 5000, 0, 0);
```

This function returns vectors containing the amount of principal paid,

```
Prpmt = [402.76  405.78  408.82  411.89  414.97  418.09
         421.22  424.38  427.56  430.77  434.00  437.26]
```

the amount of interest paid,

```
Intpmt = [34.50  31.48  28.44  25.37  22.28  19.17
          16.03  12.88   9.69   6.49   3.26   0.00]
```

the remaining balance for each period of the loan,

```
Balance = [4600.24  4197.49  3791.71  3382.89  2971.01
           2556.03  2137.94  1716.72  1292.34   864.77
           434.00    0.00]
```

and a scalar for the monthly payment.

```
Payment = 437.26
```

Pricing and Computing Yields for Fixed-Income Securities

The Securities Industry Association (SIA) has established conventions regarding bond pricing, yield calculation and quotation, time factors and accrued interest, coupon and quasi-coupon dates, and duration and convexity sensitivity measures. The Financial Toolbox includes SIA-compliant functions to compute accrued interest, determine prices and yields, as well as calculate convexity and duration of fixed-income securities. It also includes a set of functions to generate and analyze term structure of interest rates.

SIA-compliant functions can be used with U.S. Treasury bills, bonds, and notes; corporate bonds; and municipal bonds. Bonds can have long, normal or short first or last coupon periods.

The “Function Reference” identifies SIA-compliant functions. These functions have been thoroughly tested against the benchmarks found in Jan Mayle’s “Standard Securities Calculation Methods” document listed in the “Bibliography.”

Terminology

Since terminology varies among texts on this subject, here are some basic definitions that apply to these Financial Toolbox functions. The “Glossary” contains additional definitions.

The *settlement date* of a bond is the date when money first changes hands; i.e., when a buyer pays for a bond. It need not coincide with the *issue date*, which is the date a bond is first offered for sale.

The *first coupon date* and *last coupon date* are the dates when the first and last coupons are paid, respectively. Although bonds typically pay periodic annual or semi-annual coupons, the length of the first and last coupon periods may differ from the standard coupon period. The toolbox includes price and yield functions that handle these odd first and/or last periods.

Successive *quasi-coupon dates* determine the length of the standard coupon period for the fixed income security of interest, and do not necessarily coincide with actual coupon payment dates. The toolbox includes functions that calculate both actual and quasi-coupon dates for bonds with odd first and/or last periods.

Fixed-income securities can be purchased on dates that do not coincide with coupon payment dates. In this case, the bond owner is not entitled to the full

value of the coupon for that period. When a bond is purchased between coupon dates, the buyer must compensate the seller for the pro-rata share of the coupon interest earned from the previous coupon payment date. This pro-rata share of the coupon payment is called *accrued interest*. The *purchase price*, the price actually paid for a bond, is the quoted market price plus accrued interest.

The *maturity date* of a bond is the date when the issuer returns the final face value, also known as the *redemption value* or *par value*, to the buyer. The *yield-to-maturity* of a bond is the nominal compound rate of return that equates the present value of all future cash flows (coupons and principal) to the current market price of the bond.

The *period* of a bond refers to the frequency with which the issuer of a bond makes coupon payments to the holder.

Table 2-1: Period of a Bond

Period Value	Payment Schedule
0	No coupons. (Zero coupon bond.)
1	Annual
2	Semi-annual
3	Tri-annual
4	Quarterly
6	Bi-monthly
12	Monthly

The *basis* of a bond refers to the basis or day-count convention for a bond. Basis is normally expressed as a fraction in which the numerator determines the number of days between two dates, and the denominator determines the number of days in the year. For example, the numerator of *actual/actual* means that when determining the number of days between two dates, count the actual number of days; the denominator means that you use the actual

number of days in the given year in any calculations (either 365 or 366 days depending on whether or not the given year is a leap year).

Table 2-2: Basis of a Bond

Basis Value	Meaning
0 (default)	actual/actual
1	30/360
2	actual/360
3	actual/365

The *end-of-month rule* affects a bond's coupon payment structure. When the rule is in effect, a security that pays a coupon on the last actual day of a month will always pay coupons on the last day of the month. This means, for example, that a semi-annual bond that pays a coupon on February 28 in nonleap years will pay coupons on August 31 in all years and on February 29 in leap years.

Table 2-3: End-of-Month Rule

End of Month Rule Value	Meaning
1 (default)	Rule in effect.
0	Rule not in effect.

SIA Framework

Many of the fixed-income related functions in the Financial Toolbox comply with the Securities Industry Association (SIA) conventions. Although not all

SIA-compliant functions require the same input arguments, they all accept the following common set of input arguments.

Table 2-4: SIA Common Input Arguments

Input	Meaning
Settle	Settlement date
Maturity	Maturity date
Period	Coupon payment period
Basis	Day-count basis
EndMonthRule	End-of-month payment rule
IssueDate	Bond issue date
FirstCouponDate	First coupon payment date
LastCouponDate	Last coupon payment date

Of the common input arguments, only `Settle` and `Maturity` are required. All others are optional. They will be set to the default values if you do not explicitly set them. Note that, by default, the `FirstCouponDate` and `LastCouponDate` are nonapplicable. In other words, if you do not specify `FirstCouponDate` and `LastCouponDate`, the bond is assumed to have no odd first or last coupon periods. In this case, the bond is simply a standard bond with a coupon payment structure based solely on the maturity date.

SIA Default Parameter Values

To illustrate the use of default values in SIA-compliant functions, consider the `cfdates` function, which computes actual cash flow payment dates for a portfolio of fixed income securities regardless of whether the first and/or last coupon periods are normal, long, or short.

The complete calling syntax with the full input argument list is

```
CFlowDates = cfdates(Settle, Maturity, Period, Basis, ...
    EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
```

while the minimal calling syntax requires only settlement and maturity dates

```
CFlowDates = cfdates(Settle, Maturity)
```

Single Bond Example

As an example, suppose you have a bond with these characteristics

```
Settle          = '20-Sep-1999'  
Maturity        = '15-Oct-2007'  
Period          = 2  
Basis           = 0  
EndMonthRule    = 1  
IssueDate       = NaN  
FirstCouponDate = NaN  
LastCouponDate  = NaN
```

Note that `Period`, `Basis`, and `EndMonthRule` are set to their default values, and `IssueDate`, `FirstCouponDate`, and `LastCouponDate` are set to `NaN`.

Formally, a `NaN` is an IEEE arithmetic standard for *Not-a-Number* and is used to indicate the result of an undefined operation (e.g., zero divided by zero). However, `NaN` is also a very convenient placeholder. In the SIA functions of the Financial Toolbox, `NaN` indicates the presence of a nonapplicable value. It tells the SIA fixed-income functions to ignore the input value and apply the default. Setting `IssueDate`, `FirstCouponDate`, and `LastCouponDate` to `NaN` in this example tells `cfdates` to assume that the bond has been issued prior to settlement and that no odd first or last coupon periods exist.

Having set these values, all these calls to `cfdates` produce the same result.

```
cfdates(Settle, Maturity)  
cfdates(Settle, Maturity, Period)  
cfdates(Settle, Maturity, Period, [])  
cfdates(Settle, Maturity, [], Basis)  
cfdates(Settle, Maturity, [], [])  
cfdates(Settle, Maturity, Period, [], EndMonthRule)  
cfdates(Settle, Maturity, Period, [], NaN)  
cfdates(Settle, Maturity, Period, [], [], IssueDate)  
cfdates(Settle, Maturity, Period, [], [], IssueDate, [], [])  
cfdates(Settle, Maturity, Period, [], [], [], [], LastCouponDate)  
cfdates(Settle, Maturity, Period, Basis, EndMonthRule, ...  
IssueDate, FirstCouponDate, LastCouponDate)
```


Thus, leaving a particular input unspecified has the same effect as passing an empty matrix ([]) or passing a NaN – all three tell `cfdates` (and other SIA-compliant functions) to use the default value for a particular input parameter.

Bond Portfolio Example

Since the previous example included only a single bond, there was no difference between passing an empty matrix or passing a NaN for an optional input argument. For a portfolio of bonds, however, using NaN as a placeholder is the only way to specify default acceptance for some bonds while explicitly setting nondefault values for the remaining bonds in the portfolio.

Now suppose you have a portfolio of two bonds.

```
Settle    = '20-Sep-1999'
Maturity  = ['15-Oct-2007'; '15-Oct-2010']
```

These calls to `cfdates` all set the coupon period to its default value (`Period = 2`) for both bonds.

```
cfdates(Settle, Maturity, 2)
cfdates(Settle, Maturity, [2 2])
cfdates(Settle, Maturity, [])
cfdates(Settle, Maturity, NaN)
cfdates(Settle, Maturity, [NaN NaN])
cfdates(Settle, Maturity)
```

The first two calls explicitly set `Period = 2`. Since `Maturity` is a 2-by-1 vector of maturity dates, `cfdates` knows you have a two-bond portfolio.

The first call specifies a single (i.e., scalar) 2 for `Period`. Passing a scalar tells `cfdates` to apply the scalar-valued input to all bonds in the portfolio. This is an example of implicit scalar-expansion. Note that the settlement date has been implicit scalar-expanded as well.

The second call also applies the default coupon period by explicitly passing a two-element vector of 2's. The third call passes an empty matrix, which `cfdates` interprets as an invalid period, for which the default value will be used. The fourth call is similar, except that a NaN has been passed. The fifth call passes two NaN's, and has the same effect as the third. The last call passes the minimal input set.

Finally, consider the following calls to `cfdates` for the same two-bond portfolio.

```
cfdates(Settle, Maturity, [4 NaN])  
cfdates(Settle, Maturity, [4 2])
```

The first call explicitly sets `Period = 4` for the first bond and implicitly sets the default `Period = 2` for the second bond. The second call has the same effect as the first but explicitly sets the periodicity for both bonds.

The optional input `Period` has been used for illustrative purpose only. The default-handling process illustrated in the examples applies to any of the optional input arguments.

SIA Coupon Date Calculations

Calculating coupon dates, either actual or quasi dates, is notoriously complicated. The Financial Toolbox follows the SIA conventions in coupon date calculations.

The first step in finding the coupon dates associated with a bond is to determine the reference, or synchronization date (the *sync date*). Within the SIA framework, the order of precedence for determining the sync date is (1) the first coupon date, (2) the last coupon date, and finally (3) the maturity date.

In other words, an SIA-compliant function in the Financial Toolbox first examines the `FirstCouponDate` input. If `FirstCouponDate` is specified, coupon payment dates and quasi-coupon dates are computed with respect to `FirstCouponDate`; if `FirstCouponDate` is unspecified, empty (`[]`), or `NaN`, then the `LastCouponDate` is examined. If `LastCouponDate` is specified, coupon payment dates and quasi-coupon dates are computed with respect to `LastCouponDate`. If both `FirstCouponDate` and `LastCouponDate` are unspecified, empty (`[]`), or `NaN`, the `Maturity` (a required input argument) serves as the sync date.

SIA Semi-Annual Yield Conventions

Within the SIA framework, all yields and time factors for price-to-yield conversion are quoted on a semi-annual bond basis (see `bndprice`, `bndyield`, and `cfamounts`) regardless of the period of the bond's coupon payments (including zero-coupon bonds). In addition, any yield-related sensitivity (i.e., duration and convexity), when quoted on a periodic basis, assumes semi-annual coupon periods. (See `bndconvp`, `bndconvy`, `bnddurp`, and `bnddury`).

Pricing Functions

This example shows how easily you can compute the price of a bond with an odd first period using the SIA-compliant function `bndprice`. Assume you have a bond with these characteristics

```
Settle          = '11-Nov-1992';  
Maturity        = '01-Mar-2005';  
IssueDate       = '15-Oct-1992';  
FirstCouponDate = '01-Mar-1993';  
CouponRate      = 0.0785;  
Yield           = 0.0625;
```

Allow coupon payment period (`Period = 2`), day-count basis (`Basis = 0`), and end-of-month rule (`EndMonthRule = 1`) to assume the default values. Also, assume there is no odd last coupon date and that the face value of the bond is \$100. Calling the function

```
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, ...  
    Maturity, [], [], [], IssueDate, FirstCouponDate)
```

returns a price of \$113.60 and accrued interest of \$0.59.

Similar functions compute prices with regular payments, odd first and last periods, as well as prices of Treasury bills and discounted securities such as zero-coupon bonds.

Note `bndprice` and other SIA-compliant functions use nonlinear formulas to compute the price of a security. For this reason, the Financial Toolbox uses Newton's method when solving for an independent variable within a formula. See any elementary numerical methods textbook for the mathematics underlying Newton's method.

Yield Functions

To illustrate toolbox yield functions, compute the yield of a bond that has odd first and last periods and settlement in the first period. First set up variables for settlement, maturity date, issue, first coupon, and a last coupon date.

```
Settle          = '12-Jan-2000';  
Maturity        = '01-Oct-2001';
```

```
IssueDate      = '01-Jan-2000';  
FirstCouponDate = '15-Jan-2000';  
LastCouponDate  = '15-Apr-2000';
```

Assume a face value of \$100. Specify a purchase price of \$95.70, a coupon rate of 4%, quarterly coupon payments, and a 30/360 day-count convention (Basis = 1).

```
Price          = 95.7;  
CouponRate     = 0.04;  
Period         = 4;  
Basis          = 1;  
EndMonthRule   = 1;
```

Calling the function

```
Yield = bndyield(Price, CouponRate, Settle, Maturity, Period,...  
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
```

returns

Yield = 0.0659 (6.60%).

Fixed-Income Sensitivities

The toolbox includes SIA-compliant functions to perform sensitivity analysis such as convexity and the Macaulay and modified durations for fixed-income securities. The Macaulay duration of an income stream, such as a coupon bond, measures how long, on average, the owner waits before receiving a payment. It is the weighted average of the times payments are made, with the weights at time T equal to the present value of the money received at time T. The modified duration is the Macaulay duration discounted by the per-period interest rate; i.e., divided by $(1 + \text{rate}/\text{frequency})$.

To illustrate, the following example computes the annualized Macaulay and modified durations, and the periodic Macaulay duration for a bond with settlement (12-Jan-2000) and maturity (01-Oct-2001) dates as above, a 5% coupon rate, and a 4.5% yield to maturity. For simplicity, any optional input arguments assume default values (i.e., semi-annual coupons, and day-count basis = 0 (actual/actual), coupon payment structure synchronized to the maturity date, and end-of-month payment rule in effect).

```
CouponRate = 0.05;
Yield = 0.045;

[ModDuration, YearDuration, PerDuration] = bnddury(Yield,...
CouponRate, Settle, Maturity)
```

The durations are

```
ModDuration = 1.6107 (years)
YearDuration = 1.6470 (years)
PerDuration = 3.2940 (semi-annual periods)
```

Note that the semi-annual periodic Macaulay duration (PerDuration) is twice the annualized Macaulay duration (YearDuration).

Term Structure of Interest Rates

The toolbox contains several functions to derive and analyze interest rate curves, including data conversion and extrapolation, bootstrapping, and interest-rate curve conversion functions.

One of the first problems in analyzing the term structure of interest rates is dealing with market data reported in different formats. Treasury bills, for example, are quoted with bid and asked bank-discount rates. Treasury notes and bonds, on the other hand, are quoted with bid and asked prices based on \$100 face value. To examine the full spectrum of Treasury securities, analysts must convert data to a single format. Toolbox functions ease this conversion. This brief example uses only one security each; analysts often use 30, 100, or more of each.

First, capture Treasury bill quotes in their reported format

```
%      Maturity      Days  Bid    Ask    AskYield
TBill = [datetime('12/26/2000') 53    0.0503 0.0499 0.0510];
```

then capture Treasury bond quotes in their reported format

```
%      Coupon  Maturity      Bid    Ask    AskYield
TBond = [0.08875  datetime(2001,11,5) 103+4/32 103+6/32 0.0564];
```

and note that these quotes are based on a November 3, 2000 settlement date.

```
Settle = datetime('3-Nov-2000');
```

Next use the toolbox `tbl2bond` function to convert the Treasury bill data to Treasury bond format.

```
TBTBond = tbl2bond(TBill)
```

```
TBTBond =
      0      730846      99.26      99.27      0.05
```

(The second element of `TBTBond` is the serial date number for December 26, 2000.)

Now combine short-term (Treasury bill) with long-term (Treasury bond) data to set up the overall term structure.

```
TBondsAll = [TBTBond; TBond]
```

```
TBondsAll =
      0      730846      99.26      99.27      0.05
  0.09      731160     103.13     103.19      0.06
```

The toolbox provides a second data-preparation function, `tr2bonds`, to convert the bond data into a form ready for the bootstrapping functions. `tr2bonds` generates a matrix of bond information sorted by maturity date, plus vectors of prices and yields.

```
[Bonds, Prices, Yields] = tr2bonds(TBondsAll);
```

With this market data, you are now ready to use one of the toolbox bootstrapping functions to derive an implied zero curve. Bootstrapping is a process whereby you begin with known data points and solve for unknown data points using an underlying arbitrage theory. Every coupon bond can be valued as a package of zero-coupon bonds which mimic its cash flow and risk characteristics. By mapping yields-to-maturity for each theoretical zero-coupon bond, to the dates spanning the investment horizon, you can create a theoretical zero-rate curve. The toolbox provides two bootstrapping functions: `zbtprice` derives a zero curve from bond data and *prices*, and `zbtyield` derives a zero curve from bond data and *yields*. Using `zbtprice`

```
[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle)
```

```
ZeroRates =
```

```
    0.05
```

```
    0.06
```

```
CurveDates =
```

```
    730846
```

```
    731160
```

CurveDates gives the investment horizon.

```
datestr(CurveDates)
```

```
ans =
```

```
26-Dec-2000
```

```
05-Nov-2001
```

Additional toolbox functions construct discount, forward, and par yield curves from the zero curve, and vice versa.

```
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,...  
Settle);  
[FwdRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle);  
[PYldRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...  
Settle);
```

Pricing and Analyzing Equity Derivatives

These toolbox functions compute prices, sensitivities, and profits for portfolios of options or other equity derivatives. They use the Black-Scholes model for European options and the binomial model for American options. Such measures are useful for managing portfolios and for executing collars, hedges, and straddles.

Sensitivity Measures

There are six basic sensitivity measures associated with option pricing: delta, gamma, lambda, rho, theta, and vega — the “greeks.” The toolbox provides functions for calculating each sensitivity and for implied volatility.

Delta

Delta of a derivative security is the rate of change of its price relative to the price of the underlying asset. It is the first derivative of the curve that relates the price of the derivative to the price of the underlying security. When delta is large, the price of the derivative is sensitive to small changes in the price of the underlying security.

Gamma

Gamma of a derivative security is the rate of change of delta relative to the price of the underlying asset; i.e., the second derivative of the option price relative to the security price. When gamma is small, the change in delta is small. This sensitivity measure is important for deciding how much to adjust a hedge position.

Lambda

Lambda, also known as the elasticity of an option, represents the percentage change in the price of an option relative to a 1% change in the price of the underlying security.

Rho

Rho is the rate of change in option price relative to the risk-free interest rate.

Theta

Theta is the rate of change in the price of a derivative security relative to time. Theta is usually very small or negative since the value of an option tends to drop as it approaches maturity.

Vega

Vega is the rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large the security is sensitive to small changes in volatility. For example, options traders often must decide whether to buy an option to hedge against vega or gamma. The hedge selected usually depends upon how frequently one rebalances a hedge position and also upon the variance of the price of the underlying asset (the volatility). If the variance is changing rapidly, balancing against vega is usually preferable.

Implied Volatility

The implied volatility of an option is the variance that makes a call option price equal to the market price. It helps determine a market estimate for the future volatility of a stock and provides the input volatility (when needed) to the other Black-Scholes functions.

Analysis Models

Toolbox functions for analyzing equity derivatives use the Black-Scholes model for European options and the binomial model for American options. The Black-Scholes model makes several assumptions about the underlying securities and their behavior. The binomial model, on the other hand, makes far fewer assumptions about the processes underlying an option. For further explanation, please see the book by John Hull listed in the *Bibliography*.

Black-Scholes Model

Using the Black-Scholes model entails several assumptions:

- The prices of the underlying asset follow an Ito process. (See Hull, 196 - 198)
- The option can be exercised only on its expiration date (European option).
- Short selling is permitted.
- There are no transaction costs.
- All securities are divisible and pay no dividends.

- There is no riskless arbitrage.
- Trading is a continuous process.
- The risk-free interest rate is constant and remains the same for all maturities.

If any of these assumptions is untrue, Black-Scholes may not be an appropriate model.

To illustrate toolbox Black-Scholes functions, this example computes the call and put prices of a European option and its delta, gamma, lambda, and implied volatility. The asset price is \$100.00, the exercise price is \$95.00, the risk-free interest rate is 10%, the time to maturity is 0.25 years, the volatility is 0.50, and the dividend rate is 0. Simply executing the toolbox functions

```
[OptCall, OptPut] = blsprice(100, 95, 0.10, 0.25, 0.50, 0);  
[CallVal, PutVal] = blsdelta(100, 95, 0.10, 0.25, 0.50, 0);  
GammaVal = blsgamma(100, 95, 0.10, 0.25, 0.50, 0);  
VegaVal = blsvega(100, 95, 0.10, 0.25, 0.50, 0);  
[LamCall, LamPut] = blslambda(100, 95, 0.10, 0.25, 0.50, 0);
```

yields:

- The option call price OptCall = \$13.70
- The option put price OptPut = \$6.35
- delta for a call CallVal = 0.6665 and delta for a put PutVal = -0.3335
- gamma GammaVal = 0.0145
- vega VegaVal = 18.1843
- lambda for a call LamCall = 4.8664 and lambda for a put LamPut = -5.2528

Now as a computation check, find the implied volatility of the option using the call option price from blsprice.

```
Volatility = blsimpv(100, 95, 0.10, 0.25, OptCall);
```

The function returns an implied volatility of 0.500, the original blsprice input.

Binomial Model

The binomial model for pricing options or other equity derivatives assumes that the probability over time of each possible price follows a binomial distribution. The basic assumption is that prices can move to only two values,

one up and one down, over any short time period. Plotting the two values, and then the subsequent two values each, and then the subsequent two values each, and so on over time, is known as “building a binomial tree.” This model applies to American options, which can be exercised any time up to and including their expiration date.

This example prices an American call option using a binomial model. Again, the asset price is \$100.00, the exercise price is \$95.00, the risk-free interest rate is 10%, and the time to maturity is 0.25 years. It computes the tree in increments of 0.05 years, so there are $0.25/0.05 = 5$ periods in the example. The volatility is 0.50, this is a call (`flag = 1`), the dividend rate is 0, and it pays a dividend of \$5.00 after three periods (an ex-dividend date). Executing the toolbox function

```
[StockPrice, OptionPrice] = binprice(100, 95, 0.10, 0.25, ...
0.05, 0.50, 1, 0, 5.0, 3);
```

returns the tree of prices of the underlying asset

StockPrice =

100.00	111.27	123.87	137.96	148.69	166.28
0	89.97	100.05	111.32	118.90	132.96
0	0	81.00	90.02	95.07	106.32
0	0	0	72.98	76.02	85.02
0	0	0	0	60.79	67.98
0	0	0	0	0	54.36

and the tree of option values.

OptionPrice =

12.10	19.17	29.35	42.96	54.17	71.28
0	5.31	9.41	16.32	24.37	37.96
0	0	1.35	2.74	5.57	11.32
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

The output from the binomial function is a binary tree. Read the StockPrice matrix this way: column 1 shows the price for period 0, column 2 shows the up and down prices for period 1, column 3 shows the up-up, up-down, and down-down prices for period 2, etc. Ignore the zeros. The OptionPrice matrix

gives the associated option value for each node in the price tree. Ignore the zeros that correspond to a zero in the price tree.

Analyzing Portfolios

Portfolio managers concentrate their efforts on achieving the best possible trade-off between risk and return. For portfolios constructed from a fixed set of assets, the risk/return profile varies with the portfolio composition. Portfolios that maximize the return, given the risk, or, conversely, minimize the risk for the given return, are called *optimal*. Optimal portfolios define a line in the risk/return plane called the *efficient frontier*.

A portfolio may also have to meet additional requirements to be considered. Different investors have different levels of risk tolerance. Selecting the adequate portfolio for a particular investor is a difficult process. The portfolio manager can hedge the risk related to a particular portfolio along the efficient frontier with partial investment in risk-free assets. The definition of the capital allocation line, and finding where the final portfolio falls on this line, if at all, is a function of:

- The risk/return profile of each asset
- The risk-free rate
- The borrowing rate
- The degree of risk aversion characterizing an investor

The Financial Toolbox includes a set of portfolio optimization functions designed to find the portfolio that best meets investor requirements.

Portfolio Optimization Functions

The portfolio optimization functions assist portfolio managers in constructing portfolios that optimize risk and return.

Capital Allocation	
portalloc	Computes the optimal risky portfolio on the efficient frontier, based on the risk-free rate, the borrowing rate, and the investor's degree of risk aversion. Also generates the capital allocation line, which provides the optimal allocation of funds between the risky portfolio and the risk-free asset.

Efficient Frontier Computation

frontcon	Computes portfolios along the efficient frontier for a given group of assets. The computation is based on sets of constraints representing the maximum and minimum weights for each asset, and the maximum and minimum total weight for specified groups of assets.
portopt	Computes portfolios along the efficient frontier for a given group of assets. The computation is based on a set of user-specified linear constraints. Typically, these constraints are generated using the constraint specification functions described below.

Constraint Specification

portcons	Generates the portfolio constraints matrix for a portfolio of asset investments using linear inequalities. The inequalities are of the type $A * Wts' \leq b$, where Wts is a row vector of weights. The capabilities of portcons are also provided individually by the following functions.	
	pcalims	Asset minimum and maximum allocation. Generates a constraint set to fix the minimum and maximum weight for each individual asset.
	pcgcomp	Group-to-group ratio constraint. Generates a constraint set specifying the maximum and minimum ratios between pairs of groups.

	pcglims	Asset group minimum and maximum allocation. Generates a constraint set to fix the minimum and maximum total weight for each defined group of assets.
	pcpval	Total portfolio value. Generates a constraint set to fix the total value of the portfolio.

Portfolio Construction Examples

The efficient frontier computation functions require information about each asset in the portfolio. This data is entered into the function via two matrices: an expected return vector and a covariance matrix. The expected return vector contains the average expected return for each asset in the portfolio. The covariance matrix is a square matrix representing the interrelationships between pairs of assets. This information can be directly specified or can be estimated from an asset return time series with the function `ewstats`.

Efficient Frontier Example

This example computes the efficient frontier of portfolios consisting of three different assets using the function `frontcon`. To visualize the efficient frontier curve clearly, consider 10 different evenly spaced portfolios.

Assume that the expected return of the first asset is 10%, the second is 20%, and the third is 15%. The covariance is defined in the matrix `ExpCovariance`.

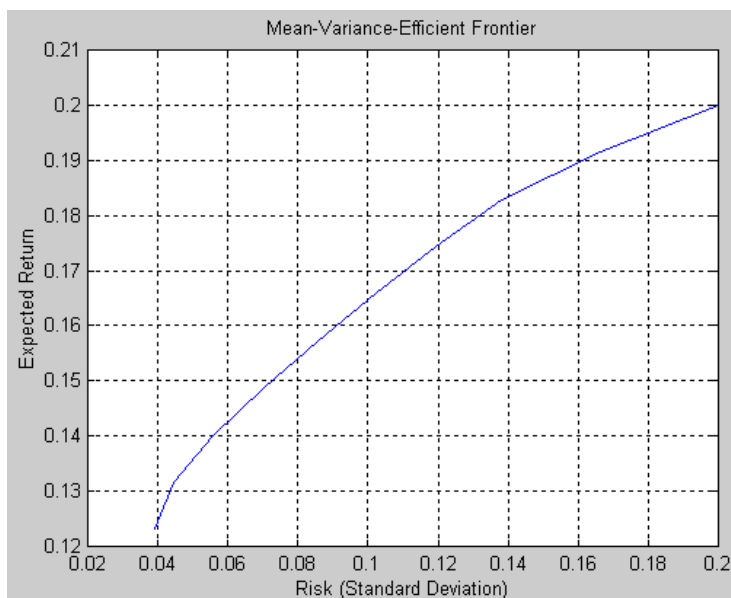
```
ExpReturn = [0.1 0.2 0.15];

ExpCovariance = [ 0.005   -0.010   0.004;
                  -0.010   0.040   -0.002;
                  0.004   -0.002   0.023];

NumPorts = 10;
```

Since there are no constraints, you can call `frontcon` directly with the data you already have. If you call `frontcon` without specifying any output arguments, you get a graph representing the efficient frontier curve.

```
frontcon (ExpReturn, ExpCovariance, NumPorts);
```



Calling `frontcon` while specifying the output arguments returns the corresponding vectors and arrays representing the risk, return, and weights for each of the 10 points computed along the efficient frontier.

```
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn,...  
ExpCovariance, NumPorts)  
PortRisk =  
    0.0392  
    0.0445  
    0.0559  
    0.0701  
    0.0858  
    0.1023  
    0.1192  
    0.1383  
    0.1661  
    0.2000
```


PortReturn =

```
0.1231
0.1316
0.1402
0.1487
0.1573
0.1658
0.1744
0.1829
0.1915
0.2000
```

PortWts =

```
0.7692  0.2308  0.0000
0.6667  0.2991  0.0342
0.5443  0.3478  0.1079
0.4220  0.3964  0.1816
0.2997  0.4450  0.2553
0.1774  0.4936  0.3290
0.0550  0.5422  0.4027
      0  0.6581  0.3419
      0  0.8291  0.1709
      0  1.0000  0.0000
```

The output data is represented row-wise. Each portfolio's risk, rate of return, and associated weights are identified as corresponding rows in the vectors and matrix.

For example, you can see from these results that the second portfolio has a risk of 0.0445, an expected return of 13.16%, and allocations of about 67% in the first asset, 30% in the second, and 3% in the third.

Portfolio Selection and Risk Aversion

One of the factors to consider when selecting the optimal portfolio for a particular investor is degree of risk aversion. This level of aversion to risk can be characterized by defining the investor's indifference curve. This curve consists of the family of risk/return pairs defining the trade-off between the expected return and the risk. It establishes the increment in return that a

particular investor will require in order to make an increment in risk worthwhile. Typical risk aversion coefficients range between 2.0 and 4.0, with the higher number representing lesser tolerance to risk. The equation used to represent risk aversion in the Financial Toolbox is

$$U = E(r) - 0.005 \cdot A \cdot \text{sig}^2$$

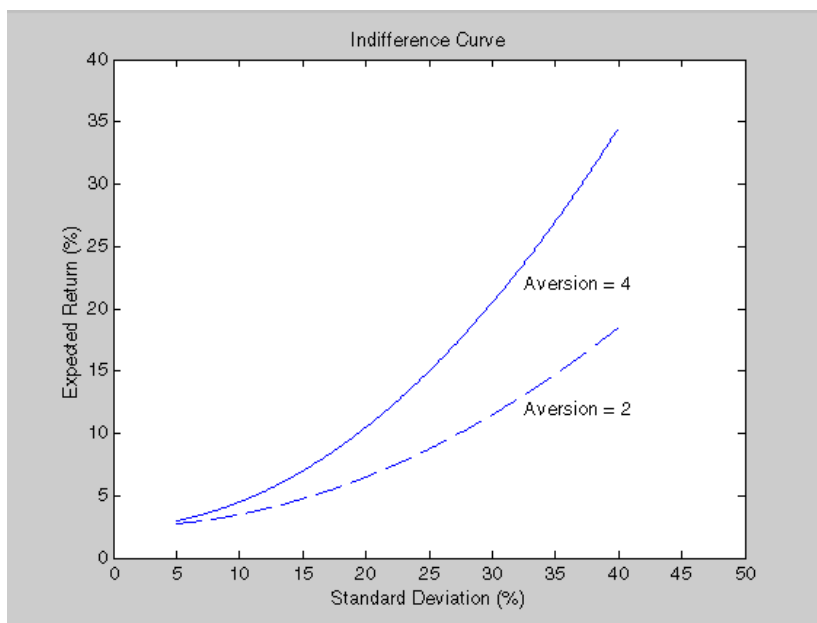
where:

U is the utility value.

E(r) is the expected return.

A is the index of investor's aversion.

sig is the standard deviation.



Optimal Risky Portfolio Example

This example computes the optimal risky portfolio on the efficient frontier based upon the risk-free rate, the borrowing rate, and the investor's degree of risk aversion. You do this with the function `portalloc`.

First generate the efficient frontier data using either `portopt` or `frontcon`. This example uses `portopt` and the same asset data from the previous example.

```
ExpReturn = [0.1 0.2 0.15];

ExpCovariance = [ 0.005   -0.010   0.004;
                  -0.010   0.040  -0.002;
                  0.004   -0.002   0.023];
```

This time consider 20 different points along the efficient frontier.

```
NumPorts = 20;

[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ...
ExpCovariance, NumPorts);
```

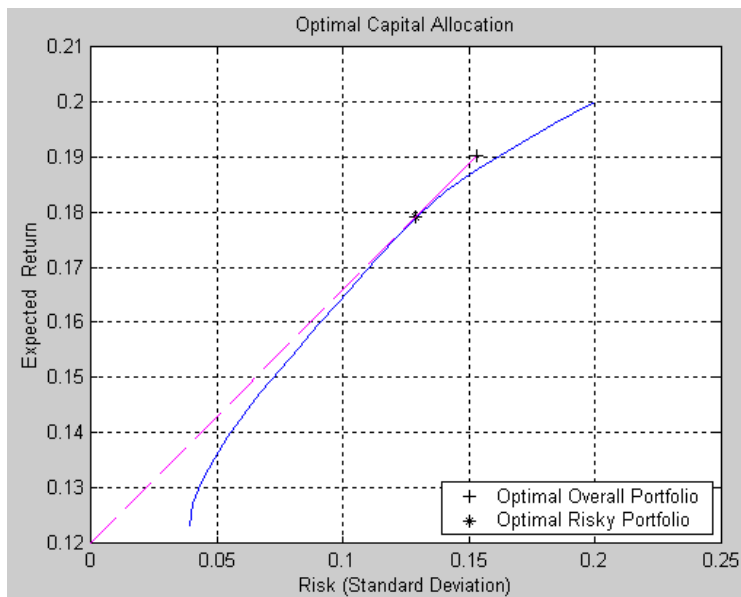
As with `frontcon`, calling `portopt` while specifying output arguments returns the corresponding vectors and arrays representing the risk, return, and weights for each of the portfolios along the efficient frontier. Use them as the first three input arguments to the function `portaloc`.

Now find the optimal risky portfolio and the optimal allocation of funds between the risky portfolio and the risk-free asset, using these values for the risk-free rate, borrowing rate and investor's degree of risk aversion.

```
RisklessRate = 0.08
BorrowRate   = 0.12
RiskAversion = 3
```

Calling `portaloc` without specifying any output arguments gives a graph displaying the critical points.

```
portaloc (PortRisk, PortReturn, PortWts, RisklessRate, ...
BorrowRate, RiskAversion);
```



Calling `portalloc` while specifying the output arguments returns the variance (`RiskyRisk`), the expected return (`RiskyReturn`), and the weights (`RiskyWts`) allocated to the optimal risky portfolio. It also returns the fraction (`RiskyFraction`) of the complete portfolio allocated to the risky portfolio, and the variance (`OverallRisk`) and expected return (`OverallReturn`) of the optimal overall portfolio. The overall portfolio combines investments in the risk-free asset and in the risky portfolio. The actual proportion assigned to each of these two investments is determined by the degree of risk aversion characterizing the investor.

```
[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, OverallRisk, ...
OverallReturn] = portalloc (PortRisk, PortReturn, PortWts, ...
RisklessRate, BorrowRate, RiskAversion)
```

```
RiskyRisk = 0.1288
RiskyReturn = 0.1791
RiskyWts = 0.0057 0.5879 0.4064
RiskyFraction = 1.1869
OverallRisk = 0.1529
OverallReturn = 0.1902
```

The value of RiskyFraction exceeds 1 (100%), implying that the risk tolerance specified allows borrowing money to invest in the risky portfolio, and that no money will be invested in the risk-free asset. This borrowed capital is added to the original capital available for investment. In this example the customer will tolerate borrowing 18.69% of the original capital amount.

Constraint Specification Example

This example computes the efficient frontier of portfolios consisting of three different assets, INTC, XON, and RD, given a list of constraints. The expected returns for INTC, XON, and RD are respectively as follows.

```
ExpReturn = [0.1 0.2 0.15];
```

The covariance matrix is

```
ExpCovariance = [ 0.005  -0.010  0.004;
                  -0.010  0.040  -0.002;
                  0.004  -0.002  0.023];
```

Constraint 1. Allow short selling up to 10% of the portfolio value in any asset but limit the investment in any one asset to 110% of the portfolio value.

Constraint 2. Consider two different sectors, technology and energy, with the table below indicating the sector each asset belongs to.

Asset	INTC	XON	RD
Sector	Technology	Energy	Energy

Constrain the investment in the Energy sector to 80% of the portfolio value, and the investment in the Technology sector to 70%.

To solve this problem, use `frontcon`, passing in a list of asset constraints. Consider eight different portfolios along the efficient frontier.

```
NumPorts = 8;
```

To introduce the asset bounds constraints specified in Constraint 1, create the matrix `AssetBounds`, where each column represents an asset. The upper row represents the lower bounds, and the lower row represents the upper bounds.

```
AssetBounds = [-0.10, -0.10, -0.10;  
               1.10,  1.10,  1.10];
```

Constraint 2 needs to be entered in two parts, the first part defining the groups, and the second part defining the constraints for each group. Given the information above, you can build a matrix of 1s and 0s indicating whether a specific asset belongs to a group. Each column represents an asset, and each row represents a group. This example has two groups: the technology group, and the energy group. Create the matrix Groups as follows.

```
Groups = [0  1  1;  
          1  0  0];
```

The GroupBounds matrix allows you to specify an upper and lower bound for each group. Each row in this matrix represents a group. The first column represents the minimum allocation, and the second column represents the maximum allocation to each group. Since the investment in the Energy sector is capped at 80% of the portfolio value, and the investment in the Technology sector is capped at 70%, create the GroupBounds matrix using this information.

```
GroupBounds = [0  0.80;  
               0  0.70];
```

Now use `frontcon` to obtain the vectors and arrays representing the risk, return, and weights for each of the eight portfolios computed along the efficient frontier.

```
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn,...  
ExpCovariance, NumPorts, [], AssetBounds, Groups, GroupBounds)
```

```
PortRisk =
```

```
0.0416  
0.0499  
0.0624  
0.0767  
0.0920  
0.1100  
0.1378  
0.1716
```

```
PortReturn =
```

```
0.1279
0.1361
0.1442
0.1524
0.1605
0.1687
0.1768
0.1850
```

```
PortWts =
```

```
0.7000    0.2582    0.0418
0.6031    0.3244    0.0725
0.4864    0.3708    0.1428
0.3696    0.4172    0.2132
0.2529    0.4636    0.2835
0.2000    0.5738    0.2262
0.2000    0.7369    0.0631
0.2000    0.9000   -0.1000
```

The output data is represented row-wise, where each portfolio's risk, rate of return, and associated weight is identified as corresponding rows in the vectors and matrix.

Linear Constraint Equations

While `frontcon` allows you to enter a fixed set of constraints related to minimum and maximum values for groups and individual assets, you often need to specify a larger and more general set of constraints when finding the optimal risky portfolio. The function `portopt` addresses this need, by accepting an arbitrary set of constraints as an input matrix.

The auxiliary function `portcons` can be used to create the matrix of constraints, with each row representing an inequality. These inequalities are of the type $A \cdot Wts' \leq b$, where A is a matrix, b is a vector, and Wts is a row vector of asset allocations. The number of columns of the matrix A , and the length of the vector Wts correspond to the number of assets. The number of rows of the matrix A , and the length of vector b correspond to the number of

constraints. This method allows you to specify any number of linear inequalities to the function `portopt`.

In actuality, `portcons` is an entry point to a set of functions that generate matrices for specific types of constraints. `portcons` allows you to specify all the constraints data at once, while the specific portfolio constraint functions allow you to build the constraints incrementally. These constraint functions are `pcpval`, `pcalims`, `pcglims`, and `pcgcomp`.

Consider an example to help understand how to specify constraints to `portopt` while bypassing the use of `portcons`. This example requires specifying the minimum and maximum investment in various groups.

Table 2-5: Maximum and Minimum Group Exposure

Group	Minimum Exposure	Maximum Exposure
North America	0.30	0.75
Europe	0.10	0.55
Latin America	0.20	0.50
Asia	0.50	0.50

Note that the minimum and maximum exposure in Asia is the same. This means that you require a fixed exposure for this group.

Also assume that the portfolio consists of three different funds. The correspondence between funds and groups is shown in Table 2-6.

Table 2-6: Group Membership

Group	Fund 1	Fund 2	Fund 3
North America	X	X	
Europe			X
Latin America	X		
Asia		X	X

Using the information in these two tables, build a mathematical representation of the constraints represented. Assume that the vector of weights representing the exposure of each asset in a portfolio is called $wts = [w_1 \ w_2 \ w_3]$.

Specifically

1. $w_1 + w_2 \geq 0.30$
2. $w_1 + w_2 \leq 0.75$
3. $w_3 \geq 0.10$
4. $w_3 \leq 0.55$
5. $w_1 \geq 0.20$
6. $w_1 \leq 0.50$
7. $w_2 + w_3 = 0.50$

Since you need to represent the information in the form $A \cdot wts \leq b$, multiply equations 1, 3 and 5 by -1 . Also turn equation 7 into a set of two inequalities: $w_2 + w_3 \geq 0.50$ and $w_2 + w_3 \leq 0.50$ (The intersection of these two inequalities is the equality itself.). Thus

1. $-w_1 - w_2 \leq -0.30$
2. $w_1 + w_2 \leq 0.75$
3. $-w_3 \leq -0.10$
4. $w_3 \leq 0.55$
5. $-w_1 \leq -0.20$
6. $w_1 \leq 0.50$
7. $-w_2 - w_3 \leq -0.50$
8. $w_2 + w_3 \leq 0.50$

Bringing these equations into matrix notation gives

```
A = [-1    -1    0;
      1     1    0;
      0     0   -1;
      0     0    1;
     -1     0    0;
      1     0    0;
      0    -1   -1;
      0     1    1]
```

```
b = [-0.30;
      0.75;
     -0.10;
      0.55;
     -0.20;
      0.50;
     -0.50;
      0.50]
```

Build the constraint matrix ConSet by concatenating the matrix A to the vector b.

```
ConSet = [A, b]
```

Specifying Additional Constraints

The example above defined a constraints matrix that specified a set of typical scenarios. It defined groups of assets, specified upper and lower bounds for total allocation in each of these groups, and it set the total allocation of one of the groups to a fixed value. Constraints like these are common occurrences. The function `portcons` was created to simplify the creation of the constraint matrix for these and other common portfolio requirements. `portcons` takes as input arguments a list of constraint-specifier strings, followed by the data necessary to build the constraint specified by the strings.

Assume that you need to add more constraints to the previous example. Specifically, add a constraint indicating that the sum of weights in any portfolio should be equal to 1, and another set of constraints (one per asset) indicating that the weight for each asset must greater than 0. This translates into five more constraint rows: two for the new equality, and three indicating that each weight must be greater or equal to 0. The total number of inequalities

in the example is now 13. Clearly, creating the constraint matrix can turn into a tedious task.

To create the new constraint matrix using `portcons`, use two separate constraint-specifier strings:

- 'Default', which indicates that each weight is greater than 0 and that the total sum of the weights adds to 1.
- 'GroupLims', which defines the minimum and maximum allocation on each group.

The only data requirement for the constraint-specifier string 'Default' is `NumAssets` (the total number of assets). The constraint-specifier string 'GroupLims' requires three different arguments: a `Groups` matrix indicating the assets that belong to each group, the `GroupMin` vector indicating the minimum bounds for each group, and the `GroupMax` vector indicating the maximum bounds for each group. Based on Table 2-6, Group Membership, build the `Group` matrix, with each row representing a group, and each column representing an asset.

```
Group = [1    1    0;
         0    0    1;
         1    0    0;
         0    1    1]
```

Table 2-5, Maximum and Minimum Group Exposure, has the information to build `GroupMin` and `GroupMax`.

```
GroupMin = [0.30  0.10  0.20  0.50];
GroupMax = [0.75  0.55  0.50  0.50];
```

Given that the number of assets is three, build the constraint matrix by calling `portcons`.

```
ConSet = portcons('Default', 3, 'GroupLims', Group, GroupMin, ...
                  GroupMax);
```

In most cases, `portcons('Default')` returns the minimal set of constraints required for calling `portopt`. If `ConSet` is not specified in the call to `portopt`, the function calls `portcons` passing 'Default' as its only specifier.

Now use `portopt` to obtain the vectors and arrays representing the risk, return, and weights for the portfolios computed along the efficient frontier.

```
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...  
ExpCovariance, [], [], ConSet)
```

```
PortRisk = 0.0586  
Port Return = 0.1375  
PortWts = 0.5 0.25 0.25
```

In this case the constraints allow only one optimum portfolio.

Solving Sample Problems

Common Problems in Finance	3-3
Sensitivity of Bond Prices to Changes in Interest Rates . . .	3-3
Constructing a Bond Portfolio to Hedge Against Duration and Convexity	3-6
Visualizing the Sensitivity of a Bond Portfolio's Price to Parallel Shifts in the Yield Curve	3-8
Constructing Greek-Neutral Portfolios of European Stock Options	3-12
Term Structure Analysis and Interest Rate Swap Pricing . .	3-15
 Producing Graphics with the Toolbox	3-19
Plotting an Efficient Frontier	3-19
Plotting Sensitivities of an Option	3-21
Plotting Sensitivities of a Portfolio of Options	3-23

This section shows how Financial Toolbox functions solve real-world problems. The examples ship with the toolbox as M-files. Try them by entering the commands directly or by executing the M-files.

This chapter contains two major topics:

- **Common Problems in Finance**
Shows how the toolbox solves real-world financial problems, specifically:
 - “Sensitivity of Bond Prices to Changes in Interest Rates” on page 3-3
 - “Constructing a Bond Portfolio to Hedge Against Duration and Convexity” on page 3-6
 - “Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve” on page 3-8
 - “Constructing Greek-Neutral Portfolios of European Stock Options” on page 3-12
 - “Term Structure Analysis and Interest Rate Swap Pricing” on page 3-15
- **Producing Graphics with the Toolbox**
Shows how the toolbox produces presentation-quality graphics by solving these problems:
 - “Plotting an Efficient Frontier” on page 3-19
 - “Plotting Sensitivities of an Option” on page 3-21
 - “Plotting Sensitivities of a Portfolio of Options” on page 3-23

Common Problems in Finance

Sensitivity of Bond Prices to Changes in Interest Rates

Macaulay and *modified duration* measure the sensitivity of a bond's price to changes in the level of interest rates. *Convexity* measures the change in duration for small shifts in the yield curve, and thus measures the second-order price sensitivity of a bond. Both measures can gauge the vulnerability of a bond portfolio's value to changes in the level of interest rates.

Alternatively, analysts can use duration and convexity to construct a bond portfolio that is partly hedged against small shifts in the term structure. If you combine bonds in a portfolio whose duration is zero, the portfolio is insulated, to some extent, against interest rate changes. If the portfolio convexity is also zero, this insulation is even better. However, since hedging costs money or reduces expected return, you need to know how much protection results from hedging duration alone compared with hedging both duration and convexity.

This example demonstrates a way to analyze the relative importance of duration and convexity for a bond portfolio using some of the SIA-compliant bond functions in the Financial Toolbox. Using duration, it constructs a first-order approximation of the change in portfolio price to a level shift in interest rates. Then, using convexity, it calculates a second-order approximation. Finally it compares the two approximations with the true price change resulting from a change in the yield curve. The example M-file is `ftspex1.m`.

Step 1. Define three bonds using values for the settlement date, maturity date, face value, and coupon rate. For simplicity, accept default values for the coupon payment periodicity (semi-annual), end-of-month payment rule (rule in effect), and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (no odd first or last coupon dates). Any inputs for which defaults are accepted are set to empty matrices ([]) as placeholders where appropriate.

```
Settle      = '19-Aug-1999';
Maturity    = ['17-Jun-2010'; '09-Jun-2015'; '14-May-2025'];
Face        = [100; 100; 1000];
CouponRate  = [0.07; 0.06; 0.045];
```

Also, specify the yield curve information.

```
Yields = [0.05; 0.06; 0.065];
```

Step 2. Use Financial Toolbox functions to calculate the price, modified duration in years, and convexity in years of each bond.

The true price is quoted (clean) price plus accrued interest.

```
[CleanPrice, AccruedInterest] = bndprice(Yields, CouponRate, ...
Settle, Maturity, 2, 0, [], [], [], [], Face);
```

```
Durations = bnddury(Yields, CouponRate, Settle, Maturity, 2, 0, ...
[], [], [], [], [], Face);
```

```
Convexities = bndconvy(Yields, CouponRate, Settle, Maturity, 2, 0, ...
[], [], [], [], [], Face);
```

```
Prices = CleanPrice + AccruedInterest;
```

Step 3. Choose a hypothetical amount by which to shift the yield curve (here, 0.2 percentage point or 20 basis points).

```
dY = 0.002;
```

Weight the three bonds equally, and calculate the actual quantity of each bond in the portfolio, which has a total value of \$100,000.

```
PortfolioPrice = 100000;
PortfolioWeights = ones(3,1)/3;
PortfolioAmounts = PortfolioPrice * PortfolioWeights ./ Prices;
```

Step 4. Calculate the modified duration and convexity of the portfolio. Note that the portfolio duration or convexity is a weighted average of the durations or convexities of the individual bonds. Calculate the first- and second-order approximations of the percent price change as a function of the change in the level of interest rates.

```
PortfolioDuration = PortfolioWeights' * Durations;
PortfolioConvexity = PortfolioWeights' * Convexities;
PercentApprox1 = -PortfolioDuration * dY * 100;
```

```
PercentApprox2 = PercentApprox1 + ...
```



```
PortfolioConvexity*dY^2*100/2.0;
```

Step 5. Estimate the new portfolio price using the two estimates for the percent price change.

```
PriceApprox1 = PortfolioPrice + ...
PercentApprox1 * PortfolioPrice/100;
```

```
PriceApprox2 = PortfolioPrice + ...
PercentApprox2 * PortfolioPrice/100;
```

Step 6. Calculate the true new portfolio price by shifting the yield curve.

```
[CleanPrice, AccruedInterest] = bndprice(Yields + dY,...
CouponRate, Settle, Maturity, 2, 0, [], [], [], [], [],...
Face);
```

```
NewPrice = PortfolioAmounts' * (CleanPrice + AccruedInterest);
```

Step 7. Compare the results. The analysis results are as follows (verify these results by running the example M-file `ftspex1.m`):

- The original portfolio price was \$100,000.
- The yield curve shifted up by 0.2 percentage point or 20 basis points.
- The portfolio duration and convexity are 10.3181 and 157.6346, respectively. These will be needed below for “Constructing a Bond Portfolio to Hedge Against Duration and Convexity”.
- The first-order approximation, based on modified duration, predicts the new portfolio price (`PriceApprox1`) will be \$97,936.37.
- The second-order approximation, based on duration and convexity, predicts the new portfolio price (`PriceApprox2`) will be \$97,967.90.
- The true new portfolio price (`NewPrice`) for this yield curve shift is \$97,967.51.
- The estimate using duration and convexity is quite good (at least for this fairly small shift in the yield curve), but only slightly better than the estimate using duration alone. The importance of convexity increases as the magnitude of the yield curve shift increases. Try a larger shift (`dY`) to see this effect.

The approximation formulas in this example consider only parallel shifts in the term structure, because both formulas are functions of dY , the change in yield. The formulas are not well-defined unless each yield changes by the same amount. In actual financial markets, changes in yield curve level typically explain a substantial portion of bond price movements. However, other changes in the yield curve, such as slope, may also be important and are not captured here. Also, both formulas give local approximations whose accuracy deteriorates as dY increases in size. You can demonstrate this by running the program with larger values of dY .

Constructing a Bond Portfolio to Hedge Against Duration and Convexity

This example constructs a bond portfolio to hedge the portfolio of “Sensitivity of Bond Prices to Changes in Interest Rates.” It assumes a long position in (holding) the portfolio, and that three other bonds are available for hedging. It chooses weights for these three other bonds in a new portfolio so that the duration and convexity of the new portfolio match those of the original portfolio. Taking a short position in the new portfolio, in an amount equal to the value of the first portfolio, partially hedges against parallel shifts in the yield curve.

Recall that portfolio duration or convexity is a weighted average of the durations or convexities of the individual bonds in a portfolio. As in the previous example, this example uses modified duration in years and convexity in years. The hedging problem therefore becomes one of solving a system of linear equations, which is very easy to do in MATLAB. The M-file for this example is `ftspex2.m`.

Step 1. Define three bonds available for hedging the original portfolio. Specify values for the settlement date, maturity date, face value, and coupon rate. For simplicity, accept default values for the coupon payment periodicity (semi-annual), end-of-month payment rule (rule in effect), and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (i.e., no odd first or last coupon dates). Set any inputs for which defaults are accepted to empty matrices (`[]`) as placeholders where appropriate. The intent is to hedge against duration and convexity as well as constrain total portfolio price.

```
Settle      = '19-Aug-1999';  
Maturity    = ['15-Jun-2005'; '02-Oct-2010'; '01-Mar-2025'];
```

```
Face      = [500; 1000; 250];
CouponRate = [0.07; 0.066; 0.08];
```

Also, specify the yield curve for each bond.

```
Yields = [0.06; 0.07; 0.075];
```

Step 2. Use Financial Toolbox functions to calculate the price, modified duration in years, and convexity in years of each bond.

The true price is quoted (clean price plus accrued interest).

```
[CleanPrice, AccruedInterest] = bndprice(Yields, CouponRate, ...
Settle, Maturity, 2, 0, [], [], [], [], [], Face);
```

```
Durations = bnddury(Yields, CouponRate, Settle, Maturity, ...
2, 0, [], [], [], [], [], [], Face);
```

```
Convexities = bndconvy(Yields, CouponRate, Settle, ...
Maturity, 2, 0, [], [], [], [], [], Face);
```

```
Prices = CleanPrice + AccruedInterest;
```

Step 3. Set up and solve the system of linear equations whose solution is the weights of the new bonds in a new portfolio with the same duration and convexity as the original portfolio. In addition, scale the weights to sum to 1; that is, force them to be portfolio weights. You can then scale this unit portfolio to have the same price as the original portfolio. Recall that the original portfolio duration and convexity are 10.3181 and 157.6346, respectively. Also, note that the last row of the linear system ensures the sum of the weights is unity.

```
A = [Durations'
      Convexities'
      1 1 1];
```

```
b = [ 10.3181
      157.6346
      1];
```

```
Weights = A\b;
```

Step 4. Compute the duration and convexity of the hedge portfolio, which should now match the original portfolio.

```
PortfolioDuration = Weights' * Durations;  
PortfolioConvexity = Weights' * Convexities;
```

Step 5. Finally, scale the unit portfolio to match the value of the original portfolio and find the number of bonds required to insulate against small parallel shifts in the yield curve.

```
PortfolioValue = 100000;  
HedgeAmounts = Weights ./ Prices * PortfolioValue;
```

Step 6. Compare the results. (Verify the analysis results by running the example M-file `ftspex2.m`.)

- As required, the duration and convexity of the new portfolio are 10.3181 and 157.6346, respectively.
- The hedge amounts for bonds 1, 2, and 3 are -57.37, 71.70, and 216.27, respectively.

Notice that the hedge matches the duration, convexity, and value (\$100,000) of the original portfolio. If you are holding that first portfolio, you can hedge by taking a short position in the new portfolio.

Just as the approximations of the first example are appropriate only for small parallel shifts in the yield curve, the hedge portfolio is appropriate only for reducing the impact of small level changes in the term structure.

Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve

Often bond portfolio managers want to consider more than just the sensitivity of a portfolio's price to a small shift in the yield curve, particularly if the investment horizon is long. This example shows how MATLAB can visualize the price behavior of a portfolio of bonds over a wide range of yield curve scenarios, and as time progresses toward maturity.

This example uses the Financial Toolbox bond pricing functions to evaluate the impact of time-to-maturity and yield variation on the price of a bond portfolio. It plots the portfolio value and shows the behavior of bond prices as yield and time vary. This example M-file is `ftspex3.m`.

Step 1. Specify values for the settlement date, maturity date, face value, coupon rate, and coupon payment periodicity of a four-bond portfolio. For simplicity, accept default values for the end-of-month payment rule (rule in effect) and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (no odd first or last coupon dates). Any inputs for which defaults are accepted are set to empty matrices ([]) as placeholders where appropriate.

```
Settle      = '15-Jan-1995';
Maturity    = datenum(['03-Apr-2020'; '14-May-2025'; ...
                      '09-Jun-2019'; '25-Feb-2019']);
Face        = [1000; 1000; 1000; 1000];
CouponRate  = [0; 0.05; 0; 0.055];
Periods     = [0; 2; 0; 2];
```

Also, specify the points on the yield curve for each bond.

```
Yields = [0.078; 0.09; 0.075; 0.085];
```

Step 2. Use Financial Toolbox functions to calculate the true bond prices as the sum of the quoted price plus accrued interest.

```
[CleanPrice, AccruedInterest] = bndprice(Yields, ...
    CouponRate, Settle, Maturity, Periods, ...
    [], [], [], [], [], [], Face);
```

```
Prices = CleanPrice + AccruedInterest;
```

Step 3. Assume the value of each bond is \$25,000, and determine the quantity of each bond such that the portfolio value is \$100,000.

```
BondAmounts = 25000 ./ Prices;
```

Step 4. Compute the portfolio price for a rolling series of settlement dates over a range of yields. The evaluation dates occur annually on January 15, beginning on 15-Jan-1995 (settlement) and extending out to 15-Jan-2018. Thus, this step evaluates portfolio price on a grid of time of progression (dT) and interest rates (dY).

```
dy = -0.05:0.005:0.05; % Yield changes

D = datevec(Settle); % Get date components
dt = datenum(D(1):2018, D(2), D(3)); % Get evaluation dates
```

```
[dT, dY] = meshgrid(dt, dy); % Create grid

NumTimes = length(dt);      % Number of time steps
NumYields = length(dy);     % Number of yield changes
NumBonds = length(Maturity); % Number of bonds

% Preallocate vector
Prices = zeros(NumTimes*NumYields, NumBonds);
```

Now that the grid and price vectors have been created, compute the price of each bond in the portfolio on the grid one bond at a time.

```
for i = 1:NumBonds

    [CleanPrice, AccruedInterest] = bndprice(Yields(i)+...
        dY(:), CouponRate(i), dT(:), Maturity(i), Periods(i),...
        [], [], [], [], [], [], Face(i));

    Prices(:,i) = CleanPrice + AccruedInterest;

end
```

Scale the bond prices by the quantity of bonds.

```
Prices = Prices * BondAmounts;
```

Reshape the bond values to conform to the underlying evaluation grid.

```
Prices = reshape(Prices, NumYields, NumTimes);
```

Step 5. Plot the price of the portfolio as a function of settlement date and a range of yields, and as a function of the change in yield (dY). This plot illustrates the interest rate sensitivity of the portfolio as time progresses (dT), under a range of interest rate scenarios. With the following graphics commands, you can visualize the three-dimensional surface relative to the current portfolio value (i.e., \$100,000).

```
figure          % Open a new figure window
surf(dt, dy, Prices) % Draw the surface
```

Add the base portfolio value to the existing surface plot.

```
hold on          % Add the current value for reference
```

```
basemesh = mesh(dt, dy, 100000*ones(NumYields, NumTimes));
```

Make it transparent, plot it so the price surface shows through, and draw a box around the plot.

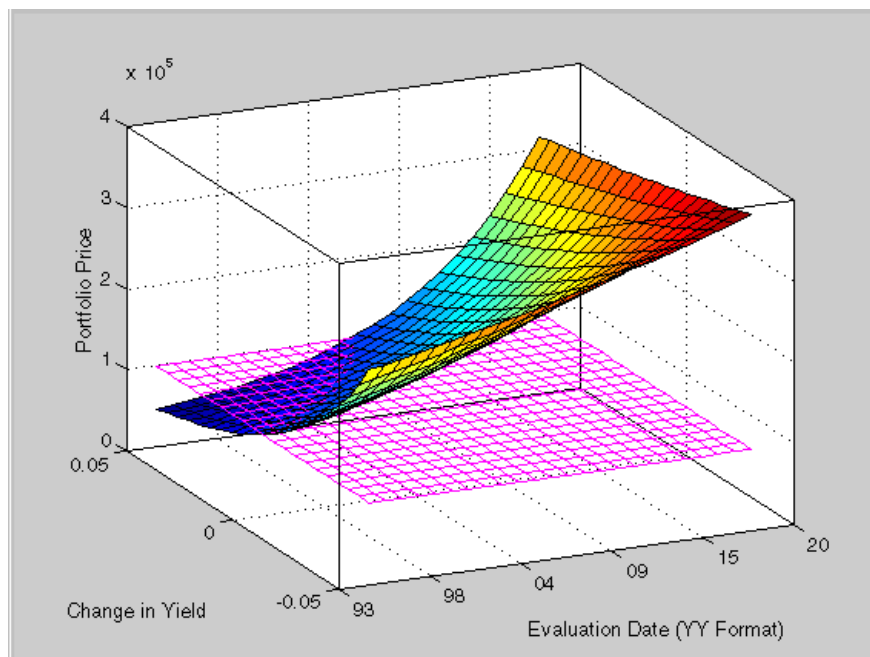
```
set(basemesh, 'facecolor', 'none');  
set(basemesh, 'edgecolor', 'm');  
set(gca, 'box', 'on');
```

Plot the x-axis using two-digit year (YY format) labels for ticks.

```
dateaxis('x', 11);
```

Add axis labels and set the three-dimensional viewpoint. MATLAB produces the figure.

```
xlabel('Evaluation Date (YY Format)');  
ylabel('Change in Yield');  
zlabel('Portfolio Price');  
hold off  
view(-25,25);
```



MATLAB's three-dimensional graphics allow you to visualize the interest rate risk experienced by a bond portfolio over time. This example assumed parallel shifts in the term structure, but it might similarly have allowed other components to vary, such as the level and slope.

Constructing Greek-Neutral Portfolios of European Stock Options

The option sensitivity measures familiar to most option traders are often referred to as the *greeks*: *delta*, *gamma*, *vega*, *lambda*, *rho*, and *theta*. Delta is the price sensitivity of an option with respect to changes in the price of the underlying asset. It represents a first-order sensitivity measure analogous to duration in fixed income markets. Gamma is the sensitivity of an option's delta to changes in the price of the underlying asset, and represents a second-order price sensitivity analogous to convexity in fixed income markets. Vega is the price sensitivity of an option with respect to changes in the volatility of the underlying asset. See "Pricing and Analyzing Equity Derivatives" on page 2-32 or the "Glossary" for other definitions.

The greeks of a particular option are a function of the model used to price the option. However, given enough different options to work with, a trader can construct a portfolio with any desired values for its greeks. For example, to insulate the value of an option portfolio from small changes in the price of the underlying asset, one trader might construct an option portfolio whose delta is zero. Such a portfolio is then said to be “delta neutral.” Another trader may wish to protect an option portfolio from larger changes in the price of the underlying asset, and so might construct a portfolio whose delta and gamma are both zero. Such a portfolio is both delta and gamma neutral. A third trader may wish to construct a portfolio insulated from small changes in the volatility of the underlying asset in addition to delta and gamma neutrality. Such a portfolio is then delta, gamma, and vega neutral.

Using the Black-Scholes model for European options, this example creates an equity option portfolio that is simultaneously delta, gamma, and vega neutral. The value of a particular greek of an option portfolio is a weighted average of the corresponding greek of each individual option. The weights are the quantity of each option in the portfolio. Hedging an option portfolio thus involves solving a system of linear equations, an easy process in MATLAB. This example M-file is `ftspex4.m`.

Step 1. Create an input data matrix to summarize the relevant information. Each row of the matrix contains the standard inputs to the Financial Toolbox Black-Scholes suite of functions: column 1 contains the current price of the underlying stock; column 2 the strike price of each option; column 3 the time to-expiry of each option in years; column 4 the annualized stock price volatility; and column 5 the annualized dividend rate of the underlying asset. Note that rows 1 and 3 are data related to call options, while rows 2 and 4 are data related to put options.

```
DataMatrix = [100.000  100  0.2  0.3  0          % Call
               119.100  125  0.2  0.2  0.025     % Put
               87.200   85  0.1  0.23  0          % Call
               301.125  315  0.5  0.25  0.0333] % Put
```

Also, assume the annualized risk-free rate is 10 percent and is constant for all maturities of interest.

```
RiskFreeRate = 0.10;
```

For clarity, assign each column of `DataMatrix` to a column vector whose name reflects the type of financial data in the column.

```
StockPrice = DataMatrix(:,1);
StrikePrice = DataMatrix(:,2);
ExpiryTime = DataMatrix(:,3);
Volatility = DataMatrix(:,4);
DividendRate = DataMatrix(:,5);
```

Step 2. Based on the Black-Scholes model, compute the prices, as well as the delta, gamma, and vega sensitivity greeks of each of the four options. Note that the functions `blsprice` and `blsdelta` have two outputs, while `blsgamma` and `blsvega` have only one. The price and delta of a call option differ from the price and delta of an otherwise equivalent put option, in contrast to the gamma and vega sensitivities, which are valid for both calls and puts.

```
[CallPrices, PutPrices] = blsprice(StockPrice, StrikePrice,...
RiskFreeRate, ExpiryTime, Volatility, DividendRate);
```

```
[CallDeltas, PutDeltas] = blsdelta(StockPrice,...
StrikePrice, RiskFreeRate, ExpiryTime, Volatility,...
DividendRate);
```

```
Gammas = blsgamma(StockPrice, StrikePrice, RiskFreeRate,...
ExpiryTime, Volatility , DividendRate)';
```

```
Vegas = blsvega(StockPrice, StrikePrice, RiskFreeRate,...
ExpiryTime, Volatility , DividendRate)';
```

Extract the prices and deltas of interest to account for the distinction between call and puts.

```
Prices = [CallPrices(1) PutPrices(2) CallPrices(3)...
PutPrices(4)];
```

```
Deltas = [CallDeltas(1) PutDeltas(2) CallDeltas(3)...
PutDeltas(4)];
```

Step 3. Now, assuming an arbitrary portfolio value of \$17,000, set up and solve the linear system of equations such that the overall option portfolio is simultaneously delta, gamma, and vega-neutral. The solution computes the value of a particular greek of a portfolio of options as a weighted average of the corresponding greek of each individual option in the portfolio. The system of

equations is solved using the backslash (\) operator discussed in “Solving Simultaneous Linear Equations” on page 1-12.

```
A = [Deltas; Gammas; Vegas; Prices];
b = [0; 0; 0; 17000];
OptionQuantities = A\b; % Quantity (number) of each option.
```

Step 4. Finally, compute the market value, delta, gamma, and vega of the overall portfolio as a weighted average of the corresponding parameters of the component options. The weighted average is computed as an inner product of two vectors.

```
PortfolioValue = Prices * OptionQuantities;
PortfolioDelta = Deltas * OptionQuantities;
PortfolioGamma = Gammas * OptionQuantities;
PortfolioVega = Vegas * OptionQuantities;
```

The example `ftspex4.m` performs these computations and displays the output on the screen.

Option	Price	Delta	Gamma	Vega	Quantity
1	6.3441	0.5856	0.0290	17.4293	22332.6131
2	6.6035	-0.6255	0.0353	20.0347	6864.0731
3	4.2993	0.7003	0.0548	9.5837	-15654.8657
4	22.7694	-0.4830	0.0074	83.5225	-4510.5153

```
Portfolio Value: $17000.00
Portfolio Delta:      0.00
Portfolio Gamma:     -0.00
Portfolio Vega :      0.00
```

You can verify that the portfolio value is \$17,000 and that the option portfolio is indeed delta, gamma, and vega neutral, as desired. Hedges based on these measures are effective only for small changes in the underlying variables.

Term Structure Analysis and Interest Rate Swap Pricing

This example illustrates some of the term-structure analysis functions found in the Financial Toolbox. Specifically, it illustrates how to derive implied zero (*spot*) and forward curves from the observed market prices of coupon-bearing

bonds. The zero and forward curves implied from the market data are then used to price an interest rate swap agreement.

In an interest rate swap, two parties agree to a periodic exchange of cash flows. One of the cash flows is based on a fixed interest rate held constant throughout the life of the swap. The other cash flow stream is tied to some variable index rate. Pricing a swap at inception amounts to finding the fixed rate of the swap agreement. This fixed rate, appropriately scaled by the notional principle of the swap agreement, determines the periodic sequence of fixed cash flows.

In general, interest rate swaps are priced from the forward curve such that the variable cash flows implied from the series of forward rates and the periodic sequence of fixed-rate cash flows have the same present value. Thus, interest rate swap pricing and term structure analysis are intimately related.

Step 1. Specify values for the settlement date, maturity dates, coupon rates, and market prices for 10 U.S. Treasury Bonds. This data allows us to price a five-year swap with net cash flow payments exchanged every six months. For simplicity, accept default values for the end-of-month payment rule (rule in effect) and day-count basis (actual/actual). To avoid issues of accrued interest, assume that all Treasury Bonds pay semi-annual coupons and that settlement occurs on a coupon payment date.

```
Settle    = datenum('15-Jan-1999');

BondData = {'15-Jul-1999'  0.06000  99.93
            '15-Jan-2000'  0.06125  99.72
            '15-Jul-2000'  0.06375  99.70
            '15-Jan-2001'  0.06500  99.40
            '15-Jul-2001'  0.06875  99.73
            '15-Jan-2002'  0.07000  99.42
            '15-Jul-2002'  0.07250  99.32
            '15-Jan-2003'  0.07375  98.45
            '15-Jul-2003'  0.07500  97.71
            '15-Jan-2004'  0.08000  98.15};
```

`BondData` is an instance of a MATLAB *cell array*, indicated by the curly braces (`{}`).

Next assign the date stored in the cell array to `Maturity`, `CouponRate`, and `Prices` vectors for further processing.

```
Maturity  = datenum(strvcat(BondData{:},1));
```

```
CouponRate = [BondData{:,2}]';
Prices      = [BondData{:,3}]';
Period      = 2; % semi-annual coupons
```

Step 2. Now that the data has been specified, use the term structure function `zbtprice` to bootstrap the zero curve implied from the prices of the coupon-bearing bonds. This implied zero curve represents the series of zero-coupon Treasury rates consistent with the prices of the coupon-bearing bonds such that arbitrage opportunities will not exist.

```
ZeroRates = zbtprice([Maturity CouponRate], Prices, Settle);
```

The zero curve, stored in `ZeroRates`, is quoted on a semi-annual bond basis (the periodic, six-month, interest rate is simply doubled to annualize). The first element of `ZeroRates` is the annualized rate over the next six months, the second element is the annualized rate over the next 12 months, and so on.

Step 3. From the implied zero curve, find the corresponding series of implied forward rates using the term structure function `zero2fwd`.

```
ForwardRates = zero2fwd(ZeroRates, Maturity, Settle);
```

The forward curve, stored in `ForwardRates`, is also quoted on a semi-annual bond basis. The first element of `ForwardRates` is the annualized rate applied to the interval between settlement and six months after settlement, the second element is the annualized rate applied to the interval from six months to 12 months after settlement, and so on. This implied forward curve is also consistent with the observed market prices such that arbitrage activities will be unprofitable. Since the first forward rate is also a zero rate, the first element of `ZeroRates` and `ForwardRates` are the same.

Step 4. Now that you have derived the zero curve, convert it to a sequence of discount factors with the term structure function `zero2disc`.

```
DiscountFactors = zero2disc(ZeroRates, Maturity, Settle);
```

Step 5. From the discount factors, compute the present value of the variable cash flows derived from the implied forward rates. For plain interest rate swaps, the notional principle remains constant for each payment date and cancels out of each side of the present value equation. The line below assumes unit notional principle.

```
PresentValue = sum((ForwardRates/Period) .* DiscountFactors);
```

Step 6. Compute the swap's price (the fixed rate) by equating the present value of the fixed cash flows with the present value of the cash flows derived from the implied forward rates. Again, since the notional principle cancels out of each side of the equation, it is simply assumed to be 1.

$$\text{SwapFixedRate} = \text{Period} * \text{PresentValue} / \text{sum}(\text{DiscountFactors});$$

The example `ftspex5.m` performs these computations and displays the output on the screen.

Zero Rates	Forward Rates
0.0614	0.0614
0.0642	0.0670
0.0660	0.0695
0.0684	0.0758
0.0702	0.0774
0.0726	0.0846
0.0754	0.0925
0.0795	0.1077
0.0827	0.1089
0.0868	0.1239

$$\text{Swap Price (Fixed Rate)} = 0.0845$$

All rates are in decimal format. The swap price, 8.45%, would likely be the mid-point between a market-maker's bid/ask quotes.

Producing Graphics with the Toolbox

The Financial Toolbox and MATLAB graphics functions work together to produce presentation quality graphics, as these examples show. The examples ship with the toolbox as M-files. Try them by entering the commands directly or by executing the M-files. For help using MATLAB plotting functions, see *Creating Plots* in the MATLAB documentation.

Plotting an Efficient Frontier

This example plots the efficient frontier of a hypothetical portfolio of three assets. It illustrates how to specify the expected returns, standard deviations, and correlations of a portfolio of assets, how to convert standard deviations and correlations into a covariance matrix, and how to compute and plot the efficient frontier from the returns and covariance matrix. The example also illustrates how to randomly generate a set of portfolio weights, and how to add the random portfolios to an existing plot for comparison with the efficient frontier. The example M-file is `ftgex1.m`.

First, specify the expected returns, standard deviations, and correlation matrix for a hypothetical portfolio of three assets. Note the symmetry of the correlation matrix.

```
Returns      = [0.1 0.15 0.12];
STDs         = [0.2 0.25 0.18];

Correlations = [ 1    0.8  0.4
                 0.8    1    0.3
                 0.4  0.3    1 ];
```

Convert the standard deviations and correlation matrix into a variance-covariance matrix with the Financial Toolbox function `corr2cov`.

```
Covariances = corr2cov(STDs, Correlations);
```

Evaluate and plot the efficient frontier at 20 points along the frontier, using the function `portopt` and the expected returns and corresponding covariance matrix. Although rather elaborate constraints can be placed on the assets in a portfolio, for simplicity accept the default constraints and scale the total value of the portfolio to 1 and constrain the weights to be positive (no short-selling).

```
portopt>Returns, Covariances, 20)
```

Now that the efficient frontier is displayed, randomly generate the asset weights for 1000 portfolios starting from the MATLAB initial state.

```
rand('state', 0)
Weights = rand(1000, 3);
```

The previous line of code generates three columns of uniformly distributed random weights, but does not guarantee they sum to 1. The following code segment normalizes the weights of each portfolio so that the total of the three weights represent a valid portfolio.

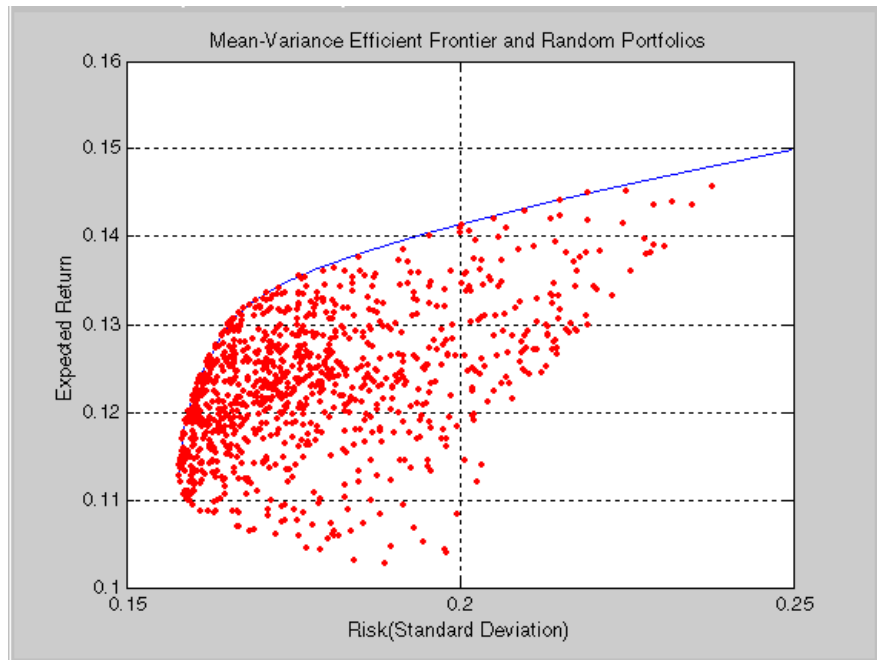
```
Total = sum(Weights, 2);    % Add the weights
Total = Total(:,ones(3,1)); % Make size-compatible matrix
Weights = Weights./Total;   % Normalize so sum = 1
```

Given the 1000 random portfolios just created, compute the expected return and risk of each portfolio associated with the weights.

```
[PortRisk, PortReturn] = portstats>Returns, Covariances, ...
                               Weights);
```

Finally, hold the current graph, and plot the returns and risks of each portfolio on top of the existing efficient frontier for comparison. After plotting, annotate the graph with a title and return the graph to default holding status (any subsequent plots will erase the existing data). The efficient frontier appears in blue, while the 1000 random portfolios appear as a set of red dots on or below the frontier.

```
hold on
plot (PortRisk, PortReturn, '.r')
title('Mean-Variance Efficient Frontier and Random Portfolios')
hold off
```

Plotting Sensitivities of an Option

This example creates a three-dimensional plot showing how gamma changes relative to price for a Black-Scholes option. Recall that gamma is the second derivative of the option price relative to the underlying security price. The plot shows a three-dimensional surface whose z-value is the gamma of an option as price (x-axis) and time (y-axis) vary. It adds yet a fourth dimension by showing option delta (the first derivative of option price to security price) as the color of the surface. This example M-file is `ftgex2.m`.

First set the price range of the options, and set the time range to one year divided into half-months and expressed as fractions of a year.

```
Range = 10:70;
Span = length(Range);
j = 1:0.5:12;
Newj = j(ones(Span,1),:)' / 12;
```

For each time period create a vector of prices from 10 to 70 and create a matrix of all ones.

```
JSpan = ones(length(j),1);  
NewRange = Range(JSpan,:);  
Pad = ones(size(Newj));
```

Call the toolbox gamma and delta sensitivity functions. Exercise price is \$40, risk-free interest rate is 10%, and volatility is 0.35 for all prices and periods. Gamma is the z-axis, delta is the color.

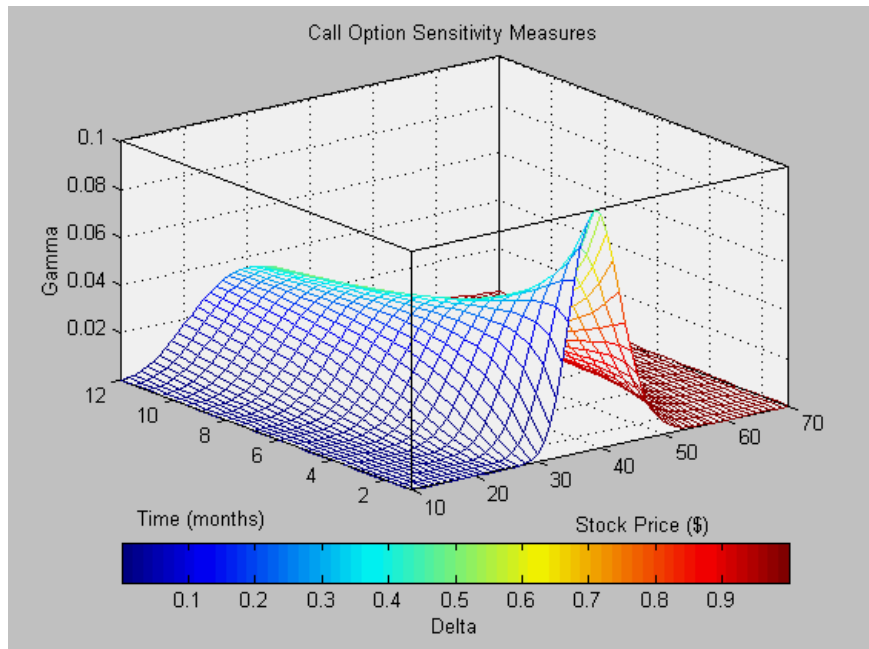
```
ZVal = blsgamma(NewRange, 40*Pad, 0.1*Pad, Newj, 0.35*Pad);  
Color = blsdelta(NewRange, 40*Pad, 0.1*Pad, Newj, 0.35*Pad);
```

Draw the surface as a mesh, add axis labels and a title. The axes range from 10 to 70, 1 to 12, and $-\infty$ to ∞ .

```
mesh(Range, j, ZVal, Color);  
xlabel('Stock Price ($)');  
ylabel('Time (months)');  
zlabel('Gamma');  
title('Call Option Sensitivity Measures');  
axis([10 70 1 12 -inf inf]);
```

Finally add a box around the whole plot, annotate the colors with a bar, and label the colorbar.

```
set(gca, 'box', 'on');  
colorbar('horiz');  
a = findobj(gcf, 'type', 'axes');  
set(get(a(2), 'xlabel'), 'string', 'Delta');
```



Plotting Sensitivities of a Portfolio of Options

This example plots gamma as a function of price and time for a portfolio of 10 Black-Scholes options. The plot shows a three-dimensional surface. For each point on the surface, the height (z-value) represents the sum of the gammas for each option in the portfolio weighted by the amount of each option. The x-axis represents changing price, and the y-axis represents time. The plot adds a fourth dimension by showing delta as surface color. This example M-file is `ftgex3.m`.

First set up the portfolio with arbitrary data. Current prices range from \$20 to \$90 for each option. Set corresponding exercise prices for each option.

```
Range = 20:90;
PLen = length(Range);
ExPrice = [75 70 50 55 75 50 40 75 60 35];
```

Set all risk-free interest rates to 10%, and set times to maturity in days. Set all volatilities to 0.35. Set the number of options of each instrument, and allocate space for matrices.

```
Rate = 0.1*ones(10,1);
Time = [36 36 36 27 18 18 18 9 9 9];
Sigma = 0.35*ones(10,1);
NumOpt = 1000*[4 8 3 5 5.5 2 4.8 3 4.8 2.5];
ZVal = zeros(36, PLen);
Color = zeros(36, PLen);
```

For each instrument, create a matrix (of size Time by PLen) of prices for each period.

```
for i = 1:10
    Pad = ones(Time(i),PLen);
    NewR = Range(ones(Time(i),1),:);
```

Create a vector of time periods 1 to Time; and a matrix of times, one column for each price.

```
T = (1:Time(i))';
NewT = T(:,ones(PLen,1));
```

Call the toolbox gamma and delta sensitivity functions to compute gamma and delta.

```
ZVal(36-Time(i)+1:36,:) = ZVal(36-Time(i)+1:36,:) ...
    + NumOpt(i) * blsgamma(NewR, ExPrice(i)*Pad, ...
    Rate(i)*Pad, NewT/36, Sigma(i)*Pad);

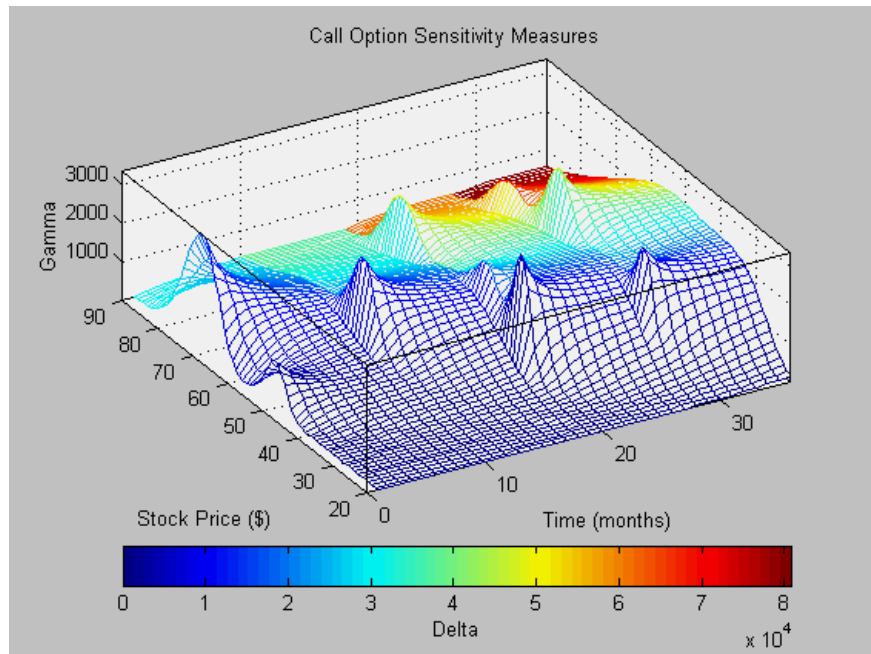
Color(36-Time(i)+1:36,:) = Color(36-Time(i)+1:36,:) ...
    + NumOpt(i) * blsdelta(NewR, ExPrice(i)*Pad, ...
    Rate(i)*Pad, NewT/36, Sigma(i)*Pad);
end
```

Draw the surface as a mesh, set the viewpoint, and reverse the x-axis because of the viewpoint. The axes range from 20 to 90, 0 to 36, and $-\infty$ to ∞ .

```
mesh(Range, 1:36, ZVal, Color);
view(60,60);
set(gca, 'xdir','reverse');
axis([20 90 0 36 -inf inf]);
```

Add a title and axis labels and draw a box around the plot. Annotate the colors with a bar and label the colorbar.

```
title('Call Option Sensitivity Measures');  
xlabel('Stock Price ($)');  
ylabel('Time (months)');  
zlabel('Gamma');  
set(gca, 'box', 'on');  
colorbar('horiz');  
a = findobj(gcf, 'type', 'axes');  
set(get(a(2), 'xlabel'), 'string', 'Delta');
```



Function Reference

Functions by Category	4-2
Handling and Converting Dates	4-2
Formatting Currency and Charting Financial Data	4-4
Analyzing and Computing Cash Flows	4-5
Fixed-Income Securities	4-7
Analyzing Portfolios	4-9
Pricing and Analyzing Derivatives	4-10
GARCH Processes	4-10
Obsolete Bond Price and Yield Functions	4-11

Functions by Category

This chapter contains detailed descriptions of all the functions in the Financial Toolbox. It first groups the functions in task categories and then explains each function in alphabetical order.

Handling and Converting Dates

Note The date functions `datenum`, `datestr`, `datevec`, `eomday`, `now`, and `weekday` now ship with basic MATLAB. They originally shipped only with the Financial Toolbox. Their descriptions remain in this manual for your convenience.

Current Time and Date	
<code>now</code>	Current date and time.
<code>today</code>	Current date.

Date and Time Components	
<code>datefind</code>	Indices of date numbers in matrix.
<code>datevec</code>	Date components.
<code>day</code>	Day of month.
<code>eomdate</code>	Last date of month.
<code>eomday</code>	Last day of month.
<code>hour</code>	Hour of date or time.
<code>lweekdate</code>	Date of last occurrence of weekday in month.
<code>minute</code>	Minute of date or time.
<code>month</code>	Month of date.
<code>months</code>	Number of whole months between dates.
<code>nweekdate</code>	Date of specific occurrence of weekday in month.
<code>second</code>	Second of date or time.
<code>weekday</code>	Day of the week.

Date and Time Components	
year	Year of date.
yeardays	Number of days in year.

Date Conversion	
datedisp	Display date entries.
datenum	Create date number.
datestr	Create date string.
m2xdate	MATLAB serial date number to Excel serial date number.
x2mdate	Excel serial date number to MATLAB serial date number.

Financial Dates	
busdate	Next or previous business day.
datemnth	Date of day in future or past month.
datewrkdy	Date of future or past workday.
days360	Days between dates based on 360-day year.
days365	Days between dates based on 365-day year.
daysact	Actual number of days between dates.
daysdif	Days between dates for any day-count basis.
fbusdate	First business date of month.
holidays	Holidays and non-trading days.
isbusday	True for dates that are business days.
lbusdate	Last business date of month.
wrkdydif	Number of working days between dates.
yearfrac	Fraction of year between dates.

Coupon Bond Dates		
accrfrac	SIA ¹	Fraction of coupon period before settlement.
cfamounts	SIA	Cash flow and time mapping for bond portfolio.
cfdates	SIA	Cash flow dates for a fixed-income security with periodic payments.
cfport		Portfolio form of cash flow amounts.
cftimes	SIA	Time factors corresponding to bond cash flow dates.
cpncount	SIA	Coupon payments remaining until maturity.
cpndaten	SIA	Next coupon date after settlement date.
cpndatenq	SIA	Next quasi coupon date for fixed income security.
cpndatep	SIA	Previous coupon date before settlement date.
cpndatepq	SIA	Previous quasi coupon date for fixed income security.
cpndaysn	SIA	Number of days between settlement date and next coupon date.
cpndaysp	SIA	Number of days between previous coupon date and settlement date.
cpnpersz	SIA	Number of days in coupon period containing settlement date.

1. The Securities Industry Association acronym, SIA, identifies SIA-compliant functions.

Formatting Currency and Charting Financial Data

Currency Formats	
cur2frac	Decimal currency value to fractional value.
cur2str	Bank formatted text.
frac2cur	Fractional currency value to decimal value.

The Financial Toolbox provides a set of functions that create several of the most commonly-used types of financial charts. The Financial Time Series Toolbox provides additional charting capabilities. Using time series data as input, the Financial Time Series Toolbox can compute the value of various

financial indicators and plot the results. Complete information may be found in the *Financial Time Series User's Guide*.

Financial Charts	
bolling	Bollinger band chart.
candle	Candlestick chart.
dateaxis	Convert serial-date axis labels to calendar-date axis labels.
highlow	High, low, open, close chart.
movavg	Leading and lagging moving averages chart.
pointfig	Point and figure chart.

Analyzing and Computing Cash Flows

Annuities	
annurate	Periodic interest rate of annuity.
annuterm	Number of periods to obtain value.

Amortization and Depreciation	
amortize	Amortization.
depfixdb	Fixed declining-balance depreciation.
depgendb	General declining-balance depreciation.
deprdv	Remaining depreciable value.
depsoyd	Sum of years' digits depreciation.
depstln	Straight-line depreciation.

Present Value	
pvinfos	Present value with fixed periodic payments.
pvinfos	Present value of varying cash flow.

Future Value	
fvdisc	Future value of discounted security.
fvfix	Future value with fixed periodic payments.
fvvar	Future value of varying cash flow.

Payment Calculations	
payadv	Periodic payment given number of advance payments.
payodd	Payment of loan or annuity with odd first period.
payper	Periodic payment of loan or annuity.
payuni	Uniform payment equal to varying cash flow.

Rates of Return	
effrr	Effective rate of return.
irr	Internal rate of return.
mirr	Modified internal rate of return.
nomrr	Nominal rate of return.
taxedrr	After-tax rate of return.
xirr	Internal rate of return for nonperiodic cash flow.

Cash Flow Sensitivities	
cfconv	Cash flow convexity.
cfdur	Cash flow duration and modified duration.

Fixed-Income Securities

Accrued Interest	
acrubond	Accrued interest of security with periodic interest payments.
acrudisc	Accrued interest of discount security paying at maturity.

Securities Industry Association (SIA)-compliant functions are identified with the SIA acronym.

Prices	
bndprice SIA	Price a fixed income security from yield to maturity.
prdisc	Price of discounted security.
prmat	Price with interest at maturity.
prtbill	Price of Treasury bill.

Term Structure of Interest Rates	
disc2zero	Zero curve given a discount curve.
fwd2zero	Zero curve given a forward curve.
prbyzero	Price bonds in a portfolio by a set of zero curves.
pyld2zero	Zero curve given a par yield curve.
tbl2bond	Treasury bond parameters given Treasury bill parameters.
tr2bonds	Term-structure parameters given Treasury bond parameters.
zbtprice	Zero curve bootstrapping from coupon bond data given price.
zbtyield	Zero curve bootstrapping from coupon bond data given yield.
zero2disc	Discount curve given a zero curve.
zero2fwd	Forward curve given a zero curve.
zero2pyld	Par yield curve given a zero curve.

Yields		
beytbill		Bond equivalent yield for Treasury bill.
bndyield	SIA	Yield to maturity for fixed income security.
discrate		Bank discount rate of a money market security.
ylddisc		Yield of discounted security.
yldmat		Yield of security with interest at maturity.
yldtbill		Yield of Treasury bill.

Interest Rate Sensitivities		
bndconvp	SIA	Bond convexity given price
bndconvy	SIA	Bond convexity given yield
bnddurp	SIA	Bond duration given price
bnddury	SIA	Bond duration given yield

Analyzing Portfolios

Portfolio Analysis	
corr2cov	Convert standard deviation and correlation to covariance.
cov2corr	Convert covariance to standard deviation and correlation coefficient.
ewstats	Expected return and covariance from return time series.
frontcon	Mean-variance efficient frontier.
pcalims	Linear inequalities for individual asset allocation.
pcgcomp	Linear inequalities for asset group comparison constraints.
pcglims	Linear inequalities for asset group minimum and maximum allocation.
pcpval	Linear inequalities for fixing total portfolio value.
portalloc	Optimal capital allocation.
portcons	Portfolio constraints.
portopt	Portfolios on constrained efficient frontier.
portrand	Randomized portfolio risks, returns, and weights.
portstats	Portfolio expected return and risk.
portsim	Random simulation of correlated asset returns.
portvrisk	Portfolio value at risk
ret2tick	Price tick series from incremental returns and initial price.
tick2ret	Incremental return series from a tick price series.

Pricing and Analyzing Derivatives

Option Valuation and Sensitivity	
bdtbond	Black-Derman-Toy pricing of option-embedded bonds.
bdttrans	Translate a tree returned by bdtbond.
binprice	Binomial put and call pricing.
blkprice	Black's option pricing.
blsdelta	Black-Scholes sensitivity to underlying price change.
blsgamma	Black-Scholes sensitivity to underlying delta change.
blsimpv	Black-Scholes implied volatility.
blslambda	Black-Scholes elasticity.
blsprice	Black-Scholes put and call pricing.
blsrho	Black-Scholes sensitivity to interest rate change.
blstheta	Black-Scholes sensitivity to time-until-maturity change.
blsvega	Black-Scholes sensitivity to underlying price volatility.
opprofit	Option profit.

GARCH Processes

The Financial Toolbox provides these representative functions to help you familiarize yourself with GARCH in the MATLAB context. The GARCH Toolbox provides a more comprehensive and integrated computing environment that includes maximum likelihood parameter estimation, volatility forecasting, Monte Carlo simulation, diagnostic and hypothesis testing, graphical analysis, and data manipulation. For information see the *GARCH Toolbox User's Guide* or the financial products Web page at <http://www.mathworks.com/products/finprod/>.

Univariate GARCH Processes	
ugarch	GARCH parameter estimation.
ugarchllf	Log-likelihood objective function.
ugarchpred	Forecast conditional variance.
ugarchsim	Simulate GARCH process.

Obsolete Bond Price and Yield Functions

The functions listed in this table are obsolete, and their descriptions have been removed from the documentation. They have been replaced with the SIA-compliant functions `bndprice` and `bndyield`. For compatibility purposes, the obsolete functions remain in the product. Type `help function_name` at the MATLAB command line for a description.

<code>prbond</code>	Price of security with regular periodic interest payments.
<code>proddf</code>	Price with odd first period.
<code>proddf1</code>	Price with odd first and last periods and settlement in first period.
<code>proddl</code>	Price with odd last period.
<code>yldbond</code>	Yield to maturity of bond.
<code>yldoddf</code>	Yield of security with odd first period.
<code>yldoddf1</code>	Yield of security with odd first and last periods and settlement in first period.
<code>yldoddl</code>	Yield of security with odd last period.

Alphabetical List of Functions

accfrac	4-16
acrbond	4-19
acrudisc	4-20
amortize	4-21
annurate	4-23
annuterm	4-24
bdtbond	4-25
bdttrans	4-33
beytbill	4-36
binprice	4-37
blkprice	4-39
blsdelta	4-41
blsgamma	4-42
blsimpv	4-43
blslambda	4-44
blsprice	4-45
blsrho	4-47
blstheta	4-48
blsvega	4-49
bndconvp	4-50
bndconvy	4-53
bnddurp	4-56
bnddury	4-59
bndprice	4-62
bndyield	4-65
bolling	4-68
busdate	4-69
candle	4-70
cfamounts	4-71
cfconv	4-76
cfdates	4-77
cfdur	4-80
cfport	4-81
cftimes	4-84
corr2cov	4-86

cov2corr	4-87
cpncount	4-88
cpndaten	4-91
cpndatenq	4-94
cpndatep	4-98
cpndatepq	4-101
cpndaysn	4-105
cpndaysp	4-108
cpnpersz	4-111
cur2frac	4-114
cur2str	4-115
dateaxis	4-116
datedisp	4-118
datefind	4-119
datemnth	4-120
datenum	4-122
datestr	4-125
datevec	4-128
datewrkdy	4-130
day	4-131
days360	4-132
days365	4-133
daysact	4-134
daysdif	4-135
depfixdb	4-136
depgendb	4-137
deprdv	4-138
depsoyd	4-139
depstln	4-140
disc2zero	4-141
discrate	4-144
effrr	4-145
eomdate	4-146
eomday	4-147
ewstats	4-148
fbusdate	4-150
frac2cur	4-151

frontcon	4-152
fvdisc	4-155
fvfix	4-156
fvvar	4-157
fwd2zero	4-159
highlow	4-163
holidays	4-164
hour	4-165
irr	4-166
isbusday	4-167
lbusdate	4-168
lweekdate	4-170
m2xdate	4-172
minute	4-174
mirr	4-175
month	4-176
months	4-177
movavg	4-178
nomrr	4-179
now	4-180
nweekdate	4-181
opprofit	4-183
payadv	4-184
payodd	4-185
payper	4-186
payuni	4-187
pcalims	4-188
pcgcomp	4-191
pcglims	4-193
pcpval	4-196
pointfig	4-198
portalloc	4-199
portcons	4-202
portopt	4-206
portrand	4-209
portsim	4-210
portstats	4-212

portvrisk	4-214
prbyzero	4-216
prdisc	4-220
prmat	4-221
prtbill	4-223
pvfix	4-224
pvvar	4-225
pyld2zero	4-227
ret2tick	4-231
second	4-233
taxedrr	4-234
tbl2bond	4-235
tick2ret	4-237
today	4-239
tr2bonds	4-240
ugarch	4-243
ugarchllf	4-245
ugarchpred	4-247
ugarchsim	4-250
weekday	4-255
wrkdydif	4-257
x2mdate	4-258
xirr	4-260
year	4-262
yeardays	4-263
yearfrac	4-264
ylddisc	4-265
yldmat	4-266
yldtbill	4-268
zbtprice	4-269
zbtyield	4-274
zero2disc	4-279
zero2fwd	4-282
zero2pyld	4-286

accrfrac

Purpose	Fraction of coupon period before settlement (SIA compliant)	
Syntax	Fraction = accrfrac(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Vector arguments must have consistent dimensions, or they must be scalars.

Description

Fraction = accrfrac(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the fraction of the coupon period before settlement. This function is used for computing accrued interest.

Examples

Given data for three bonds:

```
Settle = '14-Mar-1997';
Maturity = ['30-Nov-2000'
            '31-Dec-2000'
            '31-Jan-2001'];
Period = 2;
Basis = 0;
EndMonthRule = 1;
```

Execute the function.

```
Fraction = accrfrac(Settle, Maturity, Period, Basis,...
                    EndMonthRule)

Fraction =
    0.5714
    0.4033
    0.2320
```

accrfrac

See Also cfamounts, cfdates, cpncount, cpndaten, cpndatenq, cpndatep, cpndatepq, cpndaysn, cpndaysp, cnpersz

Purpose	Accrued interest of security with periodic interest payments		
Syntax	AccruInterest = acrubond(IssueDate, Settle, FirstCouponDate, Face, CouponRate, Period, Basis)		
Arguments	IssueDate	Enter as serial date number or date string.	
	Settle	Enter as serial date number or date string.	
	FirstCouponDate	Enter as serial date number or date string.	
	Face	Redemption (par, face) value.	
	CouponRate	Enter as decimal fraction.	
	Period	(Optional) Coupons per year. An integer. Default = 2.	
	Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.	
Description	AccruInterest = acrubond(IssueDate, Settle, FirstCouponDate, Face, CouponRate, Period, Basis) returns the accrued interest for a security with periodic interest payments. This function computes the accrued interest for securities with standard, short, and long first coupon periods.		
	Note cfamounts or accrfrac is recommended when calculating accrued interest beyond the first period.		
Examples	AccruInterest = acrubond('31-jan-1983', '1-mar-1993', ... 		

acrudisc

Purpose	Accrued interest of discount security paying at maturity
Syntax	<code>AccruInterest = acrudisc(Settle, Maturity, Face, Discount, Period, Basis)</code>
Arguments	<code>Settle</code> Enter as serial date number or date string. <code>Settle</code> must be earlier than or equal to <code>Maturity</code>. <code>Maturity</code> Enter as serial date number or date string. <code>Face</code> Redemption (par, face) value. <code>Discount</code> Discount rate of the security. Enter as decimal fraction. <code>Period</code> (Optional) Coupons per year. An integer. Default = 2. <code>Basis</code> (Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
Description	<code>AccruInterest = acrudisc(Settle, Maturity, Face, Discount, Period, Basis)</code> returns the accrued interest of a discount security paid at maturity.
Examples	<pre>AccruInterest = acrudisc('05/01/1992', '07/15/1992', ... 100, 0.1, 2, 0) AccruInterest = 2.0604 (or \$2.06)</pre>
See Also	<code>acrubond</code> , <code>prdisc</code> , <code>prmat</code> , <code>ylddisc</code> , <code>yldmat</code>
References	Mayle, <i>Standard Securities Calculation Methods</i> , Volumes I-II, 3rd edition. Formula D.

Purpose	Amortization schedule
Syntax	<code>[Principal, Interest, Balance, Payment] = amortize(Rate, NumPeriods, PresentValue, FutureValue, Due)</code>
Arguments	Rate Interest rate per period, as a decimal fraction. NumPeriods Number of payment periods. PresentValue Present value of the loan. FutureValue (Optional) Future value of the loan. Default = 0. Due (Optional) When payments are due: 0 = end of period (default), or 1 = beginning of period.
Description	<code>[Principal, Interest, Balance, Payment] = amortize(Rate, NumPeriods, PresentValue, FutureValue, Due)</code> returns the principal and interest payments of a loan, the remaining balance of the original loan amount, and the periodic payment. Principal Principal paid in each period. A 1-by-NumPeriods vector. Interest Interest paid in each period. A 1-by-NumPeriods vector. Balance Remaining balance of the loan in each payment period. A 1-by-NumPeriods vector. Payment Payment per period. A scalar.

Examples

A \$500 loan paid in six installments at an annual interest rate of 9%:

```
[Principal, Interest, Balance, Payment] = amortize(0.09/6, 6, ... 500)
```

Principal =						
	80.26	81.47	82.69	83.93	85.19	86.47
Interest =						
	7.50	6.3	5.07	3.83	2.57	1.30
Balance =						
	419.74	338.27	255.58	171.65	86.47	0.00

amortize

Payment =
87.76

See Also

annurate, annuterm, payadv, payodd, payper

Purpose	Periodic interest rate of annuity
Syntax	<code>Rate = annurate(NumPeriods, Payment, PresentValue, FutureValue, Due)</code>
Arguments	<code>NumPeriods</code> Number of payment periods. <code>Payment</code> Payment per period. <code>PresentValue</code> Present value of the loan or annuity. <code>FutureValue</code> (Optional) Future value of the loan or annuity. Default = 0. <code>Due</code> (Optional) When payments are due: 0 = end of period (default), or 1 = beginning of period.

annuterm

Purpose	Number of periods to obtain value	
Syntax	NumPeriods = annuterm(Rate, Payment, PresentValue, FutureValue, Due)	
Arguments	Rate	Interest rate per period, as a decimal fraction.
	Payment	Payment per period.
	PresentValue	Present value.
	FutureValue	(Optional) Future value. Default = 0.
	Due	(Optional) When payments are due: 0 = end of period (default), or 1 = beginning of period.
Description	NumPeriods = annuterm(Rate, Payment, PresentValue, FutureValue, Due) calculates the number of periods needed to obtain a future value. To calculate the number of periods needed to pay off a loan, enter the payment or the present value as a negative value.	
Examples	A savings account has a starting balance of \$1500. \$200 is added at the end of each month and the account pays 9% interest, compounded monthly. How many years will it take to save \$5,000?	
	NumPeriods = annuterm(0.09/12, 200, 1500, 5000, 0)	
	NumPeriods = 15.68 months or 1.31 years.	
See Also	annurate, amortize, fvfix, pvfix	

Purpose	Black-Derman-Toy pricing of option-embedded bonds
Syntax	<pre>[Price, Sensitivities, DiscTree, PriceTree] = bdtbond(OptBond, ZeroCurve, VolatilityCurve, Accuracy, CreditCurve, ComputeSensitivity)</pre>
Arguments	All arguments except Accuracy are structures. The name of the argument variable can be substituted, but the fieldnames must be reproduced exactly. The variables and the fields are listed below in Variable.fieldname format. An optional field or variable can be set to the empty matrix ([]) to invoke defaults. An optional field in a structure can also be left unspecified.

OptBond Specification of underlying bond with a possible call and put option. Fields are scalars or date strings. Type `help ftb` for detailed information on bond parameters.

OptBond.Settle: Settlement date.

OptBond.Maturity: Maturity date.

OptBond.Period: (Optional) Coupon frequency. Default = 2.

OptBond.Basis: (Optional) Market Basis. Default = 0 (actual/actual).

OptBond.EndMonthRule: (Optional) EOM rule. Default = 1 (in effect).

OptBond.FirstCouponDate: (Optional) First coupon payment if the first coupon period is irregular.

OptBond.LastCouponDate: (Optional) Last coupon payment before maturity if the last coupon period is irregular.

OptBond.IssueDate: (Optional) Issue date of the bond if the first coupon period is irregular.

OptBond.StartDate: (Future implementation; optional) Forward start date of the security if not before settlement.

OptBond.CouponRate: (Optional) Coupon payment rate.

OptBond.Face: (Optional) Maturity payment of the bond. Default = 100.

OptBond (continued)

Option specification fields: Specify fields for either a call or a put. Calls assumed held by the bond issuer; puts assumed held by the bondholder. A call therefore reduces the value of the bond to the bond holder, while a put increases the value of the bond.

`OptBond.CallStrike`: Call option strike price.

`OptBond.CallType`: 1 (American) or 0 (European). Default = 1.

`OptBond.CallExpiryDate`: Last date of exercise for an American option, or only date for a European option.

`OptBond.CallStartDate`: First date of exercise for an American option. Default is Settlement.

`OptBond.PutStrike`: Put option strike price.

`OptBond.PutType`: 1 (American) or 0 (European). Default = 1.

`OptBond.PutExpiryDate`: Last date of exercise for an American option, or only date for a European option. Default is maturity date.

`OptBond.PutStartDate`: (Optional) first date of exercise for an American option. Default is settlement date.

ZeroCurve

Zero curve of NCURVE (date, decimal rate) pairs is interpolated to cover the time span of the bond. Times before the first curve date use the first rate; times after the last curve date use the last rate.

`ZeroCurve.CurveDates`: NCURVE-by-1 vector of serial dates.

`ZeroCurve.ZeroRates`: NCURVE-by-1 vector of rates.

VolatilityCurve

Curve of instantaneous yearly volatilities of the short rates. The curve consists of NCURVE2 (date, decimal rate) pairs. The curve is interpolated to cover the time span of the bond.

VolatilityCurve.CurveDates: NCURVE2-by-1 vector of serial dates.

VolatilityCurve.VolatilityRates: NCURVE2-by-1 vector of yearly volatilities in decimal form.

Accuracy

Scalar that specifies the number of steps in the tree per coupon period. Larger numbers yield more accurate answers, but require more time and memory.

CreditCurve

(Optional) Curve of rate spreads arising from default risk. The curve has NCURVE3 (date, basis point) pairs. The curve is interpolated to cover the time span of the bond.

CreditCurve.CurveDates: NCURVE3-by-1 vector of serial dates.

CreditCurve.CreditRates: NCURVE3-by-1 vector of credit spread values in basis points (not decimal rates). The effective change to the zero rate is CreditRates/10000.

ComputeSensitivity

(Optional) Specify if bond sensitivity measures (with and without options) are to be computed. 1 indicates measure computed; 0 indicates not computed. Sensitivities found by a finite difference calculation. The default is no sensitivities, only prices returned.

ComputeSensitivity.Duration: Scalar 1 or 0.

ComputeSensitivity.Convexity: Scalar 1 or 0.

ComputeSensitivity.Vega: Scalar 1 or 0.

Description

[Price, Sensitivities, DiscTree, PriceTree] = bdtbond(OptBond, ZeroCurve, VolatilityCurve, Accuracy, CreditCurve, ComputeSensitivity) computes price and sensitivity measures of a bond with embedded call or put options. Valuation is based on the Black-Derman-Toy model for pricing interest rate options given an input yield curve (and possibly a credit spread) and volatility curve.

Price is the value of the bond with and without the options.

Price.OptionFreePrice	Scalar price of the bond without any options.
Price.OptionEmbedPrice	Scalar price (value to the holder of the bond) of the bond with options.
Price.OptionValue	Scalar value of the options to the holder of the bond.

Sensitivities refer to the effect that changes in the yield curve and volatility term structure have on option-free and option-embedded bond prices.

Sensitivities.Duration	Sensitivity of option-free bond price to parallel shifts of the yield curve.
Sensitivities.EffDuration	Sensitivity of the option-embedded price to shifts in the yield curve.
Sensitivities.Convexity	Sensitivity of Duration to shifts in the yield curve.
Sensitivities.EffConvexity	Sensitivity of EffDuration to shifts in the yield curve.
Sensitivities.Vega	Sensitivity of the option-embedded price to parallel shifts of the volatility curve.

DiscTree is the recombining binomial tree of the interest rate structure. The tree covers NPERIODS times from settlement to maturity, where there are Accuracy steps in each coupon period. The short rate at settlement and between settlement and the first time is deterministic.

<code>DiscTree.Values</code>	NSTATES-by-NPERIODS matrix of short discount factors. The NPERIODS columns of <code>Values</code> correspond to successive times. The NSTATES rows correspond to states in the rate process. Unused states are masked by NaN. Multiplication of a cash amount at time <code>Dates(i)</code> by the discount <code>Values(j,i)</code> gives the price at <code>Dates(i-1)</code> after traversing the <code>(j,i)</code> edge of the tree. The short rate <code>R(j,i)</code> prevailing at node <code>(j,i)</code> satisfies: $(1 + R(j,i)/\text{Frequency})^{-(\text{Times}(j) - \text{Times}(j-1))} = \text{Values}(j,i)$
<code>DiscTree.Times</code>	1-by-NPERIODS vector of tree node times in units of coupon intervals. (Type help <code>ftbTFactors</code> for more information.)
<code>DiscTree.Dates</code>	1-by-NPERIODS vector of tree node times as serial date numbers.
<code>DiscTree.Type</code>	'Short Discount'
<code>DiscTree.Frequency</code>	Compounding frequency of the input bond.
<code>DiscTree.ErrorFlag</code>	(0 or 1). Set to 1 if any short rate becomes negative.

`PriceTree` is the recombining binomial tree of cash amounts at tree nodes. `PriceTree` is computed from the bond cash flows and the option payoffs. The clean price of the bond is the `PriceTree` value minus the coupon payment and the accrued interest.

<code>PriceTree.Values</code>	NSTATES-by-NPERIODS matrix of price states.
<code>PriceTree.Times</code>	1-by-NPERIODS vector of tree node times in units of coupon intervals. (Type help <code>ftbTFactors</code> for more information.)
<code>PriceTree.Dates</code>	1-by-NPERIODS vector of tree node times as serial date numbers.

PriceTree.AccrInt	1-by-NPERIODS vector of accrued interest payable at each time.
PriceTree.Coupons	1-by-NPERIODS vector of coupon payments at each time.
DiscTree.Type	'Price'

Examples

Given a bond with the characteristics

```
OptBond.Settle = '15-Jul-1996';
OptBond.Maturity = '15-Jan-1998';
OptBond.CouponRate = 0.06;
OptBond.Period = 2
```

Specify an American put option on the bond. Allow the holder to put the bond between 15-Jan-1997 and maturity (15-Jan-1998) for a strike of 98.

```
OptBond.PutType = 1;
OptBond.PutStartDate = '15-Jan-1997';
OptBond.PutExpiryDate = '15-Jan-1998';
OptBond.PutStrike = 98
```

Build zero curve term structure.

```
ZeroCurve.ZeroRates = [0.05; 0.06; 0.065];
ZeroCurve.CurveDates = ['01-Jan-1996'; '01-Jan-1997';
'01-Jan-1998']
```

Build volatility curve term structure.

```
VolCurve.VolatilityRates = [0.15; 0.13];
VolCurve.CurveDates = [729025; 729756]
```

The coupon interval is 1/2 year; use 10 tree periods per year.

```
Accuracy = 5
```

Specify a constant credit spread of 200 basis points (0.02).

```
CreditCurve.CreditRates = [200];
CreditCurve.CurveDates = ['01-Jan-1996']
```

Ask for duration and vega.

bdtbond

```
SensChoice.Duration = 1;
SensChoice.Convexity = 0;
SensChoice.Vega = 1;

[Price, Sensitivities, DiscTree, PriceTree] = ...
bdtbond(OptBond, ZeroCurve, VolCurve, Accuracy, CreditCurve,...
SensChoice);

Price =
OptionFreePrice: 96.5565
OptionEmbedPrice: 97.1769
OptionValue: 0.6204

Sensitivities =
Duration: 1.3959
EffDuration: 0.6848
Convexity: NaN
EffConvexity: NaN
Vega: -0.0194
```

To look at the rate and clean price trees, use `bdttrans`.

```
bdttrans(DiscTree)
bdttrans(PriceTree)
```

See Also

`bdttrans`

Purpose	Translate a tree returned by bdtbond
Syntax	<code>bdttrans(Tree)</code> <code>[TreeMat, TreeTimes] = bdttrans(Tree)</code>
Arguments	Tree Tree structure returned by bdtbond.
Description	<code>bdttrans</code> unpacks and converts a short discount tree structure to a short rate tree, or a price tree structure to a clean price tree. The output is a matrix of tree node values and a vector of node times. <code>TreeMat</code> is an NSTATES-by-NTIMES converted matrix of short rate or clean price values at points on the tree. Time layers are columns containing the different states. Unused entries of the matrix are screened with NaN. <code>TreeTimes</code> is a 1-by-NTIMES vector of times corresponding to layers of <code>TreeMat</code>. If <code>bdttrans</code> is called without output arguments, it plots the translated tree against the time axis in coupon intervals.
Examples	Specify a bond. <pre>OptBond.Settle = '15-Jul-1996'; OptBond.Maturity = '15-Jan-1998'; OptBond.CouponRate = 0.07; OptBond.Period = 2;</pre> Specify an embedded American call option. Make the bond callable by the issuer between 15-Jan-1997 and maturity. <pre>OptBond.CallType = 1; OptBond.CallStartDate = '15-Jan-1997'; OptBond.CallExpiryDate = '15-Jan-1998'; OptBond.CallStrike = 101;</pre> Build constant zero curve structure. <pre>ZeroCurve.ZeroRates = 0.05; ZeroCurve.CurveDates = '15-Jul-1996';</pre> Build constant volatility curve structure.

```
VolCurve.VolatilityRates = 0.15;  
VolCurve.CurveDates = '15-Jul-1996';
```

Choose two tree nodes per whole coupon period.

```
Accuracy = 2;
```

Run the bdtbond function.

```
[Price, Sensitivities, DiscTree, PriceTree] = bdtbond(OptBond, ...  
ZeroCurve, VolCurve, Accuracy);
```

Convert the discount tree to a short rate tree.

```
DiscTree =  
Values: [6x7 double]  
Times: [0 0.5000 1 1.5000 2 2.5000 3]  
Dates: [729221 729313 729405 729496 729586 729678 729770]  
ErrorFlag: 0  
Type: 'Short Discount'  
Frequency: 2
```

```
[ShortRateMat, TreeTimes] = bdttrans(DiscTree)
```

```
ShortRateMat =
```

0.0500	0.0500	0.0447	0.0399	0.0357	0.0319	0.0285
NaN	NaN	0.0554	0.0494	0.0442	0.0395	0.0353
NaN	NaN	NaN	0.0613	0.0548	0.0489	0.0437
NaN	NaN	NaN	NaN	0.0679	0.0607	0.0542
NaN	NaN	NaN	NaN	NaN	0.0753	0.0672
NaN	NaN	NaN	NaN	NaN	NaN	0.0835

```
TreeTimes =
```

0	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000
---	--------	--------	--------	--------	--------	--------

Convert the price tree to a clean price tree.

```

PriceTree =
Values: [6x7 double]
Times: [0 0.5000 1 1.5000 2 2.5000 3]
Dates: [729221 729313 729405 729496 729586 729678 729770]
ErrorFlag: 0
AccrInt: [0 1.7500 0 1.7597 0 1.7500 0]
Coupons: [0 0 3.5000 0 3.5000 0 3.5000]
Type: 'Price'

[CleanPriceMat, TreeTimes] = bdttrans(PriceTree)

CleanPriceMat =

101.9512  101.4677  101.0000  101.0000  101.0000  100.9348  100.0000
      NaN      NaN  101.0000  101.0000  101.0000  100.7429  100.0000
      NaN      NaN      NaN  100.6019  100.7397  100.5060  100.0000
      NaN      NaN      NaN      NaN  100.0978  100.2139  100.0000
      NaN      NaN      NaN      NaN      NaN  99.8539  100.0000
      NaN      NaN      NaN      NaN      NaN      NaN  100.0000

TreeTimes =

0    0.5000    1.0000    1.5000    2.0000    2.5000    3.0000

```

See Also

bdtbond

beytbill

Purpose	Bond equivalent yield for Treasury bill
Syntax	<code>Yield = beytbill(Settle, Maturity, Discount)</code>
Arguments	Settle Enter as serial date number or date string. Settle must be earlier than or equal to Maturity. Maturity Enter as serial date number or date string. Discount Discount rate of the Treasury bill. Enter as decimal fraction.
Description	<code>Yield = beytbill(Settle, Maturity, Discount)</code> returns the bond equivalent yield for a Treasury bill.
Examples	The settlement date of a Treasury bill is February 11, 2000, the maturity date is August 7, 2000, and the discount rate is 5.77%. The bond equivalent yield is <pre>Yield = beytbill('2/11/2000', '8/7/2000', 0.0577)</pre> <pre>Yield = 0.0602</pre>
See Also	<code>datenum</code> , <code>prtbill</code> , <code>yldtbill</code>

Purpose	Binomial put and call pricing	
Syntax	[AssetPrice, OptionValue] = binprice(Price, Strike, Rate, Time, Increment, Volatility, Flag, DividendRate, Dividend, ExDiv)	
Arguments	Price	Underlying asset price. A scalar.
	Strike	Option exercise price. A scalar.
	Rate	Risk-free interest rate. A scalar. Enter as a decimal fraction.
	Time	Option's time until maturity in years. A scalar.
	Increment	Time increment. A scalar. Increment is adjusted so that the length of each interval is consistent with the maturity time of the option. (Increment is adjusted so that Time divided by Increment equals an integer number of increments.)
	Volatility	Asset's volatility. A scalar.
	Flag	Specifies whether the option is a call (Flag = 1) or a put (Flag = 0). A scalar.
	DividendRate	(Optional) The dividend rate, as a decimal fraction. A scalar. Default = 0. If you enter a value for DividendRate, set Dividend and ExDiv = 0 or do not enter them. If you enter values for Dividend and ExDiv, set DividendRate = 0.
	Dividend	(Optional) The dividend payment at an ex-dividend date, ExDiv. A row vector. For each dividend payment, there must be a corresponding ex-dividend date. Default = 0. If you enter values for Dividend and ExDiv, set DividendRate = 0.
	ExDiv	(Optional) Ex-dividend date, specified in number of periods. A row vector. Default = 0.
Description	[AssetPrice, OptionValue] = binprice(Price, Strike, Rate, Time, Increment, Volatility, Flag, DividendRate, Dividend, ExDiv) prices an option using the Cox-Ross-Rubinstein binomial pricing model.	
Examples	For a put option, the asset price is \$52, option exercise price is \$50, risk-free interest rate is 10%, option matures in 5 months, volatility is 40%, and there is one dividend payment of \$2.06 in 3-1/2 months:	

binprice

```
[Price, Option] = binprice(52, 50, 0.1, 5/12, 1/12, 0.4, 0, 0,...
2.06, 3.5)
```

returns the asset price and option value at each node of the binary tree:

Price =					
52.0000	58.1367	65.0226	72.7494	79.3515	89.0642
0	46.5642	52.0336	58.1706	62.9882	70.6980
0	0	41.7231	46.5981	49.9992	56.1192
0	0	0	37.4120	39.6887	44.5467
0	0	0	0	31.5044	35.3606
0	0	0	0	0	28.0688
Option =					
4.4404	2.1627	0.6361	0	0	0
0	6.8611	3.7715	1.3018	0	0
0	0	10.1591	6.3785	2.6645	0
0	0	0	14.2245	10.3113	5.4533
0	0	0	0	18.4956	14.6394
0	0	0	0	0	21.9312

See Also

blkprice, blsprice

References

Cox, J.; S. Ross; and M. Rubenstein, "Option Pricing: A Simplified Approach", *Journal of Financial Economics* 7, Sept. 1979, pp. 229 - 263

Hull, *Options, Futures, and Other Derivative Securities*, 2nd edition, Chapter 14.

Purpose	Black's option pricing
Syntax	[Call, Put] = blkprice(ForwardPrice, Strike, Rate, Time, Volatility)
Arguments	ForwardPrice Forward price of underlying asset at time zero. Must be greater than 0. You can extend Black's model to interest-rate derivatives (call and put options embedded in bonds) by calculating the forward price from the equation $f = (B - I) * \exp(r*t)$ where B is the face value of the bond, I is the present value of the coupons during the life of the option, r is the risk-free interest rate, and t is the time until maturity. Strike Strike or exercise price of the options. Must be greater than 0. Rate Risk-free interest rate (plus storage costs less any convenience yield). Must be greater than or equal to 0. Time Time until maturity of option in years. Must be greater than 0. Volatility Volatility of the price of the underlying asset. Must be greater than or equal to 0.
Description	[Call, Put] = blkprice(ForwardPrice, Strike, Rate, Time, Volatility) uses Black's model to value an option and returns the Call and Put option prices. Note This function uses normcdf, the normal cumulative distribution function in the Statistics Toolbox.
Examples	The forward price of a bond is \$95, the exercise price of the option is \$98, the risk-free interest rate is 11%, the time to maturity of the option is 3 years, and the volatility of the bond price is 2.5%.

blkprice

```
[Call, Put] = blkprice(95, 98, 0.11, 3, 0.025)
```

```
Call =  
    0.4162 (or $0.42)
```

```
Put =  
    2.5729 (or $2.57)
```

See Also binprice, blsprice

References Hull, *Options, Futures, and Other Derivative Securities*, 2nd edition, Formulas 15.7 and 15.8.

Black, “The Pricing of Commodity Contracts,” *Journal of Financial Economics*, March 3, 1976, pp. 167-179.

Purpose	Black-Scholes sensitivity to underlying price change	
Syntax	[CallDelta, PutDelta] = blsdelta(Price, Strike, Rate, Time, Volatility, DividendRate)	
Arguments	Price	Current stock price.
	Strike	Exercise price.
	Rate	Risk-free interest rate. Enter as a decimal fraction.
	Time	Time to maturity of the option, in years.
	Volatility	Standard deviation of the annualized continuously compounded rate of return of the stock, also known as volatility.
	DividendRate	(Optional) Dividend rate or foreign interest rate where applicable. Enter as a decimal fraction. Default = 0.

Description [CallDelta, PutDelta] = blsdelta(Price, Strike, Rate, Time, Volatility, DividendRate) returns delta, the sensitivity in option value to change in the underlying security price. Delta is also known as the hedge ratio.

Note This function uses normcdf, the normal cumulative distribution function in the Statistics Toolbox.

Examples

```
[CallDelta, PutDelta] = blsdelta(50, 50, 0.1, 0.25, 0.3, 0)

CallDelta =
    0.5955

PutDelta =
   -0.4045
```

See Also blsgamma, blslambda, blsprice, blsrho, blstheta, blsvega

References Hull, *Options, Futures, and Other Derivative Securities*, 2nd edition, Chapter 13.

blsgamma

Purpose	Black-Scholes sensitivity to underlying delta change	
Syntax	Gamma = blsgamma(Price, Strike, Rate, Time, Volatility, DividendRate)	
Arguments	Price	Current stock price.
	Strike	Exercise price.
	Rate	Risk-free interest rate. Enter as a decimal fraction.
	Time	Time to maturity of the option in years.
	Volatility	Standard deviation of the annualized continuously compounded rate of return of the stock (also known as the volatility).
	DividendRate	(Optional) Enter as a decimal fraction. Default = 0.
Description	Gamma = blsgamma(Price, Strike, Rate, Time, Volatility, DividendRate) returns gamma, the sensitivity of delta to change in the underlying security price.	
	Note This function uses normpdf, the normal probability density function in the Statistics Toolbox.	
Examples	Gamma = blsgamma(50, 50, 0.12, 0.25, 0.3, 0) Gamma = 0.0512	
See Also	blsdelta, blslambda, blsprice, blsrho, blstheta, blsvega	
References	Hull, <i>Options, Futures, and Other Derivative Securities</i> , 2nd edition, Chapter 13.	

Purpose	Black-Scholes implied volatility
Syntax	Volatility = blsimpv(Price, Strike, Rate, Time, Call, MaxIterations)
Arguments	Price Current asset price. Strike Exercise price. Rate Risk-free interest rate. Enter as a decimal fraction. Time Time to maturity in years. Call Call option value. MaxIterations (Optional) Maximum number of iterations used in solving for volatility using Newton's method. Default = 50.
Description	Volatility = blsimpv(Price, Strike, Rate, Time, Call, MaxIterations) returns the implied volatility of an underlying asset, using Newton's method. Note This function uses normcdf and normpdf, the normal cumulative distribution and normal probability density functions in the Statistics Toolbox.
Examples	An asset has a current price of \$100, an exercise price of \$95, the risk free interest rate is 7.5%, the time to maturity of the option is 0.25 years, and the call option has a value of \$10.00. <pre>Vol = blsimpv(100, 95, 0.075, 0.25, 10)</pre> <pre>Vol =</pre> <pre>0.3130 (or 31.3%)</pre>
See Also	blsprice
References	Bodie, Kane, and Marcus, <i>Investments</i> , page 681.

blslambda

Purpose	Black-Scholes elasticity												
Syntax	[CallEl, PutEl] = blslambda(Price, Strike, Rate, Time, Volatility, DividendRate)												
Arguments	<table><tr><td>Price</td><td>Current stock price.</td></tr><tr><td>Strike</td><td>Exercise price.</td></tr><tr><td>Rate</td><td>Risk-free interest rate. Enter as a decimal fraction.</td></tr><tr><td>Time</td><td>Time to maturity of the option in years.</td></tr><tr><td>Volatility</td><td>Standard deviation of the annualized continuously compounded rate of return of the stock (also known as the volatility).</td></tr><tr><td>DividendRate</td><td>(Optional) Dividend rate. Enter as a decimal fraction. Default = 0.</td></tr></table>	Price	Current stock price.	Strike	Exercise price.	Rate	Risk-free interest rate. Enter as a decimal fraction.	Time	Time to maturity of the option in years.	Volatility	Standard deviation of the annualized continuously compounded rate of return of the stock (also known as the volatility).	DividendRate	(Optional) Dividend rate. Enter as a decimal fraction. Default = 0.
Price	Current stock price.												
Strike	Exercise price.												
Rate	Risk-free interest rate. Enter as a decimal fraction.												
Time	Time to maturity of the option in years.												
Volatility	Standard deviation of the annualized continuously compounded rate of return of the stock (also known as the volatility).												
DividendRate	(Optional) Dividend rate. Enter as a decimal fraction. Default = 0.												

Description [CallEl, PutEl] = blslambda(Price, Strike, Rate, Time, Volatility, DividendRate) returns the elasticity of an option. CallEl is the call option elasticity or leverage factor, and PutEl is the put option elasticity or leverage factor. Elasticity (the leverage of an option position) measures the percent change in an option price per one percent change in the underlying stock price.

Note This function uses normcdf, the normal cumulative distribution function in the Statistics Toolbox.

Examples

```
[CallEl, PutEl] = blslambda(50, 50, 0.12, 0.25, 0.3)

CallEl =
    8.1274
PutEl =
   -8.6466
```

See Also blsdelta, blsgamma, blsprice, blsrho, blstheta, blsvega

References Daigler, *Advanced Options Trading*, Chapter 4.

Purpose	Black-Scholes put and call pricing
Syntax	<code>[CallPrice, PutPrice] = bbsprice(Price, Strike, Rate, Time, Volatility, DividendRate)</code>
Arguments	Price Current asset price. Strike Exercise price. Rate Risk-free interest rate. Enter as a decimal fraction. Time Time to maturity of the option in years. Volatility Standard deviation of the annualized continuously compounded rate of return of the asset (also known as the volatility). DividendRate (Optional) Dividend rate of the asset. Enter as a decimal fraction. Default = 0.
Description	<code>[CallPrice, PutPrice] = bbsprice(Price, Strike, Rate, Time, Volatility, DividendRate)</code> returns the value of call and put options using the Black-Scholes pricing formula.

Note This function uses `normcdf`, the normal cumulative distribution function in the Statistics Toolbox.

Examples	The current price of an asset is \$100, the exercise price of the option is \$95, the risk-free interest rate is 10%, the time to maturity of the option is 0.25 years, and the standard deviation of the asset is 50%. <pre>[CallPrice, PutPrice] = bbsprice(100, 95, 0.1, 0.25, 0.5) CallPrice = 13.70 PutPrice = 6.35</pre>
-----------------	--

blsprice

See Also

blkprice, blsdelta, blsgamma, blsimpv, blslambda, blsrho, blstheta, blsvega

References

Bodie, Kane, and Marcus, *Investments*, page 681.

Purpose	Black-Scholes sensitivity to interest rate change	
Syntax	[CallRho, PutRho]= blsrho(Price, Strike, Rate, Time, Volatility, DividendRate)	
Arguments	Price	Current security price.
	Strike	Exercise or strike price.
	Rate	Interest rate. Enter as a decimal fraction.
	Time	Time to maturity of the option in years.
	Volatility	Standard deviation of the annualized continuously compounded rate of return of the security (also known as the volatility).
	DividendRate	(Optional) Dividend rate of the security. Enter as a decimal fraction. Default = 0.
Description	[CallRho, PutRho]= blsrho(Price, Strike, Rate, Time, Volatility, DividendRate) returns the call option rho CallRho, and the put option rho PutRho. Rho is the rate of change in value of derivative securities with respect to interest rates.	
	Note This function uses normcdf, the normal cumulative distribution function in the Statistics Toolbox.	
Examples	<pre>[CallRho, PutRho] = blsrho(50, 50, 0.12, 0.25, 0.3, 0) CallRho = 6.6686 PutRho = -5.4619</pre>	
See Also	blsdelta, blsgamma, blslambda, blsprice, blstheta, blsvega	
References	Hull, <i>Options, Futures, and Other Derivative Securities</i> , 2nd edition, Chapter 13.	

blstheta

Purpose	Black-Scholes sensitivity to time-until-maturity change	
Syntax	[CallTheta, PutTheta] = blstheta(Price, Strike, Rate, Time, Volatility, DividendRate)	
Arguments	Price	Current stock price.
	Strike	Exercise price.
	Rate	Risk-free interest rate. Enter as a decimal fraction.
	Time	Time to maturity of the option in years.
	Volatility	Standard deviation of the annualized continuously compounded rate of return of the stock (also known as the volatility).
	DividendRate	(Optional) Enter as a decimal fraction. Default = 0.
Description	[CallTheta, PutTheta] = blstheta(Price, Strike, Rate, Time, Volatility, DividendRate) returns the call option theta CallTheta, and the put option theta PutTheta. Theta is the sensitivity in option value with respect to time.	
	Note This function uses normpdf, the normal probability density function and normcdf, the normal cumulative distribution function in the Statistics Toolbox.	
Examples	<pre>[CallTheta, PutTheta] = blstheta(50, 50, 0.12, 0.25, 0.3, 0) CallTheta = -8.9630 PutTheta = -3.1404</pre>	
See Also	blsdelta, blsgamma, blslambda, blsprice, blsrho, blsvega	
References	Hull, <i>Options, Futures, and Other Derivative Securities</i> , 2nd edition, Chapter 13.	

Purpose	Black-Scholes sensitivity to underlying price volatility	
Syntax	Vega = blsvega(Price, Strike, Rate, Time, Volatility, DividendRate)	
Arguments	Price	Current stock price.
	Strike	Exercise price.
	Rate	Risk-free interest rate. Enter as a decimal fraction.
	Time	Time to maturity of the option in years.
	Volatility	Standard deviation of the annualized continuously compounded rate of return of the stock (also known as the volatility).
	DividendRate	(Optional) Enter as a decimal fraction. Default = 0.
Description	Vega = blsvega(Price, Strike, Rate, Time, Volatility, DividendRate) returns vega, the rate of change of the option value with respect to the volatility of the underlying asset. Note This function uses normpdf, the normal probability density function in the Statistics Toolbox.	
Examples	<pre>Vega = blsvega(50, 50, 0.12, 0.25, 0.3, 0) Vega = 9.6035</pre>	
See Also	blsdelta, blsgamma, blslambda, blsprice, blsrho, blstheta	
References	Hull, <i>Options, Futures, and Other Derivative Securities</i> , 2nd edition, Chapter 13.	

bndconvp

Purpose Bond convexity given price (SIA compliant)

Syntax [YearConvexity, PerConvexity] = bndconvp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)

Arguments	Price	Clean price (excludes accrued interest).
	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

All specified arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill unspecified entries in input vectors with NaN. Dates can be serial date numbers or date strings.

Description

[YearConvexity, PerConvexity] = bndconvp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) computes the convexity of NUMBONDS fixed income securities given a clean price for each bond. This function determines the convexity for a bond whether or not the first or last coupon periods in the coupon structure are short or long (i.e., whether or not the coupon structure is synchronized to maturity). This function also determines the convexity of a zero coupon bond.

YearConvexity is the yearly (annualized) convexity; PerConvexity is the periodic convexity reported on a semi-annual bond basis (in accordance with SIA convention). Both outputs are NUMBONDS-by-1 vectors.

bndconvp

Examples

Find the convexity of three bonds given their prices.

```
Price = [106; 100; 98];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;

[YearConvexity, PerConvexity] = bndconvp(Price,...
CouponRate,Settle, Maturity, Period, Basis)

YearConvexity =

    21.4447
    21.0363
    20.8951

PerConvexity =

    85.7788
    84.1454
    83.5803
```

See Also

bndconvy, bnddurp, bnddury, cfconv, cfdur

Purpose	Bond convexity given yield (SIA compliant)	
Syntax	<code>[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)</code>	
Arguments	Yield	Yield to maturity on a semi-annual basis.
	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

All specified arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill unspecified entries in input vectors with NaN. Dates can be serial date numbers or date strings.

Description

[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) computes the convexity of NUMBONDS fixed income securities given the yield to maturity for each bond. This function determines the convexity for a bond whether or not the first or last coupon periods in the coupon structure are short or long (i.e., whether or not the coupon structure is synchronized to maturity). This function also determines the convexity of a zero coupon bond.

YearConvexity is the yearly (annualized) convexity; PerConvexity is the periodic convexity reported on a semi-annual bond basis (in accordance with SIA convention). Both outputs are NUMBONDS-by-1 vectors.

Examples

Find the convexity of a bond at three different yield values.

```
Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;

[YearConvexity, PerConvexity]=bndconvy(Yield, CouponRate,...
Settle, Maturity, Period, Basis)

YearConvexity =

    21.4825
    21.0358
    20.8885

PerConvexity =

    85.9298
    84.1434
    83.5541
```

See Also

bndconvp, bnddurp, bnddury, cfconv, cfdur

bnddurp

Purpose Bond duration given price (SIA compliant)

Syntax `[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)`

Arguments	Price	Clean price (excludes accrued interest).
	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
	Settle	Settlement date. A vector of serial date numbers or date strings. <code>Settle</code> must be earlier than or equal to <code>Maturity</code> .
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when <code>Maturity</code> is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When <code>FirstCouponDate</code> and <code>LastCouponDate</code> are both specified, <code>FirstCouponDate</code> takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

All specified arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill unspecified entries in input vectors with NaN. Dates can be serial date numbers or date strings.

Description

[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) computes the duration of NUMBONDS fixed income securities given a clean price for each bond. This function determines the Macaulay and modified duration for a bond whether or not the first or last coupon periods in the coupon structure are short or long (i.e., whether or not the coupon structure is synchronized to maturity). This function also determines the Macaulay and modified duration for a zero coupon bond.

ModDuration is the modified duration in years; YearDuration is the Macaulay duration in years; PerDuration is the periodic Macaulay duration reported on a semi-annual bond basis (in accordance with SIA convention.) Outputs are NUMBONDS-by-1 vectors.

bnddurp

Examples

Find the duration of three bonds given their prices.

```
Price = [106; 100; 98];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;

[ModDuration, YearDuration, PerDuration] = bnddurp(Price,...
CouponRate, Settle, Maturity, Period, Basis)

ModDuration =

    4.2400
    4.1925
    4.1759

YearDuration =

    4.3275
    4.3077
    4.3007

PerDuration =

    8.6549
    8.6154
    8.6014
```

See Also

bndconvp, bndconvy, bnddury

Purpose	Bond duration given yield (SIA compliant)	
Syntax	<pre>[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)</pre>	
Arguments	Yield	Yield to maturity on a semi-annual basis.
	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

All specified arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill unspecified entries in input vectors with NaN. Dates can be serial date numbers or date strings.

Description

[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) computes the Macaulay and modified duration of NUMBONDS fixed income securities given yield to maturity for each bond. This function determines the duration for a bond whether or not the first or last coupon periods in the coupon structure are short or long (i.e., whether or not the coupon structure is synchronized to maturity). This function also determines the Macaulay and modified duration for a zero coupon bond.

ModDuration is the modified duration in years; YearDuration is the Macaulay duration in years; PerDuration is the periodic Macaulay duration reported on a semi-annual bond basis (in accordance with SIA convention). Outputs are NUMBONDS-by-1 vectors.

Examples

Find the duration of a bond at three different yield values.

```
Yield = [0.04; 0.055; 0.06];  
CouponRate = 0.055;  
Settle = '02-Aug-1999';  
Maturity = '15-Jun-2004';  
Period = 2;  
Basis = 0;  
  
[ModDuration, YearDuration, PerDuration]=bnddury(Yield,...  
CouponRate, Settle, Maturity, Period, Basis)
```

ModDuration =

```
4.2444  
4.1924  
4.1751
```

YearDuration =

```
4.3292  
4.3077  
4.3004
```

PerDuration =

```
8.6585  
8.6154  
8.6007
```

See Also

bndconvp, bndconvy, bnddurp

bndprice

Purpose	Price a fixed income security from yield to maturity (SIA compliant)																
Syntax	<pre>[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, Maturity) [Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)</pre>																
Arguments	Required and optional inputs can be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Optional inputs can also be passed as empty matrices ([]) or omitted at the end of the argument list. The value NaN in any optional input invokes the default value for that entry. Dates can be serial date numbers or date strings. <table><tr><td>Yield</td><td>Bond yield to maturity on a semi-annual basis.</td></tr><tr><td>CouponRate</td><td>Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.</td></tr><tr><td>Settle</td><td>Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.</td></tr><tr><td>Maturity</td><td>Maturity date. A vector of serial date numbers or date strings.</td></tr><tr><td>Period</td><td>(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.</td></tr><tr><td>Basis</td><td>(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.</td></tr><tr><td>EndMonthRule</td><td>(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</td></tr><tr><td>IssueDate</td><td>(Optional) Date when a bond was issued.</td></tr></table>	Yield	Bond yield to maturity on a semi-annual basis.	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.	Maturity	Maturity date. A vector of serial date numbers or date strings.	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.	IssueDate	(Optional) Date when a bond was issued.
Yield	Bond yield to maturity on a semi-annual basis.																
CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.																
Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.																
Maturity	Maturity date. A vector of serial date numbers or date strings.																
Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.																
Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.																
EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.																
IssueDate	(Optional) Date when a bond was issued.																

FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

Description

[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) given bonds with SIA date parameters and semi-annual yields to maturity, returns the clean prices and accrued interest due.

Price is the clean price of the bond (current price without accrued interest).

AccruedInt is the accrued interest payable at settlement.

Price and Yield are related by the formula

$$\text{Price} + \text{Accrued_Interest} = \text{sum}(\text{Cash_Flow} * (1 + \text{Yield}/2)^{(-\text{Time})})$$

where the sum is over the bonds' cash flows and corresponding times in units of semi-annual coupon periods.

bndprice

Examples

Price a treasury bond at three different yield values.

```
Yield = [0.04; 0.05; 0.06];  
CouponRate = 0.05;  
Settle = '20-Jan-1997';  
Maturity = '15-Jun-2002';  
Period = 2;  
Basis = 0;
```

```
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,...  
    Maturity, Period, Basis)
```

```
Price =  
    104.8106  
     99.9951  
     95.4384
```

```
AccruedInt =  
     0.4945  
     0.4945  
     0.4945
```

See Also

cfamounts, bndyield

Purpose	Yield to maturity for a fixed income security (SIA compliant)																
Syntax	Yield = bndyield(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)																
Arguments	Required and optional inputs can be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Optional inputs can also be passed as empty matrices ([]) or omitted at the end of the argument list. The value NaN in any optional input invokes the default value for that entry. Dates can be serial date numbers or date strings. <table> <tr> <td>Price</td><td>Clean price of the bond (current price without accrued interest).</td></tr> <tr> <td>CouponRate</td><td>Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.</td></tr> <tr> <td>Settle</td><td>Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.</td></tr> <tr> <td>Maturity</td><td>Maturity date. A vector of serial date numbers or date strings.</td></tr> <tr> <td>Period</td><td>(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.</td></tr> <tr> <td>Basis</td><td>(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.</td></tr> <tr> <td>EndMonthRule</td><td>(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</td></tr> <tr> <td>IssueDate</td><td>(Optional) Date when a bond was issued.</td></tr> </table>	Price	Clean price of the bond (current price without accrued interest).	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.	Maturity	Maturity date. A vector of serial date numbers or date strings.	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.	IssueDate	(Optional) Date when a bond was issued.
Price	Clean price of the bond (current price without accrued interest).																
CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.																
Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.																
Maturity	Maturity date. A vector of serial date numbers or date strings.																
Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.																
Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.																
EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.																
IssueDate	(Optional) Date when a bond was issued.																

FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

Description

Yield = bndyield(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) given NUMBONDS bonds with SIA date parameters and clean prices (excludes accrued interest), returns the bond equivalent yields to maturity.

Yield is a NUMBONDS-by-1 vector of the bond equivalent yields to maturity with semi-annual compounding.

Price and Yield are related by the formula

$$\text{Price} + \text{Accrued_Interest} = \text{sum}(\text{Cash_Flow} * (1 + \text{Yield}/2)^{(-\text{Time})})$$

where the sum is over the bonds' cash flows and corresponding times in units of semi-annual coupon periods.

Examples

Compute the yield of a treasury bond at three different price values.

```
Price = [95; 100; 105];
CouponRate = 0.05;
Settle = '20-Jan-1997';
Maturity = '15-Jun-2002';
Period = 2;
Basis = 0;

[Yield] = bndyield(Price, CouponRate, Settle,...
    Maturity, Period, Basis)

Yield =
    0.0610
    0.0500
    0.0396
```

See Also

bndprice, cfamounts

bolling

Purpose Bollinger band chart

Syntax `bolling(Asset, Samples, Alpha)`
`[Movavgv, UpperBand, LowerBand] = bolling(Asset, Samples, Alpha)`

Description `bolling(Asset, Samples, Alpha)` plots Bollinger bands for given Asset data. Samples specifies the number of samples to use in computing the moving average. Alpha is the exponent used to compute the element weights of the moving average. This form of the function does not return any data.

`[Movavgv, UpperBand, LowerBand] = bolling(Asset, Samples, Alpha)` returns Movavgv with the moving average of the Asset data, UpperBand with the upper band data, and LowerBand with the lower band data. This form of the function does not plot any data.

Examples If Asset is a column vector of closing stock prices

`bolling(Asset, 20, 1)`

plots linear 20-day moving average Bollinger bands based on the stock prices.

`[Movavgv, UpperBand, LowerBand] = bolling(Asset, 20, 1)`

returns Movavgv, UpperBand, and LowerBand as (N-19)-by-1 vectors containing the moving average, upper band, and lower band data, without plotting the data.

See Also `candle`, `dateaxis`, `highlow`, `movavg`, `pointfig`

Purpose	Next or previous business day
Syntax	<code>Busday = busdate(Date, Direction, Holiday)</code>
Arguments	<code>Date</code> Reference date. Enter as serial date number or date string. <code>Direction</code> (Optional) Direction. 1 = next (default) or -1 = previous business day. <code>Holiday</code> (Optional) Vector of holidays and nontrading-day dates. All dates in <code>Holiday</code> must be the same format: either serial date numbers or date strings. (Using serial date numbers improves performance.) The <code>holidays</code> function supplies the default vector.
Description	<code>Busday = busdate(Date, Direction, Holiday)</code> returns the serial date number of the next or previous business day from the reference date. Use the function <code>datestr</code> to convert serial date numbers to formatted date strings.
Examples	<pre> Busday = busdate('3-Jul-2001', 1, holidays) Busday = 731037 datestr(Busday) ans = 05-Jul-2001 </pre>
See Also	<code>holidays</code> , <code>isbusday</code>

candle

Purpose	Candlestick chart										
Syntax	<code>candle(High, Low, Close, Open, Color)</code>										
Arguments	<table><tr><td>High</td><td>High prices for a security. A column vector.</td></tr><tr><td>Low</td><td>Low prices for a security. An column vector.</td></tr><tr><td>Close</td><td>Closing prices for a security. A column vector.</td></tr><tr><td>Open</td><td>Opening prices for a security. A column vector.</td></tr><tr><td>Color</td><td>(Optional) Candlestick color. A string. MATLAB supplies a default color if none is specified. The default color differs depending on the background color of the figure window. See <code>ColorSpec</code> in the MATLAB documentation for color names.</td></tr></table>	High	High prices for a security. A column vector.	Low	Low prices for a security. An column vector.	Close	Closing prices for a security. A column vector.	Open	Opening prices for a security. A column vector.	Color	(Optional) Candlestick color. A string. MATLAB supplies a default color if none is specified. The default color differs depending on the background color of the figure window. See <code>ColorSpec</code> in the MATLAB documentation for color names.
High	High prices for a security. A column vector.										
Low	Low prices for a security. An column vector.										
Close	Closing prices for a security. A column vector.										
Open	Opening prices for a security. A column vector.										
Color	(Optional) Candlestick color. A string. MATLAB supplies a default color if none is specified. The default color differs depending on the background color of the figure window. See <code>ColorSpec</code> in the MATLAB documentation for color names.										
Description	<code>candle(High, Low, Close, Open, Color)</code> plots a candlestick chart given column vectors with the high, low, closing, and opening prices of a security. If the closing price is greater than the opening price, the body (the region between the opening and closing price) is filled. If the opening price is greater than the closing price, the body is empty.										
Examples	Given High, Low, Close, and Open as equal-size vectors of stock price data <pre>candle(High, Low, Close, Open, 'cyan')</pre> plots a candlestick chart with cyan candles.										
See Also	<code>bolling</code> , <code>dateaxis</code> , <code>highlow</code> , <code>movavg</code> , <code>pointfig</code>										

Purpose	Cash flow and time mapping for bond portfolio (SIA compliant)	
Syntax	<pre>[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)</pre>	
Arguments	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.
Face	(Optional) Face or par value. Default = 100.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) returns matrices of cash flow amounts, cash flow dates, time factors, and cash flow flags for a portfolio of NUMBONDS fixed income securities. The elements contained in the cash flow matrix, time factor matrix, and cash flow flag matrix correspond to the cash flow dates for each security. The first element of each row in the cash flow matrix is the accrued interest payable on each bond. This is zero in the case of all zero coupon bonds. This function determines all cash flows and time mappings for a bond whether or not the coupon structure contains odd first or last periods. All output matrices are padded with NaNs as necessary to ensure that all rows have the same number of elements.

CFlowAmounts is the cash flow matrix of a portfolio of bonds. Each row represents the cash flow vector of a single bond. Each element in a column represents a specific cash flow for that bond.

CFlowDates is the cash flow date matrix of a portfolio of bonds. Each row represents a single bond in the portfolio. Each element in a column represents a cash flow date of that bond.

TFactors is the matrix of time factors for a portfolio of bonds. Each row corresponds to the vector of time factors for each bond. Each element in a column corresponds to the specific time factor associated with each cash flow of a bond. Time factors are useful in determining the present value of a stream of cash flows. The term “time factor” refers to the exponent TF in the discounting equation

$$PV = CF / (1 + z/2)^{TF}$$

where:

PV = present value of a cash flow

CF = the cash flow amount

Z = the risk-adjusted annualized rate or yield corresponding to given cash flow. The yield is quoted on a semi-annual basis.

TF = time factor for a given cash flow. Time is measured in semi-annual periods from the settlement date to the cash flow date.

CFlowFlags is the matrix of cash flow flags for a portfolio of bonds. Each row corresponds to the vector of cash flow flags for each bond. Each element in a column corresponds to the specific flag associated with each cash flow of a bond. Flags identify the type of each cash flow (e.g., nominal coupon cash flow, front or end partial or “stub” coupon, maturity cash flow). Possible values are shown in the table.

Flag	Cash Flow Type
0	Accrued interest due on a bond at settlement.
1	Initial cash flow amount smaller than normal due to “stub” coupon period. A stub period is created when the time from issue date to first coupon is shorter than normal.
2	Larger than normal initial cash flow amount because first coupon period is longer than normal.

Flag	Cash Flow Type
3	Nominal coupon cash flow amount.
4	Normal maturity cash flow amount (face value plus the nominal coupon amount).
5	End “stub” coupon amount (last coupon period abnormally short and actual maturity cash flow is smaller than normal).
6	Larger than normal maturity cash flow because last coupon period longer than normal.
7	Maturity cash flow on a coupon bond when the bond has less than one coupon period to maturity.
8	Smaller than normal maturity cash flow when bond has less than one coupon period to maturity.
9	Larger than normal maturity cash flow when bond has less than one coupon period to maturity.
10	Maturity cash flow on a zero coupon bond.

Examples

Consider a portfolio containing a corporate bond paying interest quarterly and a treasury bond paying interest semi-annually. Compute the cash flow structure and the time factors for each bond.

```
Settle = '01-Nov-1993';
Maturity = ['15-Dec-1994';'15-Jun-1995'];
CouponRate= [0.06; 0.05];
Period = [4;2];
Basis = [1;0];
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = ...
cfamounts(CouponRate,Settle, Maturity, Period, Basis)

CFlowAmounts =

    -0.7667    1.5000    1.5000    1.5000    1.5000   101.5000
    -1.8989    2.5000    2.5000    2.5000   102.5000         NaN
```



```
CFlowDates =  
  
728234      728278      728368      728460      728552      728643  
728234      728278      728460      728643      728825      NaN  
  
TFactors =  
  
0      0.2404      0.7403      1.2404      1.7403      2.2404  
0      0.2404      1.2404      2.2404      3.2404      NaN  
  
CFlowFlags =  
  
0      3      3      3      3      4  
0      3      3      3      4      NaN
```

See Also accrfrac, cfdates, cpncount, cpndaten, cpndatenq, cpndatep, cpndatepq,
cpndaysn, cpndaysp, cpnpersz

cfconv

Purpose	Cash flow convexity
Syntax	CFlowConvexity = cfconv(CashFlow, Yield)
Arguments	CashFlow A vector of real numbers. Yield Periodic yield. A scalar. Enter as a decimal fraction.
Description	CFlowConvexity = cfconv(CashFlow, Yield) returns the convexity of a cash flow in periods.
Examples	Given a cash flow of nine payments of \$2.50 and a final payment \$102.50, with a periodic yield of 2.5%: CashFlow = [2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 102.5]; Convex = cfconv(CashFlow, 0.025) Convex = 90.4493 (periods)
See Also	bndconvp, bndconvy, bnddurp, bnddury, cfdur

Purpose	Cash flow dates for a fixed-income security (SIA compliant)	
Syntax	<code>CFlowDates = cfdates(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)</code>	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Maturity contains N dates, then Settle must contain N dates or a single date.

Description

CFlowDates = cfdates(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns a matrix of cash flow dates for a bond or set of bonds. cfdates determines all cash flow dates for a bond whether or not the coupon payment structure is normal or the first and/or last coupon period is long or short.

CFlowDates is an N-row matrix of serial date numbers, padded with NaNs as necessary to ensure that all rows have the same number of elements. Use the function datestr to convert serial date numbers to formatted date strings.

Note The cash flow flags for a portfolio of bonds were formerly available as the cfdates second output argument, CFlowFlags. You can now use cfamounts to get these flags. If you specify a CFlowFlags argument, cfdates displays a message directing you to use cfamounts.

Examples

```
CFlowDates = cfdates('14 Mar 1997', '30 Nov 1998', 2, 0, 1)
CFlowDates =
    729541    729724    729906    730089
datestr(CFlowDates)
ans =
31-May-1997
30-Nov-1997
31-May-1998
30-Nov-1998
```

Given three securities with different maturity dates and the same default arguments:

```
Maturity = ['30-Sep-1997'; '31-Oct-1998'; '30-Nov-1998'];
CFlowDates = cfdates('14-Mar-1997', Maturity)
CFlowDates =
    729480    729663         NaN         NaN
    729510    729694    729875    730059
    729541    729724    729906    730089
```

Look at the cash-flow dates for the last security:

```
datestr(CFlowDates(3,:))
ans =
31-May-1997
30-Nov-1997
31-May-1998
30-Nov-1998
```

See Also

accrfrac, cfamounts, cftimes, cpncount, cpndaten, cpndatenq, cpndatep, cpndatepq, cpndaysn, cpndaysp, cpnpersz

cfdur

Purpose Cash-flow duration and modified duration

Syntax [Duration, ModDuration] = cfdur(CashFlow, Yield)

Arguments
CashFlow A vector of real numbers.
Yield Periodic yield. A scalar. Enter as a decimal fraction.

Description [Duration, ModDuration] = cfdur(CashFlow, Yield) calculates the duration and modified duration of a cash flow in periods.

Examples Given a cash flow of nine payments of \$2.50 and a final payment \$102.50, with a periodic yield of 2.5%:

```
CashFlow=[2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 102.5];
```

```
[Duration, ModDuration] = cfdur(CashFlow, 0.025)
```

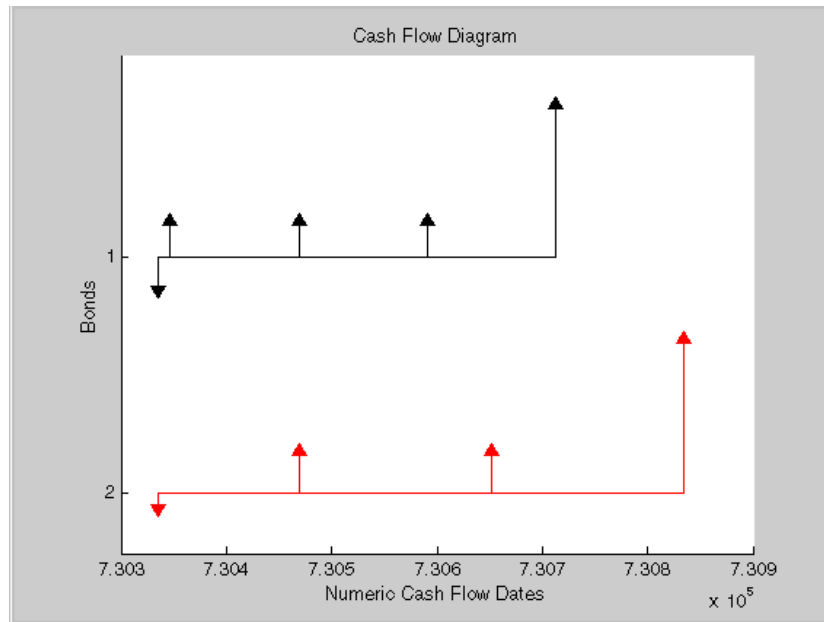
```
Duration =  
      8.9709 (periods)
```

```
ModDuration =  
      8.7521 (periods)
```

See Also bndconvp, bndconvy, bnddurp, bnddury, cfconv

Purpose	Portfolio form of cash flow amounts	
Syntax	<code>[CFBondDate, AllDates, AllTF, IndByBond] = cfport(CFlowAmounts, CFlowDates, TFactors)</code>	
Arguments	CFlowAmounts	Number of bonds (NUMBONDS) by number of cash flows (NUMCFS) matrix with entries listing cash flow amounts corresponding to each date in CFlowDates.
	CFlowDates	NUMBONDS-by-NUMCFS matrix with rows listing cash flow dates for each bond and padded with NaNs.
	TFactors	(Optional) NUMBONDS-by-NUMCFS matrix with entries listing the time between settlement and the cash flow date measured in semi-annual coupon periods.
Description	<code>[CFBondDate, AllDates, AllTF, IndByBond] = cfport(CFlowAmounts, CFlowDates, TFactors)</code> computes a vector of all cash flow dates of a bond portfolio, and a matrix mapping the cash flows of each bond to those dates. Use the matrix for pricing the bonds against a curve of discount factors. CFBondDate is a NUMBONDS by number of dates (NUMDATES) matrix of cash flows indexed by bond and by date in AllDates. Each row contains a bond's cash flow values at the indices corresponding to entries in AllDates. Other indices in the row contain zeros. AllDates is a NUMDATES-by-1 list of all dates that have any cash flow from the bond portfolio. AllTF is a NUMDATES-by-1 list of time factors corresponding to the dates in AllDates. If TFactors is not entered, AllTF contains the number of days from the first date in AllDates. IndByBond is a NUMBONDS-by-NUMCFS matrix of indices. The <i>i</i>th row contains a list of indices into AllDates where the <i>i</i>th bond has cash flows. Since some bonds have more cash flows than others, the matrix is padded with NaNs.	
Examples	Use cfamounts to calculate the cash flow amounts, cash flow dates, and time factors for each of two bonds. Then use cfplot to plot the cash flow diagram.	

```
Settle = '03-Aug-1999';
Maturity = ['15-Aug-2000'; '15-Dec-2000'];
CouponRate= [0.06; 0.05];
Period = [3;2];
Basis = [1;0];
[CFlowAmounts, CFlowDates, TFactors] = cfamounts(CouponRate,...
Settle, Maturity, Period, Basis);
cfplot(CFlowDates,CFlowAmounts)
xlabel('Numeric Cash Flow Dates')
ylabel('Bonds')
title('Cash Flow Diagram')
```



Finally, call `cfport` to map the cash flow amounts to the cash flow dates.

Each row in the resultant `CFBondDate` matrix represents a bond. Each column represents a date on which one or more of the bonds has a cash flow. A 0 means the bond did not have a cash flow on that date. The dates associated with the columns are listed in `AllDates`. For example, the first bond had a cash flow of 2.000 on 730347. The second bond had no cash flow on this date.

For each bond, IndByBond indicates the columns of CFBondDate, or dates in AllDates, for which a bond has a cash flow.

```
[CFBondDate, AllDates, AllTF, IndByBond] = ...
cfport(CFlowAmounts, CFlowDates, TFactors)
```

CFBondDate =

-1.8000	2.0000	2.0000	2.0000	0	102.0000	0
-0.6694	0	2.5000	0	2.5000	0	102.5000

AllDates =

- 730335
- 730347
- 730469
- 730591
- 730652
- 730713
- 730835

AllTF =

- 0
- 0.0663
- 0.7322
- 1.3989
- 1.7322
- 2.0663
- 2.7322

IndByBond =

1	2	3	4	6
1	3	5	7	NaN

See Also

cfamounts

cftimes

Purpose	Time factors corresponding to bond cash flow dates (SIA compliant)	
Syntax	<code>TFactors = cftimes(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)</code>	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. (Time factors are computed on an actual/actual basis. Basis is included here as an input argument to maintain interface consistency with other coupon functions.)
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Description

TFactors = cftimes(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) determines the time factors corresponding to the cash flows of a bond or set of bonds. The time factor of a cash flow is the difference between the settlement date and the cash flow date in units of semi-annual coupon periods.

Examples

```
Settle = '15-Mar-1997';
Maturity = '01-Sep-1999';
Period = 2;
TFactors = cftimes(Settle, Maturity, Period)

TFactors =

    0.9239    1.9239    2.9239    3.9239    4.9239
```

See Also

accrfrac, cfdates, cfamounts, cpncount, cpndaten, cpndatenq, cpndatep, cpndatepq, cpndaysn, cpndaysp, cpnpersz

corr2cov

Purpose

Convert standard deviation and correlation to covariance

Syntax

`ExpCovariance = corr2cov(ExpSigma, ExpCorrC)`

Arguments

ExpSigma Vector of length *n* with the standard deviations of each process. *n* is the number of random processes.

ExpCorrC (Optional) *n*-by-*n* correlation coefficient matrix. If **ExpCorrC** is not specified, the processes are assumed to be uncorrelated, and the identity matrix is used.

Description

`corr2cov` converts standard deviation and correlation to covariance.

`ExpCovariance` is an *n*-by-*n* covariance matrix, where *n* is the number of processes.

$$\text{ExpCov}(i,j) = \text{ExpCorrC}(i,j) * (\text{ExpSigma}(i) * \text{ExpSigma}(j))$$

Examples

```
ExpSigma = [0.5 2.0];
```

```
ExpCorrC = [1.0 -0.5  
            -0.5 1.0];
```

```
ExpCovariance = corr2cov(ExpSigma, ExpCorrC)
```

Expected results:

```
ExpCovariance =
```

```
    0.2500    -0.5000  
   -0.5000    4.0000
```

See Also

`corrcoef`, `cov`, `cov2corr`, `ewstats`, `std`

Purpose	Convert covariance to standard deviation and correlation coefficient
Syntax	<code>[ExpSigma, ExpCorrC] = cov2corr(ExpCovariance)</code>
Arguments	<code>ExpCovariance</code> n-by-n covariance matrix, e.g., from <code>cov</code> or <code>ewstats</code>. n is the number of random processes.
Description	<code>[ExpSigma, ExpCorrC] = cov2corr(ExpCovariance)</code> converts covariance to standard deviations and correlation coefficients. <code>ExpSigma</code> is a 1-by-n vector with the standard deviation of each process. <code>ExpCorrC</code> is an n-by-n matrix of correlation coefficients. <pre>ExpSigma(i) = sqrt(ExpCovariance(i,i)) ExpCorrC(i,j) = ExpCovariance(i,j)/(ExpSigma(i)*ExpSigma(j))</pre>
Examples	<pre>ExpCovariance = [0.25 -0.5 -0.5 4.0]; [ExpSigma, ExpCorrC] = cov2corr(ExpCovariance)</pre> Expected results: <pre>ExpSigma = 0.5000 2.0000 ExpCorrC = 1.0000 -0.5000 -0.5000 1.0000</pre>
See Also	<code>corr2cov</code> , <code>corrcoef</code> , <code>cov</code> , <code>ewstats</code> , <code>std</code>

cpncount

Purpose	Coupon payments remaining until maturity (SIA compliant)	
Syntax	NumCouponsRemaining = cpncount(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
	LastCouponDate	(Optional) Date when a bond makes its last coupon payment.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

NumCouponsRemaining = cpncount(Settle, Maturity, Period, Basis, EndMonthRule) returns the whole number of coupon payments between the settlement and maturity dates for a coupon bond or set of bonds.

Examples

```
NumCouponsRemaining = cpncount('14 Mar 1997', '30 Nov 2000', ...
    2, 0, 0)
n =
    8
```

Given three coupon bonds with different maturity dates and the same default arguments:

```
Maturity = ['30 Sep 2000'; '31 Oct 2001'; '30 Nov 2002'];
NumCouponsRemaining = cpncount('14 Sep 1997', Maturity)
NumCouponsRemaining =
    7
    9
   11
```

cpncount

See Also

accrfrac, cfamounts, cfdates, cftimes, cpndaten, cpndatenq, cpndatep, cpndatepq, cpndaysn, cpndaysp, cpnpersz

Purpose	Next coupon date for fixed-income security (SIA compliant)	
Syntax	<pre>NextCouponDate = cpndaten(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)</pre>	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

NextCouponDate = cpndaten(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the next coupon date after the settlement date. This function finds the next coupon date whether or not the coupon structure is synchronized with the maturity date.

NextCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date string.

Examples

```
NextCouponDate = cpndaten('14 Mar 1997', '30 Nov 2000', 2, 0, 0);  
datestr(NextCouponDate)
```

```
ans =  
30-May-1997
```

```
NextCouponDate = cpndaten('14 Mar 1997', '30 Nov 2000', 2, 0, 1);  
datestr(NextCouponDate)
```

```
ans =  
31-May-1997  
  
Maturity = ['30 Sep 2000'; '31 Oct 2000'; '30 Nov 2000'];  
NextCouponDate = cpndaten('14 Mar 1997', Maturity);  
datestr(NextCouponDate)  
  
ans =  
31-Mar-1997  
30-Apr-1997  
31-May-1997
```

See Also

accrfrac, cfamounts, cfdates, cftimes, cpncount, cpndatenq, cpndatep, cpndatepq, cpndaysn, cpndaysp, cpnpersz

cpndatenq

Purpose	Next quasi coupon date for fixed income security (SIA compliant)	
Syntax	NextQuasiCouponDate = cpndatenq(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
	LastCouponDate	(Optional) Date when a bond makes its last coupon payment.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate , a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (**NUMBONDS**) by 1 or 1-by-**NUMBONDS** conforming vectors or scalars. Optional arguments must be either **NUMBONDS**-by-1 or 1-by-**NUMBONDS** conforming vectors, scalars, or empty matrices. Fill unspecified entries in input vectors with the value NaN. Dates can be serial date numbers or date strings.

Description

NextQuasiCouponDate = cpndatenq(**Settle**, **Maturity**, **Period**, **Basis**, **EndMonthRule**, **IssueDate**, **FirstCouponDate**, **LastCouponDate**, **StartDate**) determines the next quasi coupon date for a portfolio of **NUMBONDS** fixed income securities whether or not the first or last coupon is normal, short, or long. For zero coupon bonds cpndatenq returns quasi coupon dates as if the bond had a semi-annual coupon structure. Successive quasi coupon dates determine the length of the standard coupon period for the fixed income security of interest and do not necessarily coincide with actual coupon payment dates.

Outputs are **NUMBONDS**-by-1 vectors

If **Settle** is a coupon date, this function never returns the settlement date. It returns the quasi coupon date strictly after settlement.

NextQuasiCouponDate is returned as a serial date number. The function **datestr** converts a serial date number to a formatted date string.

Examples

Given a pair of bonds with the characteristics:

```
Settle = str2mat('30-May-1997','10-Dec-1997');  
Maturity = str2mat('30-Nov-2002','10-Jun-2004');
```

Compute NextCouponDate for this pair of bonds.

```
NextCouponDate = cpndaten(Settle, Maturity);  
datestr(NextCouponDate)
```

```
ans =
```

```
31-May-1997  
10-Jun-1998
```

Compute the next quasi coupon dates for these two bonds.

```
NextQuasiCouponDate = cpndatenq(Settle, Maturity);  
datestr(NextQuasiCouponDate)
```

```
ans =
```

```
31-May-1997  
10-Jun-1998
```

Because no FirstCouponDate has been specified, the results are identical.

Now supply an explicit FirstCouponDate for each bond.

```
FirstCouponDate = str2mat('30-Nov-1997','10-Dec-1998');
```

Compute the next coupon dates.

```
NextCouponDate = cpndaten(Settle, Maturity, 2, 0, 1, [], ...  
FirstCouponDate);  
datestr(NextCouponDate)
```

```
ans =
```

```
30-Nov-1997  
10-Dec-1998
```

The next coupon dates are identical to the specified first coupon dates.

Now recompute the next quasi coupon dates.

```
NextQuasiCouponDate = cpndatenq(Settle, Maturity, 2, 0, 1, [], ...
FirstCouponDate);
datestr(NextQuasiCouponDate)
```

```
ans =
```

```
31-May-1997
```

```
10-Jun-1998
```

These results illustrate the distinction between actual coupon payment dates and quasi coupon dates. `FirstCouponDate` (and `LastCouponDate`, as well), when specified, is associated with an actual coupon payment and also serves as the synchronization date for determining all quasi coupon dates. Since each bond in this example pays semi-annual coupons, and the first coupon date occurs more than six months after settlement, each will have an intermediate quasi coupon date before the actual first coupon payment occurs.

See Also

`accrfrac`, `cfamounts`, `cfdates`, `cftimes`, `cpncount`, `cpndaten`, `cpndatep`, `cpndatepq`, `cpndaysn`, `cpndaysp`, `cpnpersz`

cpndatep

Purpose	Previous coupon date for fixed-income security (SIA compliant)	
Syntax	<code>PreviousCouponDate = cpndatep(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)</code>	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
	LastCouponDate	(Optional) Date when a bond makes its last coupon payment.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

PreviousCouponDate = cpndatep(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the previous coupon date on or before settlement for a portfolio of bonds. This function finds the previous coupon date whether or not the coupon structure is synchronized with the maturity date.

For zero coupon bonds the previous coupon date is the issue date, if available. However, if the issue date is not supplied, the previous coupon date for zero coupon bonds is the previous quasi coupon date calculated as if the frequency is semi-annual.

PreviousCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date string.

Examples

```
PreviousCouponDate = cpndatep('14 Mar 1997', '30 Jun 2000',...
2, 0, 0);
datestr(PreviousCouponDate)
ans =
30-Dec-1996
```

cpndatep

```
PreviousCouponDate = cpndatep('14 Mar 1997', '30 Jun 2000',...  
2, 0, 1);  
datestr(PreviousCouponDate)  
ans =  
31-Dec-1996  
  
Maturity = ['30 Apr 2000'; '31 May 2000'; '30 Jun 2000'];  
PreviousCouponDate = cpndatep('14 Mar 1997', Maturity);  
datestr(PreviousCouponDate)  
ans =  
31-Oct-1996  
30-Nov-1996  
31-Dec-1996
```

See Also

accrfrac, cfamounts, cfdates, cftimes, cpncount, cpndaten, cpndatenq,
cpndatepq, cpndaysn, cpndaysp, cnpersz

Purpose	Previous quasi coupon date for fixed income security (SIA compliant)	
Syntax	PreviousQuasiCouponDate = cpndatepq(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices. Fill unspecified entries in input vectors with the value NaN. Dates can be serial date numbers or date strings.

Description

PreviousQuasiCouponDate = cpndatepq(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) determines the previous quasi coupon date on or before settlement for a set of NUMBONDS fixed income securities. This function finds the previous quasi coupon date for a bond with a coupon structure in which the first or last period is either normal, short, or long (whether or not the coupon structure is synchronized to maturity). For zero coupon bonds this function returns quasi coupon dates as if the bond had a semi-annual coupon structure.

The term “previous quasi coupon date” refers to the previous coupon date for a bond calculated as if no issue date were specified. Although the issue date is not actually a coupon date, when issue date is specified, the previous actual coupon date for a bond is normally calculated as being either the previous coupon date or the issue date, whichever is greater. This function always returns the previous quasi coupon date regardless of issue date. If the settlement date is a coupon date, this function returns the settlement date.

PreviousQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date string.

Examples

Given a pair of bonds with the characteristics:

```
Settle = str2mat('30-May-1997','10-Dec-1997');  
Maturity = str2mat('30-Nov-2002','10-Jun-2004');
```

With no FirstCouponDate explicitly supplied, compute the PreviousCouponDate for this pair of bonds:

```
PreviousCouponDate = cpndatep(Settle, Maturity);  
datestr(PreviousCouponDate)
```

```
ans =
```

```
30-Nov-1996  
10-Dec-1997
```

Note that since the settlement date for the second bond is also a coupon date, cpndatep returns this date as the previous coupon date.

Now establish a FirstCouponDate and IssueDate for this pair of bonds:

```
FirstCouponDate = str2mat('30-Nov-1997','10-Dec-1998');  
IssueDate = str2mat('30-May-1996','10-Dec-1996');
```

Recompute the PreviousCouponDate for this pair of bonds:

```
PreviousCouponDate = cpndatep(Settle, Maturity, 2, 0, 1, ...  
IssueDate, FirstCouponDate);  
datestr(PreviousCouponDate)
```

```
ans =
```

```
30-May-1996  
10-Dec-1996
```

Since both of these bonds settled before the first coupon had been paid, cpndatep returns the IssueDate as the PreviousCouponDate.

Using the same data, compute PreviousQuasiCouponDate:

```
PreviousQuasiCouponDate = cpndatepq(Settle, Maturity, 2, 0, 1, ...  
IssueDate, FirstCouponDate);  
datestr(PreviousQuasiCouponDate)
```

```
ans =
```

```
30-Nov-1996  
10-Dec-1997
```

For the first bond the settlement date is not a normal coupon date. The PreviousQuasiCouponDate is the coupon date prior to or on the settlement date. Since the coupon structure is synchronized to FirstCouponDate, the previous quasi coupon date is 30-Nov-1996. PreviousQuasiCouponDate disregards IssueDate and FirstCouponDate in this case. For the second bond the settlement date (10-Dec-1997) occurs on a date when a coupon would normally be paid in the absence of an explicit FirstCouponDate. cpndatepq returns this date as PreviousQuasiCouponDate.

See Also

accrfrac, cfamounts, cfdates, cftimes, cpncount, cpndaten, cpndatenq, cpndatep, cpndaysn, cpndaysp, cpnpersz

Purpose	Number of days to next coupon date (SIA compliant)	
Syntax	<pre>NumDaysNext = cpndaysn(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)</pre>	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

NumDaysNext = cpndaysn(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the number of days from the settlement date to the next coupon date for a bond or set of bonds. For zero coupon bonds coupon dates are computed as if the bonds have a semi-annual coupon structure.

Examples

```
NumDaysNext = cpndaysn('14 Sep 2000', '30 Jun 2001', 2, 0, 0)
NumDaysNext =
    107

NumDaysNext = cpndaysn('14 Sep 2000', '30 Jun 2001', 2, 0, 1)
NumDaysNext =
    108

Maturity = ['30 Apr 2001'; '31 May 2001'; '30 Jun 2001'];

NumDaysNext = cpndaysn('14 Sep 2000', Maturity)
```



```
NumDaysNext =  
    47  
    77  
    108
```

See Also

accrfrac, cfamounts, cftimes, cfdates, cpncount, cpndaten, cpndatenq,
cpndatep, cpndatepq, cpndaysp, cpnpersz

cpndaysp

Purpose	Number of days since previous coupon date (SIA compliant)	
Syntax	NumDaysPrevious = cpndaysp(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
	LastCouponDate	(Optional) Date when a bond makes its last coupon payment.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be a number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

NumDaysPrevious = cpndaysp(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the number of days between the previous coupon date and the settlement date for a bond or set of bonds. When the coupon frequency is 0 (a zero coupon bond), the previous coupon date is calculated as if the frequency were semi-annual.

Examples

```
NumDaysPrevious = cpndaysp('14 Mar 2000', '30 Jun 2001', 2, 0, 0)

NumDaysPrevious =
    75

NumDaysPrevious = cpndaysp('14 Mar 2000', '30 Jun 2001', 2, 0, 1)

NumDaysPrevious =
    74

Maturity = ['30 Apr 2001'; '31 May 2001'; '30 Jun 2001'];
```

cpndaysp

```
NumDaysPrevious = cpndaysp('14 Mar 2000', Maturity)
```

```
NumDaysPrevious =
```

```
135
```

```
105
```

```
74
```

See Also

accrfrac, cfamounts, cfdates, cftimes, cpncount, cpndaten, cpndatenq, cpdatep, cpdatepq, cpndaysn, cpnpersz

Purpose	Number of days in coupon period (SIA compliant)	
Syntax	<code>NumDaysPeriod = cpnpersz(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)</code>	
Arguments	Settle	Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than or equal to Maturity.
	Maturity	Maturity date. A vector of serial date numbers or date strings.
	Period	(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.
	Basis	(Optional) Day-count basis of the bond. A vector of integers. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
	EndMonthRule	(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
	IssueDate	(Optional) Date when a bond was issued.
	FirstCouponDate	(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate	(Optional) Last coupon date of a bond prior to the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.
StartDate	(Future implementation; optional) Date when a bond actually starts (the date from which a bond's cash flows can be considered). To make an instrument forward-starting, specify this date as a future date. If StartDate is not explicitly specified, the effective start date is the settlement date.

Required arguments must be a number of bonds (NUMBONDS) by 1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Description

NumDaysPeriod = cpnpersz(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the number of days in the coupon period containing the settlement date. For zero coupon bonds coupon dates are computed as if the bonds have a semi-annual coupon structure.

Examples

```
NumDaysPeriod = cpnpersz('14 Sep 2000', '30 Jun 2001', 2, 0, 0)
```

```
NumDaysPeriod =  
183
```

```
NumDaysPeriod = cpnpersz('14 Sep 2000', '30 Jun 2001', 2, 0, 1)
```

```
NumDaysPeriod =  
184
```

```
Maturity = ['30 Apr 2001'; '31 May 2001'; '30 Jun 2001'];
```

```
NumDaysPeriod = cpnpersz('14 Sep 2000', Maturity)
```

```
NumDaysPeriod =  
    184  
    183  
    184
```

See Also

accrfrac, cfamounts, cfdates, cpncount, cpndaten, cpndatenq, cpndatep,
cpndatepq, cpndaysn, cpndaysp

cur2frac

Purpose	Decimal currency values to fractional values
Syntax	Fraction = cur2frac(Decimal, Denominator)
Description	Fraction = cur2frac(Decimal, Denominator) converts decimal currency values to fractional values. Fraction is returned as a string.
Examples	<pre>Fraction = cur2frac(12.125, 8)</pre> returns Fraction = 12.1, a string.
See Also	cur2str, frac2cur

Purpose	Bank formatted text
Syntax	<code>String = cur2str(Value, Digits)</code>
Description	<code>String = cur2str(Value, Digits)</code> returns the given value in bank format. By default, <code>Digits = 2</code> . A negative <code>Digits</code> rounds the value to the left of the decimal point. <code>String</code> is returned as a string with a leading dollar sign (\$). Negative numbers are displayed in parentheses.
Examples	<pre>String = cur2str(-8264, 2)</pre> <pre>returns String = (\$8264.00)</pre>
See Also	<code>cur2frac</code> , <code>frac2cur</code>

dateaxis

Purpose Convert serial-date axis labels to calendar-date axis labels

Syntax dateaxis(Aksis, DateForm, StartDate)

Arguments

Aksis (Optional) Determines which axis tick labels—x, y, or z—to replace. Enter as a string. Default = 'x'.

DateForm (Optional) Specifies which date format to use. Enter as an integer from 0 to 16. If no DateForm argument is entered, this function determines the date format based on the span of the axis limits. For example, if the difference between the axis minimum and maximum is less than 15, the tick labels are converted to three-letter day-of-the-week abbreviations (DateForm = 8). See DateForm format descriptions below.

StartDate (Optional) Assigns the date to the first axis tick value. Enter as a string. The tick values are treated as serial date numbers. The default StartDate is the lower axis limit converted to the appropriate date number. For example, a tick value of 1 is converted to the date 01-Jan-0000. Entering StartDate as '06-apr-1999' assigns the date April 6, 1999 to the first tick value and the axis tick labels are set accordingly.

Description dateaxis(Aksis, DateForm, StartDate) replaces axis tick labels with date labels on a graphic figure.

See the MATLAB set command for information on modifying the axis tick values and other axis parameters.

DateForm	Format	Description
0	01-Mar-1999 15:45:17	day-month-year hour:minute:second
1	01-mar-1999	day-month-year
2	03/01/99	month/day/year
3	Mar	month, three letters

DateForm	Format	Description
4	M	month, single letter
5	3	month
6	03/01	month/day
7	1	day of month
8	Wed	day of week, three letters
9	W	day of week, single letter
10	1999	year, four digits
11	99	year, two digits
12	Mar99	month year
13	15:45:17	hour:minute:second
14	03:45:17 PM	hour:minute:second AM or PM
15	15:45	hour:minute
16	03:45 PM	hour:minute AM or PM

Examples

```
dateaxis('x') or dateaxis
```

converts the x-axis labels to an automatically determined date format.

```
dateaxis('y', 6)
```

converts the y-axis labels to the month/day format.

```
dateaxis('x', 2, '03/03/1999')
```

converts the x-axis labels to the month/day/year format. The minimum x-tick value is treated as March 3, 1999.

See Also

bolling, candle, datenum, datestr, highlow, movavg, pointfig

datedisp

Purpose	Display date entries				
Syntax	<code>datedisp(NumMat, DateForm)</code> <code>CharMat = datedisp(NumMat, DateForm)</code>				
Arguments	<table><tr><td>NumMat</td><td>Numeric matrix to display</td></tr><tr><td>DateForm</td><td>(Optional) Date format. See <code>datestr</code> for available and default format flags.</td></tr></table>	NumMat	Numeric matrix to display	DateForm	(Optional) Date format. See <code>datestr</code> for available and default format flags.
NumMat	Numeric matrix to display				
DateForm	(Optional) Date format. See <code>datestr</code> for available and default format flags.				

Description

`datedisp(NumMat, DateForm)` displays a matrix with the serial dates formatted as date strings, using a matrix with mixed numeric entries and serial date number entries. Integers between `datenum('01-Jan-1900')` and `datenum('01-Jan-2200')` are assumed to be serial date numbers, while all other values are treated as numeric entries.

`CharMat` is a character array representing `NumMat`. If no output variable is assigned, the function prints the array to the display.

```
Examples

NumMat = [730730, 0.03, 1200 730100;
          730731, 0.05, 1000 NaN]

NumMat =

    1.0e+05 *

    7.3073    0.0000    0.0120    7.3010
    7.3073    0.0000    0.0100         NaN

datedisp(NumMat)

01-Sep-2000    0.03    1200    11-Dec-1998
02-Sep-2000    0.05    1000         NaN
```

See Also `datestr`

Purpose Indices of date numbers in matrix

Syntax `Indices = datefind(Subset, Superset, Tolerance)`

Arguments

Subset	Subset matrix of date numbers used to find matching date numbers in Superset. These date numbers must be a nonrepeating subset of those in Superset.
Superset	Superset matrix of nonrepeating date numbers whose elements are sought.
Tolerance	(Optional) Tolerance (+/-) for matching the date numbers in Superset. A positive integer. Default = 0.

Description `Indices = datefind(Subset, Superset, Tolerance)` returns a vector of indices to the date numbers in Superset that are present in Subset, plus or minus the Tolerance. If no date numbers match, `Indices = []`.

Although this function was designed for use with sequential date numbers, you can use it with any nonrepeating integers.

Examples

```
Superset = datenum(1999, 7, 1:31);
Subset = [datenum(1999, 7, 10); datenum(1999, 7, 20)];
Indices = datefind(Subset, Superset, 1)
Indices =
     9
    10
    11
    19
    20
    21
```

See Also `datenum`

datemnth

Purpose Date of day in future or past month

Syntax `TargetDate = datemnth(StartDate, NumberMonths, DayFlag, Basis, EndMonthRule)`

Arguments

<code>StartDate</code>	Enter as serial date numbers or date strings.
<code>NumberMonths</code>	Vector containing number of months in future (positive) or past (negative). Values must be in integer form.
<code>DayFlag</code>	(Optional) Vector containing values that specify how the actual day number for the target date in future or past month is determined. 0 (default) = day number should be the day in the future or past month corresponding to the actual day number of the start date. 1 = day number should be the first day of the future or past month. 2 = day number should be the last day of the future or past month. This flag has no effect if <code>EndMonthRule</code> is set to 1.
<code>Basis</code>	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
<code>EndMonthRule</code>	(Optional) End-of-month rule. A vector. 1 = rule in effect, meaning that if you are beginning on the last day of a month, and the month has 30 or fewer days, you will end on the last actual day of the future or past month regardless of whether that month has 28, 29, 30 or 31 days) 0 = rule off (default), meaning that the rule is not in effect.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if `StartDate` is an n-row character array of date strings, then `NumberMonths` must be an n-by-1 vector of integers or a single integer. `TargetDate` is then an n-by-1 vector of date numbers.

Description `TargetDate = datemnth(StartDate, NumberMonths, DayFlag, Basis, EndMonthRule)` returns the serial date number of the target date in the future or past.

Use `datestr` to convert serial date numbers to formatted date strings.

Examples

```
Day = datemnth('3 jun 2001', 6, 0, 0, 0)
Day =
    731188
datestr(Day)
ans =
03-Dec-2001

Day = datemnth('3 jun 2001', 6, 1, 0, 1); datestr(Day)
ans =
01-Dec-2001

Day = datemnth('31 jan 2001', 5, 0, 0, 0); datestr(Day)
ans =
30-Jun-2001

Day = datemnth('31 jan 2001', 5, 1, 0, 0); datestr(Day)
ans =
01-Jun-2001

Day = datemnth('31 jan 2001', 5, 1, 0, 1); datestr(Day)
ans =
30-Jun-2001

Day = datemnth('31 jan 2001', 5, 2, 0, 1); datestr(Day)
ans =
30-Jun-2001

Months = [1; 3; 5; 7; 9];
Day = datemnth('31 jan 2001', Months); datestr(Day)
ans =
28-Feb-2001
30-Apr-2001
30-Jun-2001
31-Aug-2001
31-Oct-2001
```

See Also

datestr, datevec, days360, days365, daysact, daysdif, wrkdydif

datenum

Purpose Create date number

Syntax

```
DateNumber = datenum(DateString)
DateNumber = datenum(DateString, Pivot)
DateNumber = datenum(Year, Month, Day)
DateNumber = datenum(Year, Month, Day, Hour, Minute, Second)
```

Description `DateNumber = datenum(DateString)` returns a serial date number given a date string. Date numbers are the number of days that has passed since a base date. *In MATLAB, date number 1 is January 1, 0000 A.D.* If the input includes time components, the date number includes a fractional component. If the input is only a time component, the date number is only a fractional time component.

The date string can be any of several forms:

```
'19-may-1999'
'may 19, 1999'
'19-may-99'
'19-may' (current year assumed)
'5/19/99'
'5/19' (current year assumed)
'19-may-1999, 18:37'
'19-may-1999, 6:37 pm'
'5/19/99/18:37'
'5/19/99/6:37 pm'
'18:37'
```

Unless you specify a pivot year, date strings with two-character years, e.g., 12-june-12, are assumed to lie within the 100-year period centered about the current year.

`DateNumber = datenum(DateString, Pivot)` assumes that two-character years lie within the 100-year period beginning with the pivot year. The default pivot year is the current year minus 50 years.

`DateNumber = datetime(Year, Month, Day)` returns a serial date number given year, month, and day integers.

`DateNumber = datetime(Year, Month, Day, Hour, Minute, Second)` returns a serial date number given year, month, day, hour, minute, and second integers.

Note This function now ships with basic MATLAB. It originally shipped only with the Financial Toolbox. This description remains here for your convenience.

Examples

```
DateNumber = datetime('19-may-1999')  
DateNumber = 730259
```

```
DateNumber = datetime('5/19/99')  
DateNumber = 730259
```

```
DateNumber = datetime('19-may-1999, 6:37 pm')  
DateNumber = 730259.78
```

```
DateNumber = datetime('5/19/99/18:37')  
DateNumber = 730259.78
```

```
DateNumber = datetime('6:37 pm')  
DateNumber = 0.78
```

```
DateNumber = datetime(1999, 5, 19)  
DateNumber = 730259
```

```
DateNumber = datetime(1999, 1:6, 19)  
DateNumber = [730139 730170 730198 730229 730259 730290]
```

```
DateNumber = datetime(1999, 5, 19, 18, 37, 0)  
DateNumber = 730259.78
```

```
DateNumber = datetime(730259)  
DateNumber = 730259
```

datenum

The next example demonstrates the use of the pivot year in interpreting date strings with two-character years.

```
DateNumber = datenum('12-june-12')
DateNumber =
    735032
datestr(735032)
ans =
    12-Jun-2012

DateNumber = datenum('12-june-12',1900)
DateNumber =
    698507
datestr(698507)
ans =
    12-Jun-1912
```

See Also

`datedisp`, `datestr`, `datevec`, `daysact`, `now`, `today`

Purpose Create date string

Syntax

```
DateString = datestr(Date, DateForm)
DateString = datestr(Date, DateForm, Pivot)
DateString = datestr(Date)
```

Description DateString = datestr(Date, DateForm) converts a date number or a date string to a date string. DateForm specifies the format of DateString. Date strings with two-character years, e.g., 12-june-12, are assumed to lie within the 100-year period centered about the current year.

DateString = datestr(Date, DateForm, Pivot) assumes that two-character years lie within the 100-year period beginning with the pivot year. The default pivot year is the current year minus 50 years.

Note MATLAB's internal date handling and calculations generate no ambiguous values. However, whenever possible, programmers should use date strings containing four-digit years or serial date numbers.

DateString = datestr(Date) assumes DateForm is 1, 16, or 0 depending on whether the date number Date contains a date, time, or both, respectively. If Date is a date string, the function assumes DateForm is 1.

DateForm	Format	Example
0	'dd-mmm-yyyy HH:MM:SS'	01-Mar-2000 15:45:17
1	'dd-mmm-yyyy'	01-Mar-2000
2	'mm/dd/yy'	03/01/00
3	'mmm'	Mar
4	'm'	M
5	'mm'	03
6	'mm/dd'	03/01

DateForm	Format	Example
7	'dd'	01
8	'ddd'	Wed
9	'd'	W
10	'yyyy'	2000
11	'yy'	00
12	'mmyy'	Mar00
13	'HH:MM:SS'	15:45:17
14	'HH:MM:SS PM'	3:45:17 PM
15	'HH:MM'	15:45
16	'HH:MM PM'	3:45 PM
17	'QQ-YY'	Q1-01
18	'QQ'	Q1
19	'dd/mm'	01/03
20	'dd/mm/yy'	01/03/00
21	'mmm.dd.yyyy HH:MM:SS'	Mar.01,2000 15:45:17
22	'mmm.dd.yyyy'	Mar.01.2000
23	'mm/dd/yyyy'	03/01/2000
24	'dd/mm/yyyy'	01/03/2000
25	'yy/mm/dd'	00/03/01
26	'yyyy/mm/dd'	2000/03/01
27	'QQ-YYYY'	Q1-2001
28	'mmyyyy'	Mar2000

Note This function now ships with basic MATLAB. It originally shipped only with the Financial Toolbox. This description remains here for your convenience.

Examples

```
DateString = datestr(730123, 1)
DateString = 03-Jan-1999

DateString = datestr(730123, 2)
DateString = 01/03/99

DateString = datestr(730123, 12)
DateString = Jan99

DateString = datestr(730123.776, 0)
DateString = 03-Jan-1999 18:37:26

DateString = datestr('1/03', 1) (assuming the current year is 1999)
DateString = 03-Jan-1999

DateString = datestr(730123)
DateString = 03-Jan-1999

DateString = datestr([730123 730154 730182 730213 730243 730274])
DateString =
03-Jan-1999
03-Feb-1999
03-Mar-1999
03-Apr-1999
03-May-1999
03-Jun-1999

DateString = datestr('1/03')
DateString = 03-Jan-1999 (assuming the current year is 1999)
```

See Also

dateaxis, datedisp, datenum, datevec, daysact, now, today

datevec

Purpose	Date components
Syntax	<pre>DateVector = datevec(Date) DateVector = datevec(Date, Pivot) [Year, Month, Day, Hour, Minute, Second] = datevec(Date)</pre>
Description	<code>DateVector = datevec(Date)</code> converts a date number or a date string to a date vector whose elements are [Year Month Day Hour Minute Second]. The first five elements are integers, the sixth is a floating-point number. Date strings with two-character years, e.g., 12-june-12, are assumed to lie within the 100-year period centered about the current year. <code>DateVector = datevec(Date, Pivot)</code> assumes that two-character years lie within the 100-year period beginning with the pivot year. The default pivot year is the current year minus 50 years. Note MATLAB's internal date handling and calculations generate no ambiguous values. However, whenever possible, programmers should use date strings containing four-digit years or serial date numbers . <code>[Year, Month, Day, Hour, Minute, Second] = datevec(Date)</code> converts a date number or a date string to a date vector and returns the components of the date vector as individual variables. Note This function now ships with basic MATLAB. It originally shipped only with the Financial Toolbox. This description remains here for your convenience.

Examples	<pre>DateVec = datevec('28-Jul-00') DateVec = 2000 7 28 0 0 0 DateVec = datevec(730695) DateVec = 2000 7 28 0 0 0</pre>
----------	--

```

DateVec = datevec(730695.776)

DateVec =
    2000     7    28    18    37    26.4

[Year, Month, Day, Hour, Minute, Second] = datevec(730695.776)

Year =
    2000

Month =
     7

Day =
    28

Hour =
    18

Minute =
    37

Second =
    26.4

[Year, Month, Day] = datevec(730695:730697)

Year =
    2000    2000    2000

Month =
     7       7       7

Day =
    28     29     30

```

See Also

datenum, datestr, now, today

datewrkdy

Purpose Date of future or past workday

Syntax `EndDate = datewrkdy(StartDate, NumberWorkDays, NumberHolidays)`

Arguments

<code>StartDate</code>	Start date vector. Enter as serial date numbers or date strings.
<code>NumberWorkDays</code>	Vector containing number of work or business days in future (positive) or past (negative), including the starting date.
<code>NumberHolidays</code>	Vector containing values for the number of holidays within <code>NumberWorkDays</code> . <code>NumberHolidays</code> and <code>NumberWorkDays</code> must have the same sign.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if `StartDate` is an n-row character array of date strings, then `NumberWorkDays` must be an n-by-1 vector of integers or a single integer. `EndDate` is then an n-by-1 vector of date numbers.

Description `EndDate = datewrkdy(StartDate, NumberWorkDays, NumberHolidays)` returns the serial number of the date a given number of workdays before or after the start date.

Use `datestr` to convert serial date numbers to formatted date strings.

Examples

```
Workday = datewrkdy('12-dec-2000', 16, 2);
datestr(Workday)
ans =
04-Jan-2001
NumDays = [16; 20; 44];
Workdays = datewrkdy('12-dec-2000', NumDays, 2);
datestr(Workdays)
ans =
4-Jan-2001
10-Jan-2001
13-Feb-2001
```

See Also `busdate`, `holidays`, `isbusday`, `wrkdydif`

Purpose	Day of month
Syntax	<code>DayMonth = day(Date)</code>
Description	<code>DayMonth = day(Date)</code> returns the day of the month given a serial date number or date string.
Examples	<pre>DayMonth = day(730544) or DayMonth = day('2/28/00') returns DayMonth = 28</pre>
See Also	<code>datevec</code> , <code>eomday</code> , <code>month</code> , <code>year</code>

days360

Purpose Days between dates based on 360-day year

Syntax NumDays = days360(StartDate, EndDate)

Arguments

StartDate	Enter as serial date numbers or date strings.
EndDate	Enter as serial date numbers or date strings.

Either input can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an n-by-1 vector of integers or a single integer. NumDays is then an n-by-1 vector of date numbers.

Description NumDays = days360(StartDate, EndDate) returns the number of days between StartDate and EndDate based on a 360-day year (i.e., all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative.

Examples

```
NumDays = days360('15-jan-2000', '15-mar-2000')

NumDays =
    60

MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days360('15-jan-2000', MoreDays)

NumDays =
    60
    90
   150
```

See Also days365, daysact, daysdif, wrkdydif, yearfrac

References Addendum to Securities Industry Association, *Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures*, Vol. 2, Spring 1995.

Purpose Days between dates based on 365-day year

Syntax NumDays = days365(StartDate, EndDate)

Arguments

StartDate	Enter as serial date numbers or date strings.
EndDate	Enter as serial date numbers or date strings.

Either input can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an n-by-1 vector of integers or a single integer. NumDays is then an n-by-1 vector of date numbers.

Description NumDays = days365(StartDate, EndDate) returns the number of days between dates StartDate and EndDate based on a 365-day year. (All months contain their actual number of days. February always contains 28 days.) If EndDate is earlier than StartDate, NumDays is negative. Enter dates as serial date numbers or date strings.

Examples

```
NumDays = days365('15-jan-2000', '15-mar-2000')

NumDays =
    59

MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days365('15-jan-2000', MoreDays)

NumDays =
    59
    90
   151
```

See Also days360, daysact, daysdif, wrkdydif, yearfrac

daysact

Purpose Actual number of days between dates

Syntax NumDays = daysact(StartDate, EndDate)

Arguments

StartDate	Enter as serial date numbers or date strings.
EndDate	(Optional) Enter as serial date numbers or date strings.

Either input argument can contain multiple values, but if so, the other input must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an n-row character array of date strings or a single date. NumDays is then an n-by-1 vector of numbers.

Description NumDays = daysact(StartDate, EndDate) returns the actual number of days between two dates. Enter dates as serial date numbers or date strings. NumDays is negative if EndDate is earlier than StartDate.

NumDays = daysact(StartDate) returns the actual number of days between the MATLAB base date and StartDate. In MATLAB, the base date 1 is 1-Jan-0000 A.D. See datenum for a similar function.

Examples

```
NumDays = daysact('7-sep-2002', '25-dec-2002')
NumDays =
    109

NumDays = daysact('9/7/2002')
NumDays =
    731466

MoreDays = ['09/07/2002'; '10/22/2002'; '11/05/2002'];
NumDays = daysact(MoreDays, '12/25/2002')
NumDays =
    109
     64
     50
```

See Also datenum, datevec, days360, days365, daysdif

Purpose Days between dates for any day-count basis

Syntax NumDays = daysdif(StartDate, EndDate, Basis)

Arguments

StartDate	Enter as serial date numbers or date strings.
EndDate	Enter as serial date numbers or date strings.
Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

Any input argument can contain multiple values, but if so, the other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an n-row character array of date strings or a single date. NumDays is then an n-by-1 vector of numbers.

Description NumDays = daysdif(StartDate, EndDate, Basis) returns the number of days between dates StartDate and EndDate using the given day-count basis. Enter dates as serial date numbers or date strings.

This function is a helper function for the bond pricing and yield functions. It is designed to make the code more readable and to eliminate redundant calls within if statements.

Examples

```
NumDays = daysdif('3/1/99', '3/1/00', 1)
NumDays =
    360

MoreDays = ['3/1/2001'; '3/1/2002'; '3/1/2003'];
NumDays = daysdif('3/1/98', MoreDays)
NumDays =
    1096
    1461
    1826
```

See Also datenum, days360, days365, daysact, wrkdydif, yearfrac

depfixdb

Purpose	Fixed declining-balance depreciation schedule				
Syntax	Depreciation = depfixdb(Cost, Salvage, Life, Period, Month)				
Arguments	Cost	Initial value of the asset.			
	Salvage	Salvage value of the asset.			
	Life	Life of the asset in years.			
	Period	Number of years to calculate.			
	Month	(Optional) Number of months in the first year of asset life. Default = 12.			
Description	Depreciation = depfixdb(Cost, Salvage, Life, Period, Month) calculates the fixed declining-balance depreciation for each period.				
Examples	A car is purchased for \$11,000 with a salvage value of \$1500 and a lifetime of eight years. To calculate the depreciation for the first five years				
	Depreciation = depfixdb(11000, 1500, 8, 5)				
	returns				
	Depreciation =				
	2425.08	1890.44	1473.67	1148.78	895.52
See Also	depgendb, deprdrv, depsoyd, depstln				

Purpose	General declining-balance depreciation schedule				
Syntax	Depreciation = depgendb(Cost, Salvage, Life, Factor)				
Arguments	Cost	Cost of the asset.			
	Salvage	Estimated salvage value of the asset.			
	Life	Number of periods over which the asset is depreciated.			
	Factor	Depreciation factor. Factor = 2 uses the double-declining-balance method.			
Description	Depreciation = depgendb(Cost, Salvage, Life, Factor) calculates the declining-balance depreciation for each period.				
Examples	A car is purchased for \$11,000 and is to be depreciated over five years. The estimated salvage value is \$1000. Using the double-declining-balance method, the function calculates the depreciation for each year and returns the remaining depreciable value at the end of the life of the car.				
	Depreciation = depgendb(11000, 1000, 5, 2)				
	returns				
	Depreciation =	4400.00	2640.00	1584.00	950.40
See Also	depfixdb, deprdv, depsoyd, depstln				

deprdv

Purpose	Remaining depreciable value	
Syntax	Value = deprdv(Cost, Salvage, Accum)	
Arguments	Cost	Cost of the asset.
	Salvage	Salvage value of the asset.
	Accum	Accumulated depreciation of the asset for prior periods.
Description	Value = deprdv(Cost, Salvage, Accum) returns the remaining depreciable value for an asset.	
Examples	The cost of an asset is \$13,000 with a life of 10 years. The salvage value is \$1000. First find the accumulated depreciation with the straight-line depreciation function, depstln. Then find the remaining depreciable value after six years.	
	Accum = depstln(13000, 1000, 10) * 6	
	Accum = 7200.00	
	Value = deprdv(13000, 1000, 7200)	
	Value = 4800.00	
See Also	depfixdb, depgendb, depsoyd, depstln	

Purpose Sum of years' digits depreciation

Syntax Sum = depsoyd(Cost, Salvage, Life)

Arguments

Cost	Cost of the asset.
Salvage	Salvage value of the asset.
Life	Depreciable life of the asset in years.

Description Sum = depsoyd(Cost, Salvage, Life) calculates the depreciation for an asset using the sum of years' digits method. Sum is a 1-by-Life vector of depreciation values with each element corresponding to a year of the asset's life.

Examples The cost of an asset is \$13,000 with a life of 10 years. The salvage value of the asset is \$1000.

```
Sum = depsoyd(13000, 1000, 10)'
```

returns

```
Sum =
    2181.82
    1963.64
    1745.45
    1527.27
    1309.09
    1090.91
     872.73
     654.55
     436.36
     218.18
```

See Also depfixdb, depgendb, deprdv, depstln

depstln

Purpose	Straight-line depreciation schedule	
Syntax	Depreciation = depstln(Cost, Salvage, Life)	
Arguments	Cost	Cost of the asset.
	Salvage	Salvage value of the asset.
	Life	Depreciable life of the asset in years.
Description	Depreciation = depstln(Cost, Salvage, Life) calculates straight-line depreciation for an asset.	
Examples	The cost of an asset is \$13,000 with a life of 10 years. The salvage value of the asset is \$1000.	
	Depreciation = depstln(13000, 1000, 10)	
	returns	
	Depreciation = 1200	
See Also	depfixdb, depgendb, deprdv, depsoyd	

Purpose	Zero curve given a discount curve	
Syntax	<code>[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates, Settle, OutputCompounding, OutputBasis)</code>	
Arguments	DiscRates	Column vector of discount factors, as decimal fractions. In aggregate, the factors in DiscRates constitute a discount curve for the investment horizon represented by CurveDates.
	CurveDates	Column vector of maturity dates (as serial date numbers) that correspond to the discount factors in DiscRates.
	Settle	Serial date number that is the common settlement date for the discount rates in DiscRates.
	OutputCompounding	(Optional) Output compounding. A scalar that sets the compounding frequency per year for annualizing the output zero rates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding 365 daily compounding -1 continuous compounding
	OutputBasis	(Optional) Output day-count basis for annualizing the output zero rates. Allowed values are: 0 actual/actual (default) 1 30/360 2 actual/360 3 actual/365

disc2zero

Description

`[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates, Settle, OutputCompounding, OutputBasis)` returns a zero curve given a discount curve and its maturity dates.

ZeroRates Column vector of decimal fractions. In aggregate, the rates in **ZeroRates** constitute a zero curve for the investment horizon represented by **CurveDates**. The zero rates are the yields to maturity on theoretical zero-coupon bonds.

CurveDates Column vector of maturity dates (as serial date numbers) that correspond to the zero rates. This vector is the same as the input vector **CurveDates**.

Examples

Given discount factors **DiscRates** over a set of maturity dates **CurveDates**, and a settlement date **Settle**:

```
DiscRates = [0.9996
              0.9947
              0.9896
              0.9866
              0.9826
              0.9786
              0.9745
              0.9665
              0.9552
              0.9466];

CurveDates = [datenum('06-Nov-2000')
              datenum('11-Dec-2000')
              datenum('15-Jan-2001')
              datenum('05-Feb-2001')
              datenum('04-Mar-2001')
              datenum('02-Apr-2001')
              datenum('30-Apr-2001')
              datenum('25-Jun-2001')
              datenum('04-Sep-2001')
              datenum('12-Nov-2001')];

Settle = datenum('03-Nov-2000');
```

Set daily compounding for the output zero curve, on an actual/365 basis.

```
OutputCompounding = 365;  
OutputBasis = 3;
```

Execute the function

```
[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates,...  
Settle, OutputCompounding, OutputBasis)
```

which returns the zero curve ZeroRates at the maturity dates CurveDates:

```
ZeroRates =  
    0.0487  
    0.0510  
    0.0523  
    0.0524  
    0.0530  
    0.0526  
    0.0530  
    0.0532  
    0.0549  
    0.0536
```

```
CurveDates =  
    730796  
    730831  
    730866  
    730887  
    730914  
    730943  
    730971  
    731027  
    731098  
    731167
```

For readability, DiscRates and ZeroRates are shown here only to the basis point. However, MATLAB computed them at full precision. If you enter DiscRates as shown, ZeroRates may differ due to rounding.

See Also

zero2disc and other functions for Term Structure of Interest Rates

discrate

Purpose Bank discount rate of a money market security

Syntax `DiscRate = discrate(Settle, Maturity, Face, Price, Basis)`

Arguments

Settle	Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.
Maturity	Enter as serial date number or date string.
Face	Redemption (par, face) value.
Price	Price of the security.
Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

Description `DiscRate = discrate(Settle, Maturity, Face, Price, Basis)` finds the bank discount rate of a security. The bank discount rate normalizes by the face value of the security (e.g., U. S. Treasury Bills) and understates the true yield earned by investors.

Examples

```
DiscRate = discrate('12-jan-2000', '25-jun-2000', 100, 97.74, 0)
```

returns

```
DiscRate =  
    0.0501
```

a discount rate of 5.01%.

See Also `acrudisc`, `fvdisc`, `prdisc`, `ylddisc`

References Mayle, *Standard Securities Calculation Methods*, Volumes I-II, 3rd edition. Formula 1.

Purpose	Effective rate of return
Syntax	Return = effrr(Rate, NumPeriods)
Arguments	Rate Annual percentage rate. Enter as a decimal fraction. NumPeriods Number of compounding periods per year, an integer.
Description	Return = effrr(Rate, NumPeriods) calculates the annual effective rate of return. Compounding continuously returns Return equivalent to $(e^{\text{Rate}} - 1)$.
Examples	Find the effective annual rate of return based on an annual percentage rate of 9% compounded monthly. <pre>Return = effrr(0.09, 12)</pre> returns <pre>Return =</pre> 0.0938 or 9.38%
See Also	nomrr

eomdate

Purpose Last date of month

Syntax DayMonth = eomdate(Year, Month)

Description DayMonth = eomdate(Year, Month) returns the serial date number of the last date of the month for the given year and month. Enter Year as a four-digit integer; enter Month as an integer from 1 to 12.

Either input argument can contain multiple values, but if so, the other input must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. DayMonth is then a 1-by-n vector of date numbers.

Use the function `datestr` to convert serial date numbers to formatted date strings.

Examples

```
DayMonth = eomdate(2001, 2)
DayMonth =
    730910
datestr(DayMonth)
ans =
    28-Feb-2001

Year = [2002 2003 2004 2005];
DayMonth = eomdate(Year, 2)
DayMonth =
    731275    731640    732006    732371

datestr(DayMonth)

ans =
    28-Feb-2002
    28-Feb-2003
    29-Feb-2004
    28-Feb-2005
```

See Also day, eomday, lbusdate, month, year

Purpose Last day of month

Syntax Day = eomday(Year, Month)

Description Day = eomday(Year, Month) returns the last day of the month for the given year and month. Enter Year as a four-digit integer; enter Month as an integer from 1 to 12.

Either input argument can contain multiple values, but if so, the other input must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. Day is then a 1-by-n vector of days.

Note This function now ships with basic MATLAB. It originally shipped only with the Financial Toolbox. This description remains here for your convenience.

Examples Day = eomday(2000, 2)

Day =
29

See Also day, eomdate, month

Purpose	Expected return and covariance from return time series	
Syntax	<code>[ExpReturn, ExpCovariance, NumEffObs] = ewstats(RetSeries, DecayFactor, WindowLength)</code>	
Arguments	RetSeries	Return Series: number of observations (NUMOBS) by number of assets (NASSETS) matrix of equally spaced incremental return observations. The first row is the oldest observation, and the last row is the most recent.
	DecayFactor	(Optional) Controls how much less each observation is weighted than its successor. The k th observation back in time has weight DecayFactor^k . DecayFactor must lie in the range: $0 < \text{DecayFactor} \leq 1$. Default = 1, the equally weighted linear moving average model (BIS).
	WindowLength	(Optional) Number of recent observations in the computation. Default = NUMOBS.
Description	<code>[ExpReturn, ExpCovariance, NumEffObs] = ewstats(RetSeries, DecayFactor, WindowLength)</code> computes estimated expected returns, estimated covariance matrix, and the number of effective observations.	
	ExpReturn is a 1-by-NASSETS vector of estimated expected returns.	
	ExpCovariance is an NASSETS-by-NASSETS estimated covariance matrix. The standard deviations of the asset return processes are given by:	
	$\text{STDVec} = \text{sqrt}(\text{diag}(\text{ExpCovariance}))$	
	The correlation matrix is:	
	$\text{CorrMat} = \text{ExpCovariance} ./ (\text{STDVec} * \text{STDVec}')$	
	NumEffObs is the number of effective observations = $(1 - \text{DecayFactor}^{\text{WindowLength}}) / (1 - \text{DecayFactor})$.	
	A smaller DecayFactor or WindowLength emphasizes recent data more strongly but uses less of the available data set.	

Examples

```
RetSeries = [ 0.24 0.08
              0.15 0.13
              0.27 0.06
              0.14 0.13 ];

DecayFactor = 0.98;

[ExpReturn, ExpCovariance] = ewstats(RetSeries, DecayFactor)

ExpReturn =

    0.1995    0.1002

ExpCovariance =

    0.0032   -0.0017
   -0.0017    0.0010
```

See Also

cov, mean

fbusdate

Purpose First business date of month

Syntax `Date = fbusdate(Year, Month, Holiday)`

Arguments

Year	Enter as four-digit integer.
Month	Enter as integer from 1 to 12.
Holiday	(Optional) Vector of holidays and nontrading-day dates. All dates in Holiday must be the same format: either serial date numbers or date strings. (Using date numbers improves performance.) The holidays function supplies the default vector.

Description `Date = fbusdate(Year, Month, Holiday)` returns the serial date number for the first business date of the given year and month. Holiday specifies nontrading days.

Year and Month can contain multiple values. If one contains multiple values, the other must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. Date is then a 1-by-n vector of date numbers.

Use the function `datestr` to convert serial date numbers to formatted date strings.

Examples

```
Date = fbusdate(2001, 11); datestr(Date)
ans =
01-Nov-2001

Year = [2002 2003 2004];
Date = fbusdate(Year, 11); datestr(Date)

ans =
01-Nov-2002
03-Nov-2003
01-Nov-2004
```

See Also `busdate`, `eomdate`, `holidays`, `isbusday`, `lbusdate`

Purpose	Fractional currency value to decimal value
Syntax	<code>Decimal = frac2cur(Fraction, Denominator)</code>
Description	<code>Decimal = frac2cur(Fraction, Denominator)</code> converts a fractional currency value to a decimal value. <code>Fraction</code> is the fractional currency value input as a string, and <code>Denominator</code> is the denominator of the fraction.
Examples	<pre>Decimal = frac2cur('12.1', 8) returns Decimal = 12.1250</pre>
See Also	<code>cur2frac</code> , <code>cur2str</code>

Purpose	Mean-variance efficient frontier	
Syntax	<code>[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, NumPorts, PortReturn, AssetBounds, Groups, GroupBounds)</code>	
Arguments	ExpReturn	1 by number of assets (NASSETS) vector specifying the expected (mean) return of each asset.
	ExpCovariance	NASSETS-by-NASSETS matrix specifying the covariance of asset returns.
	NumPorts	(Optional) Number of portfolios generated along the efficient frontier. Returns are equally spaced between the maximum possible return and the minimum risk point. If NumPorts is empty (entered as []), frontcon computes 10 equally spaced points. When entering a target rate of return (PortReturn), enter NumPorts as an empty matrix [].
	PortReturn	(Optional) Vector of length equal to the number of portfolios (NPORTS) containing the target return values on the frontier. If PortReturn is not entered or [], NumPorts equally spaced returns between the minimum and maximum possible values are used.
	AssetBounds	(Optional) 2-by-NASSETS matrix containing the lower and upper bounds on the weight allocated to each asset in the portfolio. Default lower bound = all 0s (no short-selling). Default upper bound = all 1s (any asset may constitute the entire portfolio).

Groups	(Optional) Number of groups (NGROUPS)-by-NASSETS matrix specifying NGROUPS asset groups or classes. Each row specifies a group. $\text{Groups}(i, j) = 1$ (j th asset belongs in the i th group). $\text{Groups}(i, j) = 0$ (j th asset not a member of the i th group).
GroupBounds	(Optional) NGROUPS-by-2 matrix specifying, for each group, the lower and upper bounds of the total weights of all assets in that group. Default lower bound = all 0s. Default upper bound = all 1s.

Description

`[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, NumPorts, PortReturn, AssetBounds, Groups, GroupBounds)` returns the mean-variance efficient frontier with user-specified asset constraints, covariance, and returns. For a collection of NASSETS risky assets, computes a portfolio of asset investment weights that minimize the risk for given values of the expected return. The portfolio risk is minimized subject to constraints on the asset weights or on groups of asset weights.

PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.

PortReturn is a NPORTS-by-1 vector of the expected return of each portfolio.

PortWts is an NPORTS-by-NASSETS matrix of weights allocated to each asset. Each row represents a portfolio. The total of all weights in a portfolio is 1.

frontcon generates a plot of the efficient frontier if you invoke it without output arguments.

The asset returns are assumed to be jointly normal, with expected mean returns of ExpReturn and return covariance ExpCovariance. The variance of a portfolio with 1-by-NASSETS weights PortWts is given by $\text{PortVar} = \text{PortWts} * \text{ExpCovariance} * \text{PortWts}'$. The portfolio expected return is $\text{PortReturn} = \text{dot}(\text{ExpReturn}, \text{PortWts})$.

Examples

Given three assets with expected returns of

```
ExpReturn = [0.1 0.2 0.15];
```

and expected covariance of

frontcon

```
ExpCovariance = [ 0.0100    -0.0061    0.0042  
                 -0.0061     0.0400   -0.0252  
                 0.0042    -0.0252    0.0225];
```

compute the mean-variance efficient frontier for four points.

```
NumPorts = 4;  
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn,...  
ExpCovariance, NumPorts)
```

```
PortRisk =
```

```
0.0426  
0.0483  
0.1089  
0.2000
```

```
PortReturn =
```

```
0.1569  
0.1713  
0.1856  
0.2000
```

```
PortWts =
```

```
0.2134    0.3518    0.4348  
0.0096    0.4352    0.5552  
0         0.7128    0.2872  
0         1.0000         0
```

See Also

ewstats, portopt, portstats

Purpose	Future value of discounted security										
Syntax	<code>FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)</code>										
Arguments	<table><tr><td>Settle</td><td>Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.</td></tr><tr><td>Maturity</td><td>Maturity date. Enter as serial date number or date string.</td></tr><tr><td>Price</td><td>Price (present value) of the security.</td></tr><tr><td>Discount</td><td>Bank discount rate of the security. Enter as decimal fraction.</td></tr><tr><td>Basis</td><td>(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.</td></tr></table>	Settle	Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.	Maturity	Maturity date. Enter as serial date number or date string.	Price	Price (present value) of the security.	Discount	Bank discount rate of the security. Enter as decimal fraction.	Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
Settle	Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.										
Maturity	Maturity date. Enter as serial date number or date string.										
Price	Price (present value) of the security.										
Discount	Bank discount rate of the security. Enter as decimal fraction.										
Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.										
Description	<code>FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)</code> finds the amount received at maturity for a fully vested security.										
Examples	Using this data <pre>Settle = '02/15/2001'; Maturity = '05/15/2001'; Price = 100; Discount = 0.0575; Basis = 2;</pre> <code>FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)</code> returns <pre>FutureVal = 101.44</pre>										
See Also	<code>acrudisc</code> , <code>discrate</code> , <code>prdisc</code> , <code>ylddisc</code>										
References	Mayle, <i>Standard Securities Calculation Methods</i> , Volumes I-II, 3rd edition.										

fvfix

Purpose	Future value with fixed periodic payments										
Syntax	<code>FutureVal = fvfix(Rate, NumPeriods, Payment, PresentVal, Due)</code>										
Arguments	<table><tr><td><code>rate</code></td><td>Periodic interest rate, as a decimal fraction.</td></tr><tr><td><code>NumPeriods</code></td><td>Number of periods.</td></tr><tr><td><code>Payment</code></td><td>Periodic payment.</td></tr><tr><td><code>PresentVal</code></td><td>(Optional) Initial value. Default = 0.</td></tr><tr><td><code>Due</code></td><td>(Optional) When payments are due or made: 0 = end of period (default), or 1 = beginning of period.</td></tr></table>	<code>rate</code>	Periodic interest rate, as a decimal fraction.	<code>NumPeriods</code>	Number of periods.	<code>Payment</code>	Periodic payment.	<code>PresentVal</code>	(Optional) Initial value. Default = 0.	<code>Due</code>	(Optional) When payments are due or made: 0 = end of period (default), or 1 = beginning of period.
<code>rate</code>	Periodic interest rate, as a decimal fraction.										
<code>NumPeriods</code>	Number of periods.										
<code>Payment</code>	Periodic payment.										
<code>PresentVal</code>	(Optional) Initial value. Default = 0.										
<code>Due</code>	(Optional) When payments are due or made: 0 = end of period (default), or 1 = beginning of period.										

Description	<code>FutureVal = fvfix(Rate, NumPeriods, Payment, PresentVal, Due)</code> returns the future value of a series of equal payments.
--------------------	--

Examples	A savings account has a starting balance of \$1500. \$200 is added at the end of each month for 10 years and the account pays 9% interest compounded monthly. Using this data <pre>FutureVal = fvfix(0.09/12, 12*10, 200, 1500, 0)</pre> returns <pre>FutureVal = 42379.89</pre>
-----------------	--

See Also	<code>fvvar</code> , <code>pvfix</code> , <code>pvvar</code>
-----------------	--

Purpose	Future value of varying cash flow	
Syntax	<code>FutureVal = fvvar(CashFlow, Rate, IrrCFDates)</code>	
Arguments	CashFlow	A vector of varying cash flows. Include the initial investment as the initial cash flow value (a negative number).
	Rate	Periodic interest rate. Enter as a decimal fraction.
	IrrCFDates	(Optional) For irregular (nonperiodic) cash flows, a vector of dates on which the cash flows occur. Enter dates as serial date numbers or date strings. Default assumes CashFlow contains regular (periodic) cash flows.

Description `FutureVal = fvvar(CashFlow, Rate, IrrCFDates)` returns the future value of a varying cash flow.

Examples This cash flow represents the yearly income from an initial investment of \$10,000. The annual interest rate is 8%.

Year 1	\$2000
Year 2	\$1500
Year 3	\$3000
Year 4	\$3800
Year 5	\$5000

For the future value of this regular (periodic) cash flow

```
FutureVal = fvvar([-10000 2000 1500 3000 3800 5000], 0.08)
```

returns

```
FutureVal =
    2520.47
```

An investment of \$10,000 returns this irregular cash flow. The original investment and its date are included. The periodic interest rate is 9%.

Cash flow	Dates
(\$10000)	January 12, 2000
\$2500	February 14, 2001
\$2000	March 3, 2001
\$3000	June 14, 2001
\$4000	December 1, 2001

To calculate the future value of this irregular (nonperiodic) cash flow

```
CashFlow = [-10000, 2500, 2000, 3000, 4000];  
IrrCFDates = ['01/12/2000'  
              '02/14/2001'  
              '03/03/2001'  
              '06/14/2001'  
              '12/01/2001'];
```

```
FutureVal = fvvar(CashFlow, 0.09, IrrCFDates)
```

returns

```
FutureVal =  
          170.66
```

See Also

fvfix, irr, payuni, pvfix, pvvar

Purpose	Zero curve given a forward curve	
Syntax	<code>[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis)</code>	
Arguments	ForwardRates	A number of bonds (NUMBONDS) by 1 vector of annualized implied forward rates, as decimal fractions. In aggregate, the rates in ForwardRates constitute an implied forward curve for the investment horizon represented by CurveDates.
	CurveDates	A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the forward rates.
	Settle	A serial date number that is the common settlement date for the forward rates.
	OutputCompounding	(Optional) Output compounding. A scalar that sets the compounding frequency per year for annualizing the output zero rates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding 365 daily compounding -1 continuous compounding
	OutputBasis	(Optional) Output day-count basis for annualizing the output zero rates. Allowed values are: 0 actual/actual (default) 1 30/360 2 actual/360

	3 actual/365
InputCompounding	(Optional) A scalar that indicates the compounding frequency per year used for annualizing the input forward rates. Allowed values are the same as for OutputCompounding. Default = OutputCompounding.
InputBasis	(Optional) Input day-count basis used for annualizing the forward rates. Allowed values are the same as for OutputBasis. Default = OutputBasis.

Description

[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis) returns a zero curve given an implied forward curve and its maturity dates.

ZeroRates A NUMBONDS-by-1 vector of decimal fractions. In aggregate, the rates in ZeroRates constitute a zero curve for the investment horizon represented by CurveDates.

CurveDates A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the zero rates in ZeroRates. This vector is the same as the input vector CurveDates.

Examples

Given an implied forward curve ForwardRates over a set of maturity dates CurveDates, and a settlement date Settle:

```
ForwardRates = [0.0469
                0.0519
                0.0549
                0.0535
                0.0558
                0.0508
                0.0560
                0.0545
                0.0615
                0.0486];
```

```
CurveDates = [datenum('06-Nov-2000')
               datenum('11-Dec-2000')
               datenum('15-Jan-2001')
               datenum('05-Feb-2001')
               datenum('04-Mar-2001')
               datenum('02-Apr-2001')
               datenum('30-Apr-2001')
               datenum('25-Jun-2001')
               datenum('04-Sep-2001')
               datenum('12-Nov-2001')];
```

```
Settle = datenum('03-Nov-2000');
```

Set daily compounding for the zero curve, on an actual/365 basis. The forward curve was compounded annually on an actual/actual basis.

```
OutputCompounding = 365;
OutputBasis = 3;
InputCompounding = 1;
InputBasis = 0;
```

Execute the function

```
[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates,...
Settle, OutputCompounding, OutputBasis, InputCompounding,...
InputBasis)
```

which returns the zero curve ZeroRates at the maturity dates CurveDates:

```
ZeroRates =
    0.0457
    0.0501
    0.0516
    0.0517
    0.0523
    0.0517
    0.0521
    0.0523
    0.0540
    0.0528
```

fwd2zero

```
CurveDates =  
    730796  
    730831  
    730866  
    730887  
    730914  
    730943  
    730971  
    731027  
    731098  
    731167
```

For readability, ForwardRates and ZeroRates are shown here only to the basis point. However, MATLAB computed them at full precision. If you enter ForwardRates as shown, ZeroRates may differ due to rounding.

See Also

zero2fwd and other functions for Term Structure of Interest Rates

Purpose	High, low, open, close chart										
Syntax	<pre>highlow(High, Low, Close, Open, Color)</pre> Handles = <code>highlow(High, Low, Close, Open, Color)</code>										
Arguments	<table> <tr> <td>High</td><td>High prices for a security. A column vector.</td></tr> <tr> <td>Low</td><td>Low prices for a security. A column vector.</td></tr> <tr> <td>Close</td><td>Closing prices for a security. A column vector.</td></tr> <tr> <td>Open</td><td>(Optional) Opening prices for a security. A column vector. To specify Color when Open is unknown, enter Open as an empty matrix [].</td></tr> <tr> <td>Color</td><td>(Optional) Vertical line color. A string. MATLAB supplies a default color if none is specified. The default color differs depending on the background color of the figure window. See ColorSpec in the MATLAB documentation for color names.</td></tr> </table>	High	High prices for a security. A column vector.	Low	Low prices for a security. A column vector.	Close	Closing prices for a security. A column vector.	Open	(Optional) Opening prices for a security. A column vector. To specify Color when Open is unknown, enter Open as an empty matrix [].	Color	(Optional) Vertical line color. A string. MATLAB supplies a default color if none is specified. The default color differs depending on the background color of the figure window. See ColorSpec in the MATLAB documentation for color names.
High	High prices for a security. A column vector.										
Low	Low prices for a security. A column vector.										
Close	Closing prices for a security. A column vector.										
Open	(Optional) Opening prices for a security. A column vector. To specify Color when Open is unknown, enter Open as an empty matrix [].										
Color	(Optional) Vertical line color. A string. MATLAB supplies a default color if none is specified. The default color differs depending on the background color of the figure window. See ColorSpec in the MATLAB documentation for color names.										
Description	<code>highlow(High, Low, Close, Open, Color)</code> plots the high, low, opening, and closing prices of an asset. Plots are vertical lines whose top is the high, bottom is the low, open is a short horizontal tick to the left, and close is a short horizontal tick to the right. <code>Handles = highlow(High, Low, Close, Open, Color)</code> plots the figure and returns the handles of the lines.										
Examples	The high, low, and closing prices for an asset are stored in equal-length vectors <code>AssetHi</code>, <code>AssetLo</code>, and <code>AssetCl</code> respectively <pre>highlow(AssetHi, AssetLo, AssetCl, [], 'cyan')</pre> plots the price data using cyan lines.										
See Also	<code>bolling</code> , <code>candle</code> , <code>dateaxis</code> , <code>movavg</code> , <code>pointfig</code>										

holidays

Purpose	Holidays and nontrading days				
Syntax	<code>Holidays = holidays(StartDate, EndDate)</code>				
Arguments	<table><tr><td><code>StartDate</code></td><td>Start date vector. Enter as serial date numbers or date strings.</td></tr><tr><td><code>EndDate</code></td><td>End date vector. Enter as serial date numbers or date strings.</td></tr></table>	<code>StartDate</code>	Start date vector. Enter as serial date numbers or date strings.	<code>EndDate</code>	End date vector. Enter as serial date numbers or date strings.
<code>StartDate</code>	Start date vector. Enter as serial date numbers or date strings.				
<code>EndDate</code>	End date vector. Enter as serial date numbers or date strings.				
Description	<code>Holidays = holidays(StartDate, EndDate)</code> returns a vector of serial date numbers corresponding to the holidays and nontrading days between <code>StartDate</code> and <code>EndDate</code>, inclusive. <code>Holidays = holidays</code> returns a vector of serial date numbers corresponding to all holidays and nontrading days. As shipped, this function contains all holidays and special nontrading days for the New York Stock Exchange between 1950 and 2030, inclusive (681 dates). You can edit the <code>holidays.m</code> file to contain your own holidays and nontrading days. By definition, holidays and nontrading days are those that occur on weekdays.				
Examples	<pre>Holidays = holidays('jan 1 2001', 'jun 23 2001') returns Holidays = 730852 730901 730954 730999 which are the serial date numbers for 01-Jan-2001 (New Year's Day) 19-Feb-2001 (President's Day) 13-Apr-2001 (Good Friday) 28-May-2001 (Memorial Day)</pre>				
See Also	<code>busdate</code> , <code>fbusdate</code> , <code>isbusday</code> , <code>lbusdate</code>				

Purpose	Hour of date or time
Syntax	Hour = hour(Date)
Description	Hour = hour(Date) returns the hour of the day given a serial date number or a date string.
Examples	<pre>Hour = hour(730473.5584278936) or Hour = hour('19-dec-1999, 13:24:08.17')</pre> returns <pre>Hour = 13</pre>
See Also	datevec, minute, second

irr

Purpose	Internal rate of return										
Syntax	<code>Return = irr(CashFlow)</code>										
Description	<code>Return = irr(CashFlow)</code> calculates the internal rate of return for a series of periodic cash flows. <code>CashFlow</code> is the cash flow vector. The first entry in <code>CashFlow</code> is the initial investment. If the cash flow payments are monthly, multiply the resulting rate of return by 12 for the annual rate of return. This function calculates only positive rates of return; for negative rates of return, <code>Return = NaN</code> .										
Examples	This cash flow represents the yearly income from an initial investment of \$100,000: <table><tr><td>Year 1</td><td>\$10,000</td></tr><tr><td>Year 2</td><td>\$20,000</td></tr><tr><td>Year 3</td><td>\$30,000</td></tr><tr><td>Year 4</td><td>\$40,000</td></tr><tr><td>Year 5</td><td>\$50,000</td></tr></table> To calculate the internal rate of return on the investment <pre>Return = irr([-100000 10000 20000 30000 40000 50000])</pre> returns <pre>Return = 0.1201 (12.01%)</pre>	Year 1	\$10,000	Year 2	\$20,000	Year 3	\$30,000	Year 4	\$40,000	Year 5	\$50,000
Year 1	\$10,000										
Year 2	\$20,000										
Year 3	\$30,000										
Year 4	\$40,000										
Year 5	\$50,000										
See Also	<code>effrr</code> , <code>mirr</code> , <code>nomrr</code> , <code>taxedrr</code> , <code>xirr</code>										
References	Brealey and Myers, <i>Principles of Corporate Finance</i> , Chapter 5										

Purpose True for dates that are business days

Syntax `Busday = isbusday(Date, Holiday)`

Arguments

Date	Date(s) being checked. Enter as a serial date number or date string. Date can contain multiple dates, but they must all be in the same format.
Holiday	(Optional) Vector of holidays and nontrading-day dates. All dates in Holiday must be the same format: either serial date numbers or date strings. (Using date numbers improves performance.) The holidays function supplies the default vector.

Description `Busday = isbusday(Date, Holiday)` returns logical true (1) if Date is a business day and logical false (0) otherwise.

Examples

```
Busday = isbusday('16 jun 2001')
Busday =

    0

Date = ['15 feb 2001'; '16 feb 2001'; '17 feb 2001'];

Busday = isbusday(Date)

Busday =
     1
     1
     0
```

See Also `busdate`, `fbusdate`, `holidays`, `lbusdate`

lbusdate

Purpose Last business date of month

Syntax `Date = lbusdate(Year, Month, Holiday)`

Arguments

Year	Enter as four-digit integer.
Month	Enter as integer from 1 to 12.
Holiday	(Optional) Vector of holidays and nontrading-day dates. All dates in Holiday must be the same format: either serial date numbers or date strings. (Using date numbers improves performance.) The holidays function supplies the default vector.

Description `Date = lbusdate(Year, Month, Holiday)` returns the serial date number for the last business date of the given year and month. Holiday specifies nontrading days.

Year and Month can contain multiple values. If one contains multiple values, the other must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. Date is then a 1-by-n vector of date numbers.

Use the function `datestr` to convert serial date numbers to formatted date strings.

Examples

```
Date = lbusdate(2001, 5)
Date =

    731002

datestr(Date)

ans =
31-May-2001

Year = [2001 2002 2003];
```

```
Date = lbusdate(Year, 5); datestr(Date)
```

```
ans =
```

```
31-May-2001
```

```
31-May-2002
```

```
30-May-2003
```

See Also

busdate, eomdate, fbusdate, holidays, isbusday

lweekdate

</

Examples

To find the last Monday in June 2001:

```
LastDate = lweekdate(2, 2001, 6); datestr>LastDate)
ans =
25-Jun-2001
```

To find the last Monday in a week that also contains a Friday in June 2001:

```
LastDate = lweekdate(2, 2001, 6, 6); datestr>LastDate)
ans =
25-Jun-2001
```

To find the last Monday in May for 2001, 2002, and 2003:

```
Year = [2001:2003];
LastDate = lweekdate(2, Year, 5)
LastDate =
           730999           731363           731727
datestr>LastDate)
ans =
28-May-2001
27-May-2002
26-May-2003
```

See Also

eomdate, lbusdate, nweekdate

m2xdate

Purpose MATLAB serial date number to Excel serial date number

Syntax `DateNum = m2xdate(MATLABDateNumber, Convention)`

Arguments

<code>MATLABDateNumber</code>	A vector or scalar of MATLAB serial date numbers.
<code>Convention</code>	(Optional) Excel date system. A vector or scalar. When <code>Convention = 0</code> (default), the Excel 1900 date system is in effect. When <code>Convention = 1</code> , the Excel 1904 date system is used. In the Excel 1900 date system, the Excel serial date number 1 corresponds to January 1, 1900 A.D. In the Excel 1904 date system, date number 0 is January 1, 1904 A.D.

Vector arguments must have consistent dimensions.

Description `DateNum = m2xdate(MATLABDateNumber, Convention)` converts MATLAB serial date numbers to Excel serial date numbers. MATLAB date numbers start with 1 = January 1, 0000 A.D., hence there is a difference of 693961 relative to the 1900 date system, or 695422 relative to the 1904 date system. This function is useful with MATLAB Excel Link.

Examples

Given MATLAB date numbers for Christmas 2001 through 2004

```
DateNum = datenum(2001:2004, 12, 25)
DateNum =
    731210    731575    731940    732306
```

convert them to Excel date numbers in the 1904 system

```
ExDate = m2xdate(DateNum, 1)
ExDate =
    35788    36153    36518    36884
```

or the 1900 system

```
ExDate = m2xdate(DateNum)
ExDate =
    37250    37615    37980    38346
```

See Also

[datenum](#), [datestr](#), [x2mdate](#)

minute

Purpose Minute of date or time

Syntax Minute = minute(Date)

Description Minute = minute(Date) returns the minute given a serial date number or a date string.

Examples

```
Minute = minute(731204.5591223380)
```

or

```
Minute = minute('19-dec-2001, 13:25:08.17')
```

returns

```
Minute =  
25
```

See Also datevec, hour, second

Purpose	Modified internal rate of return	
Syntax	Return = mirr(CashFlow, FinRate, Reinvest)	
Arguments	CashFlow	Vector of cash flows. The first entry is the initial investment.
	FinRate	Finance rate for negative cash flow values. Enter as decimal fraction.
	Reinvest	Reinvestment rate for positive cash flow values, as a decimal fraction.
Description	Return = mirr(CashFlow, FinRate, Reinvest) calculates the modified internal rate of return for a series of periodic cash flows. This function calculates only positive rates of return; for negative rates of return, Return = 0.	
Examples	This cash flow represents the yearly income from an initial investment of \$100,000. The finance rate is 9% and the reinvestment rate is 12%.	
	Year 1	\$20,000
	Year 2	(\$10,000)
	Year 3	\$30,000
	Year 4	\$38,000
	Year 5	\$50,000
	To calculate the modified internal rate of return on the investment	
	Return = mirr([-100000 20000 -10000 30000 38000 50000], 0.09, ... 0.12)	
	returns	
	Return = 0.0832 (8.32%)	
See Also	annurate, effrr, irr, nomrr, pvvar, xirr	
References	Brealey and Myers, <i>Principles of Corporate Finance</i> , Chapter 5	

month

Purpose	Month of date
Syntax	<code>[MonthNum, MonthString] = month(Date)</code>
Description	<code>[MonthNum, MonthString] = month(Date)</code> returns the month in numeric and string form given a serial date number or a date string.
Examples	<pre>[MonthNum, MonthString] = month(730368) or [MonthNum, MonthString] = month('05-Sep-1999')</pre> returns <pre>MonthNum = 9 MonthString = Sep</pre>
See Also	<code>datevec</code> , <code>day</code> , <code>year</code>

Purpose	Number of whole months between dates
Syntax	<code>Months = months(StartDate, EndDate, EndMonthFlag)</code>
Arguments	StartDate Enter as serial date numbers or date strings. EndDate Enter as serial date numbers or date strings. EndMonthFlag (Optional) end-of-month flag. If StartDate and EndDate are end-of-month dates and EndDate has fewer days than StartDate, EndMonthFlag = 1 (default) treats EndDate as the end of a whole month, while EndMonthFlag = 0 does not.
Description	<code>Months = months(StartDate, EndDate, EndMonthFlag)</code> returns the number of whole months between StartDate and EndDate. If EndDate is earlier than StartDate, Months is negative. Enter dates as serial date numbers or date strings. Any input argument can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an n-row character array of date strings or a single date. Months is then an n-by-1 vector of numbers.
Examples	<pre>Months = months('may 31 2000', 'jun 30 2000', 1) Months = 1 Months = months('may 31 2000','jun 30 2000', 0) Months = 0 Dates = ['mar 31 2002'; 'apr 30 2002'; 'may 31 2002']; Months = months(Dates, 'jun 30 2002') Months = 3 2 1</pre>
See Also	<code>yearfrac</code>

movavg

Purpose	Leading and lagging moving averages chart	
Syntax	<code>movavg(Asset, Lead, Lag, Alpha)</code> <code>[Short, Long] = movavg(Asset, Lead, Lag, Alpha)</code>	
Arguments	Asset	Security data, usually a vector of time-series prices.
	Lead	Number of samples to use in leading average calculation. A positive integer. Lead must be less than or equal to Lag.
	Lag	Number of samples to use in the lagging average calculation. A positive integer.
	Alpha	(Optional) Control parameter that determines the type of moving averages. 0 = simple moving average (default), 0.5 = square root weighted moving average, 1 = linear moving average, 2 = square weighted moving average, etc. To calculate the exponential moving average, set Alpha = 'e'.
Description	<code>movavg(Asset, Lead, lag, Alpha)</code> plots leading and lagging moving averages. <code>[Short, Long] = movavg(Asset, Lead, lag, Alpha)</code> returns the leading Short and lagging Long moving average data without plotting it.	
Examples	If Asset is a vector of stock price data <code>movavg(Asset, 3, 20, 1)</code> plots linear three-sample leading and 20-sample lagging moving averages.	
See Also	<code>bolling</code> , <code>candle</code> , <code>dateaxis</code> , <code>highlow</code> , <code>pointfig</code>	

Purpose	Nominal rate of return
Syntax	<code>Return = nomrr(Rate, NumPeriods)</code>
Arguments	<code>Rate</code> Effective annual percentage rate. Enter as a decimal fraction. <code>NumPeriods</code> Number of compounding periods per year, an integer.
Description	<code>Return = nomrr(Rate, NumPeriods)</code> calculates the nominal rate of return.
Examples	To find the nominal annual rate of return based on an effective annual percentage rate of 9.38% compounded monthly <pre>Return = nomrr(0.0938, 12)</pre> returns <pre>Return = 0.0900 (9.0%)</pre>
See Also	<code>effrr, irr, mirr, taxedrr, xirr</code>

now

Purpose Current date and time

Syntax Datenum = now

Description Datenum = now returns the current date and time as a serial date number.

Note This function now ships with basic MATLAB. It originally shipped only with the Financial Toolbox. This description remains here for your convenience.

Examples

```
Datenum = now

Datenum =

    730695.5942469908 (on July 28, 2000 at 2:15 PM)
```

See Also date, datenum, today

Purpose	Date of specific occurrence of weekday in month														
Syntax	<code>Date = nweekdate(n, Weekday, Year, Month, Same)</code>														
Arguments	n Nth occurrence of the weekday in a month. Enter as integer from 1 through 5. Weekday Weekday whose date you seek. Enter as integer from 1 through 7. <table> <tr><td>1</td><td>Sunday</td></tr> <tr><td>2</td><td>Monday</td></tr> <tr><td>3</td><td>Tuesday</td></tr> <tr><td>4</td><td>Wednesday</td></tr> <tr><td>5</td><td>Thursday</td></tr> <tr><td>6</td><td>Friday</td></tr> <tr><td>7</td><td>Saturday</td></tr> </table> Year Year. Enter as a four-digit integer. Month Month. Enter as an integer from 1 through 12. Same (Optional) Weekday that must occur in the same week with Weekday. Enter as an integer from 0 through 7, where 0 = ignore (default) and 1 through 7 are as for Weekday.	1	Sunday	2	Monday	3	Tuesday	4	Wednesday	5	Thursday	6	Friday	7	Saturday
1	Sunday														
2	Monday														
3	Tuesday														
4	Wednesday														
5	Thursday														
6	Friday														
7	Saturday														
Description	<code>Date = nweekdate(n, Weekday, Year, Month, Same)</code> returns the serial date number for the specific occurrence of the weekday in the given year and month, and in a week that also contains the weekday Same. If <code>n</code> is larger than the last occurrence of Weekday, <code>Date = 0</code>. Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if <code>Year</code> is a 1-by-<code>n</code> vector of integers, then <code>Month</code> must be a 1-by-<code>n</code> vector of integers or a single integer. <code>Date</code> is then a 1-by-<code>n</code> vector of date numbers. Use the function <code>datestr</code> to convert serial date numbers to formatted date strings.														

nweekdate

Examples

To find the first Thursday in May 2001:

```
Date = nweekdate(1, 5, 2001, 5); datestr(Date)
ans =
03-May-2001
```

To find the first Thursday in a week that also contains a Wednesday in May 2001:

```
Date = nweekdate(2, 5, 2001, 5, 4); datestr(Date)
ans =
10-May-2001
```

To find the third Monday in February for 2001, 2002, and 2003:

```
Year = [2001:2003];
Date = nweekdate(3, 2, Year, 2)

Date =
      730901      731265      731629

datestr(Date)

ans =

19-Feb-2001
18-Feb-2002
17-Feb-2003
```

See Also

fbusdate, lbusdate, lweekdate

Purpose Option profit

Syntax Profit = opprofit(AssetPrice, Strike, Cost, PosFlag, OptType)

Arguments

AssetPrice	Asset price.
Strike	Strike or exercise price.
Cost	Cost of the option.
PosFlag	Option position. 0 = long, 1 = short.
OptType	Option type. 0 = call option, 1 = put option.

Description Profit = opprofit(AssetPrice, Strike, Cost, PosFlag, OptType)
returns the profit of an option.

Examples Buying (going long on) a call option with a strike price of \$90 on an underlying asset with a current price of \$100 for a cost of \$4

```
Profit = opprofit(100, 90, 4, 0, 0)
```

returns

```
Profit =
    6.00
```

a profit of \$6 if the option is exercised under these conditions.

See Also binprice, blsprice

payadv

Purpose	Periodic payment given number of advance payments		
Syntax	Payment = payadv(Rate, NumPeriods, PresentValue, FutureValue, Advance)		
Arguments	Rate	Lending or borrowing rate per period. Enter as a decimal fraction. Must be greater than or equal to 0.	
	NumPeriods	Number of periods in the life of the instrument.	
	PresentValue	Present value of the instrument.	
	FutureValue	Future value or target value to be attained after NumPeriods periods.	
	Advance	Number of advance payments. If the payments are made at the beginning of the period, add 1 to Advance.	
Description	Payment = payadv(Rate, NumPeriods, PresentValue, FutureValue, Advance) returns the periodic payment given a number of advance payments.		
Examples	The present value of a loan is \$1000.00 and it will be paid in full in 12 months. The annual interest rate is 10% and three payments are made at closing time. Using this data		
	Payment = payadv(0.1/12, 12, 1000, 0, 3)		
	returns		
	Payment =		
	85.94		
	for the periodic payment.		
See Also	amortize, payodd, payper		

Purpose	Payment of loan or annuity with odd first period
Syntax	<code>Payment = payodd(Rate, NumPeriods, PresentValue, FutureValue, Days)</code>
Arguments	<code>rate</code> Interest rate per period. Enter as a decimal fraction. <code>NumPeriods</code> Number of periods in the life of the instrument. <code>PresentValue</code> Present value of the instrument. <code>FutureValue</code> Future value or target value to be attained after <code>NumPeriods</code> periods. <code>Days</code> Actual number of days until the first payment is made.

payper

Purpose	Periodic payment of loan or annuity
Syntax	<code>Payment = payper(Rate, NumPeriods, PresentValue, FutureValue, Due)</code>
Arguments	Rate Interest rate per period. Enter as a decimal fraction. NumPeriods Number of payment periods in the life of the instrument. PresentValue Present value of the instrument. FutureValue (Optional) Future value or target value to be attained after NumPeriods periods. Default = 0. Due (Optional) When payments are due: 0 = end of period (default), or 1 = beginning of period.
Description	<code>Payment = payper(Rate, NumPeriods, PresentValue, FutureValue, Due)</code> returns the periodic payment of a loan or annuity.
Examples	Find the monthly payment for a three-year loan of \$9000 with an annual interest rate of 11.75% <pre>Payment = payper(0.1175/12, 36, 9000, 0, 0)</pre> returns <pre>Payment =</pre> <pre>297.86</pre>
See Also	<code>amortize</code> , <code>fvfix</code> , <code>payadv</code> , <code>payodd</code> , <code>pvfix</code>

Purpose	Uniform payment equal to varying cash flow	
Syntax	Series = payuni(CashFlow, Rate)	
Arguments	CashFlow	A vector of varying cash flows. Include the initial investment as the initial cash flow value (a negative number).
	Rate	Periodic interest rate. Enter as a decimal fraction.
Description	Series = payuni(CashFlow, Rate) returns the uniform series value of a varying cash flow.	
Examples	This cash flow represents the yearly income from an initial investment of \$10,000. The annual interest rate is 8%.	
	Year 1	\$2000
	Year 2	\$1500
	Year 3	\$3000
	Year 4	\$3800
	Year 5	\$5000
	To calculate the uniform series value	
	Series = payuni([-10000 2000 1500 3000 3800 5000], 0.08)	
	returns	
	Series =	
	429.63	
See Also	fvfix, fvvar, irr, pvfix, pvvar	

pcalims

Purpose Linear inequalities for individual asset allocation

Syntax [A,b] = pcalims(AssetMin, AssetMax, NumAssets)

Arguments

AssetMin	Scalar or NASSETS vector of minimum allocations in each asset. NaN indicates no constraint.
AssetMax	Scalar or NASSETS vector of maximum allocations in each asset. NaN indicates no constraint.
NumAssets	(Optional) Number of assets. Default = length of AssetMin or AssetMax.

Description [A,b] = pcalims(AssetMin, AssetMax, NumAssets) specifies the lower and upper bounds of portfolio allocations in each of NumAssets available asset investments.

A is a matrix and b a vector such that $A * PortWts' \leq b$, where PortWts is a 1-by-NASSETS vector of asset allocations.

If pcalims is called with fewer than two output arguments, the function returns A concatenated with b [A,b].

Examples Set the minimum weight in every asset to 0 (no short-selling), and set the maximum weight of IBM to 0.5 and CSCO to 0.8, while letting the maximum weight in INTC float.

Asset	IBM	INTC	CSCO
Min. Wt.	0	0	0
Max. Wt.	0.5		0.8

```
AssetMin = 0
AssetMax = [0.5 NaN 0.8]
[A,b] = pcalims(AssetMin, AssetMax)
```

```
A =
    1     0     0
    0     0     1
   -1     0     0
    0    -1     0
    0     0    -1
```

```
b =
    0.5000
    0.8000
         0
         0
         0
```

Portfolio weights of 50% in IBM and 50% in INTC satisfy the constraints.
Set the minimum weight in every asset to 0 and the maximum weight to 1.

Asset	IBM	INTC	CSCO
Min. Wt.	0	0	0
Max. Wt.	1	1	1

```
AssetMin = 0
AssetMax = 1
NumAssets = 3
```

pcalims

```
[A,b] = pcalims(AssetMin, AssetMax, NumAssets)
```

```
A =
```

```
    1    0    0
    0    1    0
    0    0    1
   -1    0    0
    0   -1    0
    0    0   -1
```

```
b =
```

```
    1
    1
    1
    0
    0
    0
```

Portfolio weights of 50% in IBM and 50% in INTC satisfy the constraints.

See Also

pcgcomp, pcglims, pcpval, portcons, portopt

Purpose Linear inequalities for asset group comparison constraints

Syntax `[A,b] = pcgcomp(GroupA, AtoBmin, AtoBmax, GroupB)`

Arguments

GroupA	Number of groups (NGROUPS) by number of assets (NASSETS)
GroupB	specifications of groups to compare. Each row specifies a group. For a specific group, $\text{Group}(i,j) = 1$ if the group contains asset j ; otherwise, $\text{Group}(i,j) = 0$.
AtoBmin	Scalar or NGROUPS-long vectors of minimum and maximum
AtoBmax	ratios of allocations in GroupA to allocations in GroupB. NaN indicates no constraint between the two groups. Scalar bounds are applied to all group pairs. The total number of assests allocated to GroupA divided by the total number of assests allocated to GroupB is $\geq \text{AtoBmin}$ and $\leq \text{AtoBmax}$.

Description `[A,b] = pcgcomp(GroupA, AtoBmin, AtoBmax, GroupB)` specifies that the ratio of allocations in one group to allocations in another group is at least AtoBmin to 1 and at most AtoBmax to 1. Comparisons can be made between an arbitrary number of group pairs NGROUPS comprising subsets of NASSETS available investments.

A is a matrix and b a vector such that $A \cdot \text{PortWts}' \leq b$, where PortWts is a 1-by-NASSETS vector of asset allocations.

If pcgcomp is called with fewer than two output arguments, the function returns A concatenated with b `[A,b]`.

Examples

Asset	INTC	XON	RD
Region	North America	North America	Europe
Sector	Technology	Energy	Energy

Group	Min. Exposure	Max. Exposure
North America	0.30	0.75
Europe	0.10	0.55
Technology	0.20	0.50
Energy	0.20	0.80

Make the North American energy sector compose exactly 20% of the North American investment.

```
%          INTC  XON  RD
GroupA = [   0    1   0  ]; % North American Energy
GroupB = [   1    1   0  ]; % North America

AtoBmin = 0.20;
AtoBmax = 0.20;

[A,b] = pcgcomp(GroupA, AtoBmin, AtoBmax, GroupB)

A =

    0.2000    -0.8000     0
   -0.2000     0.8000     0

b =

    0
    0
```

Portfolio weights of 40% for INTC, 10% for XON, and 50% for RD satisfy the constraints.

See Also pcalims, pcglims, pcpval, portcons, portopt

Purpose Linear inequalities for asset group minimum and maximum allocation

Syntax `[A,b] = pcglims(Groups, GroupMin, GroupMax)`

Arguments

Groups	Number of groups (NGROUPS) by number of assets (NASSETS) specifications of which assets belong to which group. Each row specifies a group. For a specific group, $\text{Group}(i,j) = 1$ if the group contains asset j ; otherwise, $\text{Group}(i,j) = 0$.
GroupMin	Scalar or NGROUPS-long vectors of minimum and maximum
GroupMax	combined allocations in each group. NaN indicates no constraint. Scalar bounds are applied to all groups.

Description `[A,b] = pcglims(Groups, GroupMin, GroupMax)` specifies minimum and maximum allocations to groups of assets. An arbitrary number of groups, NGROUPS, comprising subsets of NASSETS investments, is allowed.

A is a matrix and b a vector such that $A \cdot \text{PortWts}' \leq b$, where PortWts is a 1-by-NASSETS vector of asset allocations.

If pcglims is called with fewer than two output arguments, the function returns A concatenated with b `[A,b]`.

Examples

Asset	INTC	XON	RD
Region	North America	North America	Europe
Sector	Technology	Energy	Energy

Group	Min. Exposure	Max. Exposure
North America	0.30	0.75
Europe	0.10	0.55
Technology	0.20	0.50
Energy	0.50	0.50

Set the minimum and maximum investment in various groups.

```
%      INTC  XON  RD
Groups = [  1   1   0 ; % North America
           0   0   1 ; % Europe
           1   0   0 ; % Technology
           0   1   1 ]; % Energy
```

```
GroupMin = [0.30
            0.10
            0.20
            0.50];
```

```
GroupMax = [0.75
            0.55
            0.50
            0.50];
```

```
[A,b] = pcglims(Groups, GroupMin, GroupMax)
```

A =

```
-1    -1     0
 0     0    -1
-1     0     0
 0    -1    -1
 1     1     0
 0     0     1
 1     0     0
 0     1     1
```


b =

```
-0.3000  
-0.1000  
-0.2000  
-0.5000  
0.7500  
0.5500  
0.5000  
0.5000
```

Portfolio weights of 50% in INTC, 25% in XON, and 25% in RD satisfy the constraints.

See Also

pcalims, pcgcomp, pcpval, portcons, portopt

pcpval

Purpose Linear inequalities for fixing total portfolio value

Syntax `[A,b] = pcpval(PortValue, NumAssets)`

Arguments

PortValue	Scalar total value of asset portfolio (sum of the allocations in all assets). PortValue = 1 specifies weights as fractions of the portfolio and return and risk numbers as rates instead of value.
NumAssets	Number of available asset investments.

Description `[A,b] = pcpval(PortValue, NumAssets)` scales the total value of a portfolio of NumAssets assets to PortValue. All portfolio weights, bounds, return, and risk values except ExpReturn and ExpCovariance (see portopt) are in terms of PortValue.

A is a matrix and b a vector such that $A \cdot \text{PortWts}' \leq b$, where PortWts is a 1-by-NASSETS vector of asset allocations.

If pcpval is called with fewer than two output arguments, the function returns A concatenated with b [A,b].

Examples Scale the value of a portfolio of three assets to 1, so all return values are rates and all weight values are in fractions of the portfolio.

```
PortValue = 1;  
NumAssets = 3;
```

```
[A,b] = pcpval(PortValue, NumAssets)
```

```
A =
```

```
    1    1    1  
   -1   -1   -1
```

```
b =
```

```
    1  
   -1
```

Portfolio weights of 40%, 10%, and 50% in the three assets satisfy the constraints.

See Also

`pcalims`, `pcgcomp`, `pcglims`, `portcons`, `portopt`

pointfig

Purpose	Point and figure chart
Syntax	<code>pointfig(Asset)</code>
Description	<code>pointfig(Asset)</code> plots a point and figure chart for a vector of price data <code>Asset</code> . Upward price movements are plotted as X's and downward price movements are plotted as O's.
See Also	<code>bolling</code> , <code>candle</code> , <code>dateaxis</code> , <code>highlow</code> , <code>movavg</code>

Purpose	Optimal capital allocation	
Syntax	<pre>[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn, PortWts, RisklessRate, BorrowRate, RiskAversion)</pre>	
Arguments	PortRisk	Standard deviation of each portfolio. A number of portfolios (NPORTS) by 1 vector.
	PortReturn	Expected return of each portfolio. An NPORTS-by-1 vector.
	PortWts	Weights allocated to each asset. An NPORTS by number of assets (NASSETS) matrix of weights allocated to each asset. Each row represents a different portfolio. Total of all weights in a portfolio is 1.
	RisklessRate	Risk-free rate. A decimal number.
	BorrowRate	(Optional) Borrowing rate. A decimal number. If borrowing is not desired, or not an option, set to NaN (default)
	RiskAversion	(Optional) Coefficient of investor's degree of risk aversion. Higher numbers indicate greater risk aversion. Typical coefficients range between 2.0 and 4.0 (Default = 3).
Description	[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn, PortWts, RisklessRate, BorrowRate, RiskAversion) computes the optimal risky portfolio, and the optimal allocation of funds between the risky portfolio and the risk-free asset. RiskyRisk is the standard deviation of the optimal risky portfolio. RiskyReturn is the expected return of the optimal risky portfolio. RiskyWts is a 1-by-NASSETS vector of weights allocated to the optimal risky portfolio. The total of all weights in the portfolio is 1. RiskyFraction is the fraction of the complete portfolio allocated in the risky portfolio. OverallRisk is the standard deviation of the optimal overall portfolio. OverallReturn is the expected rate of return of the optimal overall portfolio.	

Examples

Generate the efficient frontier from the asset data.

```
ExpReturn = [0.1 0.2 0.15];

ExpCovariance = [0.005   -0.010   0.004
                 -0.010   0.040  -0.002
                 0.004   -0.002   0.023];

[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance);
```

Find the optimal risky portfolio and allocate capital. The risk free investment return is 8%, and the borrowing rate is 12%.

```
RisklessRate = 0.08;
BorrowRate   = 0.12;
RiskAversion = 3;

[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, ...
OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn,...
PortWts, RisklessRate, BorrowRate, RiskAversion)

RiskyRisk =

    0.1283

RiskyReturn =

    0.1788

RiskyWts =

    0.0265    0.6023    0.3712

RiskyFraction =

    1.1898

OverallRisk =

    0.1527
```

OverallReturn =

0.1899

See Also frontcon, portrand, portstats

References Bodie, Kane, and Marcus, *Investments*, Chapters 6 and 7.

portcons

Purpose Portfolio constraints

Syntax ConSet = portcons(varargin)

Description Using linear inequalities, portcons generates a matrix of constraints for a portfolio of asset investments. The matrix ConSet is defined as $\text{ConSet} = [A \ b]$. A is a matrix and b a vector such that $A \cdot \text{PortWts}' \leq b$ sets the value, where PortWts is a 1 by number of assets (NASSETS) vector of asset allocations.

`ConSet = portcons('ConstType', Data1, ..., DataN)` creates a matrix ConSet, based on the constraint type ConstType, and the constraint parameters Data1, ..., DataN.

`ConSet = portcons('ConstType1', Data11, ..., Data1N, 'ConstType2', Data21, ..., Data2N, ...)` creates a matrix ConSet, based on the constraint types ConstTypeN, and the corresponding constraint parameters DataN1, ..., DataNN.

Constraint Type	Description	Values
Default	All allocations are ≥ 0 ; no short selling allowed. Combined value of portfolio allocations normalized to 1.	NumAssets (required). Scalar representing number of assets in portfolio.
PortValue	Fix total value of portfolio to PVal.	PVal (required). Scalar representing total value of portfolio. NumAssets (required). Scalar representing number of assets in portfolio. See pcpval.

Constraint Type	Description	Values
AssetLims	Minimum and maximum allocation per asset.	AssetMin (required). Scalar or vector of length NASSETS, specifying minimum allocation per asset. AssetMax (required). Scalar or vector of length NASSETS, specifying maximum allocation per asset. NumAssets (optional). See pcalims.
GroupLims	Minimum and maximum allocations to asset group.	Groups (required). NGROUPS-by-NASSETS matrix specifying which assets belong to each group. GroupMin (required). Scalar or a vector of length NGROUPS, specifying minimum combined allocations in each group. GroupMax (required). Scalar or a vector of length NGROUPS, specifying maximum combined allocations in each group. See pcglims.

Constraint Type	Description	Values
GroupComparison	Group-to-group comparison constraints.	GroupA (required). GroupB (required). NGROUPS-by-NASSETS matrices specifying groups to compare. AtoBmin (required). Scalar or vector of length NGROUPS specifying minimum ratios of allocations in GroupA to allocations in GroupB. AtoBmax (required). Scalar or vector of length NGROUPS specifying maximum ratios of allocations in GroupA to allocations in GroupB. See pcgcomp .
Custom	Custom linear inequality constraints $A*PortWts' \leq b$.	A (required). NCONSTRAINTS-by-NASSETS matrix, specifying weights for each asset in each inequality equation. b (required). Vector of length NCONSTRAINTS specifying the right hand sides of the inequalities.

Examples

Constrain a portfolio of three assets:

Asset	IBM	CPQ	XON
Group	A	A	B
Min. Wt.	0	0	0
Max. Wt.	0.5	0.9	0.8

```
NumAssets = 3;
PVal = 1; % Scale portfolio value to 1.
AssetMin = 0;
AssetMax = [0.5 0.9 0.8];
GroupA = [1 1 0];
GroupB = [0 0 1];
AtoBmax = 1.5 % Value of assets in Group A at most 1.5 times value
              % in group B.
```

```
ConSet = portcons('PortValue', PVal, NumAssets, 'AssetLims', ...
AssetMin, AssetMax, NumAssets, 'GroupComparison', GroupA, NaN, ...
AtoBmax, GroupB)
```

ConSet =

```
1.0000    1.0000    1.0000    1.0000
-1.0000   -1.0000   -1.0000   -1.0000
1.0000         0         0    0.5000
         0    1.0000         0    0.9000
         0         0    1.0000    0.8000
-1.0000         0         0         0
         0   -1.0000         0         0
         0         0   -1.0000         0
1.0000    1.0000   -1.5000         0
```

Portfolio weights of 30% in IBM, 30% in CPQ, and 40% in XON satisfy the constraints.

See Also

pcalims, pcgcomp, pcglims, pcpval, portopt

portopt

Purpose	Portfolios on constrained efficient frontier	
Syntax	<code>[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, NumPorts, PortReturn, ConSet)</code>	
Arguments	<code>ExpReturn</code>	1 by number of assets (NASSETS) vector specifying the expected (mean) return of each asset.
	<code>ExpCovariance</code>	NASSETS-by-NASSETS matrix specifying the covariance of the asset returns.
	<code>NumPorts</code>	(Optional) Number of portfolios generated along the efficient frontier. Returns are equally spaced between the maximum possible return and the minimum risk point. If <code>NumPorts</code> is empty (entered as <code>[]</code>), computes 10 equally spaced points.
	<code>PortReturn</code>	(Optional) Expected return of each portfolio. A number of portfolios (NPORTS) by 1 vector. If not entered or empty, <code>NumPorts</code> equally spaced returns between the minimum and maximum possible values are used.
	<code>ConSet</code>	(Optional) Constraint matrix for a portfolio of asset investments, created using <code>portcons</code> . If not specified, a default is created.

Description

`[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, NumPorts, PortReturn, ConSet)` returns the mean-variance efficient frontier with user-specified covariance, returns, and asset constraints (`ConSet`). Given a collection of NASSETS risky assets, computes a portfolio of asset investment weights that minimize the risk for given values of the expected return. The portfolio risk is minimized subject to constraints on the total portfolio value, the individual asset minimum and maximum allocation, the asset group minimum and maximum allocation, or the asset group-to-group comparison.

`PortRisk` is an NPORTS-by-1 vector of the standard deviation of each portfolio.

`PortReturn` is an NPORTS-by-1 vector of the expected return of each portfolio.

`PortWts` is an NPORTS-by-NASSETS matrix of weights allocated to each asset. Each row represents a portfolio. The total of all weights in a portfolio is 1.

If portopt is invoked without output arguments, it returns a plot of the efficient frontier.

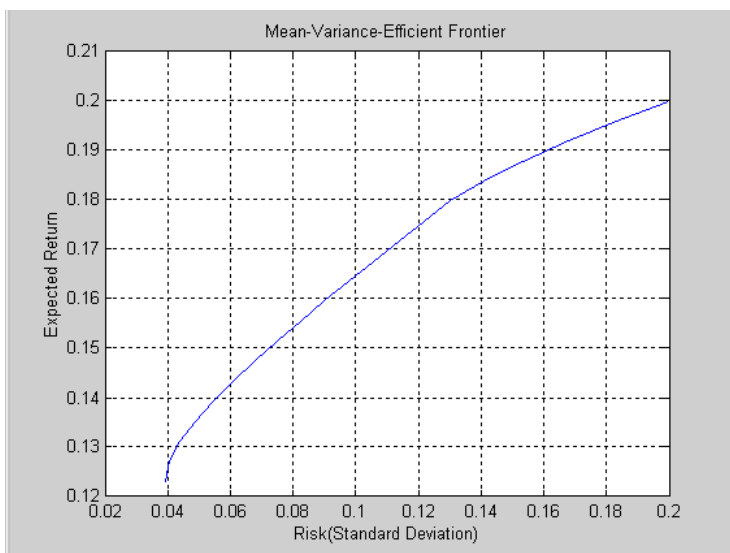
Examples

Plot the risk-return efficient frontier of portfolios allocated among three assets. Connect 20 portfolios along the frontier having evenly spaced returns. By default, choose among portfolios without short-selling and scale the value of the portfolio to 1.

```
ExpReturn = [0.1 0.2 0.15];

ExpCovariance = [0.005    -0.010    0.004
                 -0.010    0.040   -0.002
                 0.004   -0.002    0.023];

NumPorts = 20;
portopt(ExpReturn, ExpCovariance, NumPorts)
```



Return the two efficient portfolios that have returns of 16% and 17%. Limit to portfolios that have at least 20% of the allocation in the first asset, and cap the total value in the first and third assets at 50% of the portfolio.

```
ExpReturn = [0.1 0.2 0.15];

ExpCovariance = [0.005   -0.010   0.004
                 -0.010   0.040  -0.002
                 0.004   -0.002   0.023];

PortReturn = [0.16
              0.17];

NumAssets = 3;

AssetMin = [0.20 NaN NaN];

Group      = [1    0    1];

GroupMax = 0.50;

ConSet = portcons('Default', NumAssets, 'AssetLims', AssetMin, ...
                  NaN, 'GroupLims', Group, NaN, GroupMax);

[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ...
ExpCovariance, [], PortReturn, ConSet)

PortRisk =

    0.0919
    0.1138

PortReturn =

    0.1600
    0.1700

PortWts =

    0.3000    0.5000    0.2000
    0.2000    0.6000    0.2000
```

See Also

ewstats, frontcon, portcons, portstats

Purpose	Randomized portfolio risks, returns, and weights	
Syntax	<pre>[PortRisk, PortReturn, PortWts] = portrand(Asset, Return, Points) portrand(Asset, Return, Points)</pre>	
Arguments	Asset	Matrix of time series data. Each row is an observation and each column represents a single security.
	Return	(Optional) Row vector where each column represents the rate of return for the corresponding security in Asset. By default, Return is computed by taking the average value of each column of Asset.
	Points	(Optional) Scalar that specifies how many random points should be generated. Default = 1000.
Description	<pre>[PortRisk, PortReturn, PortWts] = portrand(Asset, Return, Points)</pre> returns the risks, rates of return, and weights of random portfolio configurations.	
	PortRisk	Points-by-1 vector of standard deviations.
	PortReturn	Points-by-1 vector of expected rates of return.
	PortWts	Points by number of securities matrix of asset weights. Each row of PortWts is a different portfolio configuration.
	<pre>portrand(Asset, Return, Points)</pre> plots the points representing each portfolio configuration. It does not return any data to the MATLAB workspace.	
See Also	frontcon	
References	Bodie, Kane, and Marcus, <i>Investments</i> , Chapter 7.	

portsim

Purpose	Random simulation of correlated asset returns	
Syntax	<code>RetSeries = portsim(ExpReturn, ExpCovariance, NumObs, RetIntervals, NumSim)</code>	
Arguments	<code>ExpReturn</code>	1 by number of assets (NASSETS) vector specifying the expected (mean) return of each asset.
	<code>ExpCovariance</code>	NASSETS-by-NASSETS matrix of asset-asset covariances. The standard deviations of the returns are: <code>ExpSigma = sqrt(diag(ExpCovariance))</code> .
	<code>NumObs</code>	(Optional) Number of consecutive observations in the return time series. If <code>NumObs</code> is entered as the empty matrix <code>[]</code> , the length of <code>RetIntervals</code> is used.
	<code>RetIntervals</code>	(Optional) Scalar or number of observations (NUMOBS) by 1 vector of interval times between observations. If <code>RetIntervals</code> is not specified, all intervals are assumed to have length 1.
	<code>NumSim</code>	(Optional) Number of separate simulations of the NUMOBS observations to perform. Default = 1.

Description

`portsim` simulates returns of NASSETS assets over consecutive observation intervals. Returns are simulated as the increments of constant drift and volatility Brownian processes.

`RetSeries` is a NUMOBS-by-NASSETS-by-NUMSIM array of incremental return observations. The return over an interval of length `DT` is given by `ExpReturn*DT + ExpSigma*sqrt(DT)*randn`, where `randn` provides a random scalar whose value changes each time `randn` is referenced.

The returns realized from portfolios listed in `PortWts` are given by:
`PortReturn = PortWts * RetSeries(:, :, 1)'`, where `PortWts` is a matrix in which each row contains the asset allocations of a portfolio. Each row of `PortReturn` corresponds to one of the portfolios identified in `PortWts`, and each column corresponds to one of the observations in `RetSeries`. See `portopt` and `portstats` for portfolio specification and optimization.

Examples

Create sample returns for three stocks over 10 periods.

```
ExpReturn = [0.1 0.2 0.15];

ExpCovariance = [0.005   -0.010   0.004
                 -0.010   0.040  -0.002
                  0.004   -0.002   0.023];

NumObs = 10;

RetSeries = portsim(ExpReturn, ExpCovariance, NumObs)

RetSeries =
```

0.1429	0.2626	0.2365
0.0821	0.1599	-0.1796
0.0054	0.6126	0.1072
0.1719	-0.0669	0.1913
0.1518	-0.0843	0.0442
0.0112	0.2709	0.1501
0.0409	0.1683	0.1932
0.1485	0.2522	0.2774
0.0463	0.3222	0.0954
0.1990	0.1024	0.3843

Note RetSeries is different each time this example is executed. The portsim function uses random number generation.

See Also

ewstats, portopt, portstats, randn, ret2tick

portstats

Purpose Portfolio expected return and risk

Syntax [PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance, PortWts)

Arguments

ExpReturn	1 by number of assets (NASSETS) vector specifying the expected (mean) return of each asset.
ExpCovariance	NASSETS-by-NASSETS matrix specifying the covariance of the asset returns.
PortWts	(Optional) Number of portfolios (NPORTS) by NASSETS matrix of weights allocated to each asset. Each row represents a different weighting combination. Default = 1/NASSETS (equally weighted).

Description [PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance, PortWts) computes the expected rate of return and risk for a portfolio of assets.

PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.

PortReturn is an NPORTS-by-1 vector of the expected return of each portfolio.

Examples

```
ExpReturn = [0.1 0.2 0.15];
```

```
ExpCovariance = [0.0100    -0.0061    0.0042  
                -0.0061    0.0400   -0.0252  
                0.0042   -0.0252    0.0225 ];
```

```
PortWts=[0.4 0.2 0.4; 0.2 0.4 0.2];
```

```
[PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance,...  
PortWts)
```

```
PortRisk =
```

```
0.0560  
0.0550
```

PortReturn =

0.1400

0.1300

See Also

frontcon

portvrisk

Purpose Portfolio value at risk

Syntax `ValueAtRisk = portvrisk(PortReturn, PortRisk, RiskThreshold, PortValue)`

Arguments

<code>PortReturn</code>	Number of portfolios (NPORTS) by 1 vector or scalar of the expected return of each portfolio over the period.
<code>PortRisk</code>	NPORTS-by-1 vector or scalar of the standard deviation of each portfolio over the period.
<code>RiskThreshold</code>	(Optional) NPORTS-by-1 vector or scalar specifying the loss probability. Default = 0.05 (5%).
<code>PortValue</code>	(Optional) NPORTS-by-1 vector or scalar specifying the total value of asset portfolio. Default = 1.

Description `ValueAtRisk = portvrisk(PortReturn, PortRisk, RiskThreshold, PortValue)` returns the maximum potential loss in the value of a portfolio over one period of time, given the loss probability level `RiskThreshold`.

`ValueAtRisk` is an NPORTS-by-1 vector of the estimated maximum loss in the portfolio, predicted with a confidence probability of 1- `RiskThreshold`.

If `PortValue` is not given, `ValueAtRisk` is presented on a per-unit basis. A value of 0 indicates no losses.

Examples This example computes `ValueAtRisk` on a per-unit basis.

```
PortReturn = 0.29/100;  
PortRisk = 3.08/100;  
RiskThreshold = [0.01;0.05;0.10];  
PortValue = 1;  
ValueAtRisk = portvrisk(PortReturn,PortRisk,...  
RiskThreshold,PortValue)  
ValueAtRisk =  
  
    0.0688  
    0.0478  
    0.0366
```

This example computes ValueAtRisk with actual values.

```
PortReturn = [0.29/100;0.30/100];
PortRisk = [3.08/100;3.15/100];
RiskThreshold = 0.10;
PortValue = [1000000000;5000000000];
ValueAtRisk = portvrisk(PortReturn,PortRisk,...
RiskThreshold,PortValue)
ValueAtRisk =

    1.0e+007 *
    3.6572
    1.8684
```

See Also

frontcon, portopt

Purpose	Price bonds in a portfolio by a set of zero curves													
Syntax	BondPrices = prbyzero(Bonds, Settle, ZeroRates, ZeroDates)													
Arguments	Bonds	Coupon bond information used to compute prices. A number of bonds (NUMBONDS) by 6 matrix where each row describes a bond. The first two columns are required; the rest are optional but must be added in order. All rows in Bonds must have the same number of columns. Columns are [Maturity CouponRate Face Period Basis EndMonthRule] where: <table><tr><td>Maturity</td><td>Maturity date as a serial date number or date string</td></tr><tr><td>CouponRate</td><td>Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond</td></tr><tr><td>Face</td><td>(Optional) Face or par value of the bond. Default = 100.</td></tr><tr><td>Period</td><td>(Optional) Coupons per year of the bond. Allowed values are 0,1, 2, 3, 4, 6, and 12. Default = 2.</td></tr><tr><td>Basis</td><td>(Optional) Day-count basis of the bond. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.</td></tr><tr><td>EndMonthRule</td><td>(Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.</td></tr></table>	Maturity	Maturity date as a serial date number or date string	CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond	Face	(Optional) Face or par value of the bond. Default = 100.	Period	(Optional) Coupons per year of the bond. Allowed values are 0,1, 2, 3, 4, 6, and 12. Default = 2.	Basis	(Optional) Day-count basis of the bond. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.	EndMonthRule	(Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
Maturity	Maturity date as a serial date number or date string													
CouponRate	Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond													
Face	(Optional) Face or par value of the bond. Default = 100.													
Period	(Optional) Coupons per year of the bond. Allowed values are 0,1, 2, 3, 4, 6, and 12. Default = 2.													
Basis	(Optional) Day-count basis of the bond. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.													
EndMonthRule	(Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.													
	Settle	Serial date number of the settlement date.												

ZeroRates NUMDATES-by-NUMCURVES matrix of observed zero rates, as decimal fractions. Each column represents a rate curve. Each row represents an observation date.

ZeroDates NUMDATES-by-1 column of dates for observed zeros

Description

`BondPrices = prbyzero(Bonds, Settle, ZeroRates, ZeroDates)` computes the bond prices in a portfolio using a set of zero curves.

`BondPrices` is a NUMBONDS-by-NUMCURVES matrix of clean bond prices. Each column is derived from the corresponding zero curve in `ZeroRates`.

Examples

This example uses `zbtprice` to compute a zero curve given a portfolio of coupon bonds and their prices. It then reverses the process, using the zero curve as input to `prbyzero` to compute the prices.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
          datenum('7/1/2000') 0.06 100 2 0 0;
          datenum('7/1/2000') 0.09375 100 6 1 0;
          datenum('6/30/2001') 0.05125 100 1 3 1;
          datenum('4/15/2002') 0.07125 100 4 1 0;
          datenum('1/15/2000') 0.065 100 2 0 0;
          datenum('9/1/1999') 0.08 100 3 3 0;
          datenum('4/30/2001') 0.05875 100 2 0 0;
          datenum('11/15/1999') 0.07125 100 2 0 0;
          datenum('6/30/2000') 0.07 100 2 3 1;
          datenum('7/1/2001') 0.0525 100 2 3 0;
          datenum('4/30/2002') 0.07 100 2 0 0];
```

```
Prices = [ 99.375;
           99.875;
           105.75 ;
           96.875;
           103.625;
           101.125;
           103.125;
           99.375;
           101.0 ;
           101.25 ;
```

```
96.375;  
102.75 ];
```

```
Settle = datenum('12/18/1997');
```

Set semi-annual compounding for the zero curve, on an actual/365 basis.
Derive the zero curve within 50 iterations.

```
OutputCompounding = 2;  
OutputBasis = 3;  
MaxIterations = 50;
```

Execute `zbtprice`

```
[ZeroRates, ZeroDates] = zbtprice(Bonds, Prices, Settle,...  
    OutputCompounding, OutputBasis, MaxIterations)
```

which returns the zero curve at the maturity dates.

```
ZeroRates =
```

```
0.0616  
0.0609  
0.0658  
0.0590  
0.0648  
0.0655  
0.0606  
0.0601  
0.0642  
0.0621  
0.0627
```

```
ZeroDates =
```

```
729907  
730364  
730439  
730500  
730667  
730668  
730971
```



```
731032
731033
731321
731336
```

Now execute prbyzero

```
BondPrices = prbyzero(Bonds, Settle, ZeroRates, ZeroDates)
```

which returns

```
BondPrices =
```

```
99.38
98.80
106.83
96.88
103.62
101.13
103.12
99.36
101.00
101.25
96.37
102.74
```

In this example zbtprice and prbyzero do not exactly reverse each other. Many of the bonds have the end-of-month rule off (`EndMonthRule = 0`). The rule subtly affects the time factor computation. If you set the rule on (`EndMonthRule = 1`) everywhere in the Bonds matrix, then prbyzero returns the original prices, except when the two incompatible prices fall on the same maturity date.

See Also

tr2bonds, zbtprice

prdisc

Purpose Price of discounted security

Syntax `Price = prdisc(Settle, Maturity, Face, Discount, Basis)`

Arguments

Settle	Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.
Maturity	Enter as serial date number or date string.
Face	Redemption (par, face) value.
Discount	Bank discount rate of the security. Enter as decimal fraction.
Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

Description `Price = prdisc(Settle, Maturity, Face, Discount, Basis)` returns the price of a security whose yield is quoted as a bank discount rate (e.g., U. S. Treasury Bills).

Examples Using this data

```
Settle = '10/14/2000';
Maturity = '03/17/2001';
Face = 100;
Discount = 0.087;
Basis = 2;

Price = prdisc(Settle, Maturity, Face, Discount, Basis)
```

returns

```
Price =

    96.2783
```

See Also `acrudisc`, `bndprice`, `discrate`, `prmat`, `ylldisc`

References Mayle, *Standard Securities Calculation Methods*, Volumes I-II, 3rd edition. Formula 2.

Purpose	Price with interest at maturity	
Syntax	<code>[Price, AccruInterest] = prmat(Settle, Maturity, Issue, Face, CouponRate, Yield, Basis)</code>	
Arguments	Settle	Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.
	Maturity	Enter as serial date number or date string.
	Issue	Enter as serial date number or date string.
	Face	Redemption (par, face) value.
	CouponRate	Enter as decimal fraction.
	Yield	Annual yield. Enter as decimal fraction.
	Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

Description `[Price, AccruInterest] = prmat(Settle, Maturity, Issue, Face, CouponRate, Yield, Basis)` returns the price and accrued interest of a security that pays interest at maturity. This function also applies to zero-coupon bonds or pure discount securities by setting `CouponRate = 0`.

Examples Using this data

```
Settle = '02/07/2002';
Maturity = '04/13/2002';
Issue = '10/11/2001';
Face = 100;
CouponRate = 0.0608;
Yield = 0.0608;
Basis = 1;
```

```
[Price, AccruInterest] = prmat(Settle, Maturity, Issue, Face,...
    CouponRate, Yield, Basis)
```

returns

```
Price =
    99.9784
```

prmat

AccruInterest =

1.9591

See Also

acrubond, acrudisc, bndprice, prdisc, yldmat

References

Mayle, *Standard Securities Calculation Methods*, Volumes I-II, 3rd edition.
Formula 4.

Purpose	Price of Treasury bill		
Syntax	Price = prtbill(Settle, Maturity, Face, Discount)		
Arguments	Settle	Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.	
	Maturity	Enter as serial date number or date string.	
	Face	Redemption (par, face) value.	
	Discount	Discount rate of the Treasury bill. Enter as decimal fraction.	
Description	Price = prtbill(Settle, Maturity, Face, Discount) returns the price for a Treasury bill.		
Examples	The settlement date of a Treasury bill is February 10, 2002, the maturity date is August 6, 2002, the discount rate is 3.77%, and the par value is \$1000. Using this data		
	<pre>Price = prtbill('2/10/2002', '8/6/2002', 1000, 0.0377)</pre> returns <pre>Price = 981.4642</pre>		
See Also	beytbill, yldtbill		
References	Bodie, Kane, and Marcus, <i>Investments</i> , pages 41-43.		

pvfix

Purpose	Present value with fixed periodic payments	
Syntax	<code>PresentVal = pvfix(Rate, NumPeriods, Payment, ExtraPayment, Due)</code>	
Arguments	<code>rate</code>	Periodic interest rate, as a decimal fraction.
	<code>NumPeriods</code>	Number of periods.
	<code>Payment</code>	Periodic payment.
	<code>ExtraPayment</code>	(Optional) Payment received other than <code>Payment</code> in the last period. Default = 0.
	<code>Due</code>	(Optional) When payments are due or made: 0 = end of period (default), or 1 = beginning of period.
Description	<code>PresentVal = pvfix(Rate, NumPeriods, Payment, ExtraPayment, Due)</code> returns the present value of a series of equal payments.	
Examples	\$200 is paid monthly into a savings account earning 6%. The payments are made at the end of the month for five years. To find the present value of these payments	
	<code>PresentVal = pvfix(0.06/12, 5*12, 200, 0, 0)</code>	
	returns	
	<code>PresentVal =</code> <code>10345.11</code>	
See Also	<code>fvfix</code> , <code>fvvar</code> , <code>payper</code> , <code>pvvar</code>	

Purpose	Present value of varying cash flow	
Syntax	<code>PresentVal = pvvar(CashFlow, Rate, IrrCFDates)</code>	
Arguments	CashFlow	A vector of varying cash flows. Include the initial investment as the initial cash flow value (a negative number).
	Rate	Periodic interest rate. Enter as a decimal fraction.
	IrrCFDates	(Optional) For irregular (nonperiodic) cash flows, a vector of dates on which the cash flows occur. Enter dates as serial date numbers or date strings. Default assumes CashFlow contains regular (periodic) cash flows.

Description `PresentVal = pvvar(CashFlow, Rate, IrrCFDates)` returns the net present value of a varying cash flow.

Examples This cash flow represents the yearly income from an initial investment of \$10,000. The annual interest rate is 8%.

Year 1	\$2000
Year 2	\$1500
Year 3	\$3000
Year 4	\$3800
Year 5	\$5000

To calculate the net present value of this regular cash flow

```
PresentVal = pvvar([-10000 2000 1500 3000 3800 5000], 0.08)
```

returns

```
PresentVal =  
  
1715.39
```

An investment of \$10,000 returns this irregular cash flow. The original investment and its date are included. The periodic interest rate is 9%.

Cash flow	Dates
(\$10000)	January 12, 1987
\$2500	February 14, 1988
\$2000	March 3, 1988
\$3000	June 14, 1988
\$4000	December 1, 1988

To calculate the net present value of this irregular cash flow

```
CashFlow = [-10000, 2500, 2000, 3000, 4000];  
IrrCFDates = ['01/12/1987'  
              '02/14/1988'  
              '03/03/1988'  
              '06/14/1988'  
              '12/01/1988'];  
  
PresentVal = pvvar(CashFlow, 0.09, IrrCFDates)
```

returns

```
PresentVal =  
  
          142.16
```

See Also

fvfix, fvvar, irr, payuni, pvfix

Purpose	Zero curve given a par yield curve	
Syntax	<pre>[ZeroRates, CurveDates] = pyld2zero(ParRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis, MaxIterations)</pre>	
Arguments	ParRates	Column vector of annualized implied par yield rates, as decimal fractions. (Par yields = coupon rates.) In aggregate, the yield rates in ParRates constitute an implied par yield curve for the investment horizon represented by CurveDates.
	CurveDates	Column vector of maturity dates (as serial date numbers) that correspond to the par rates.
	Settle	A serial date number that is the common settlement date for the par rates.
	OutputCompounding	(Optional) Output compounding. A scalar that sets the compounding frequency per year for annualizing the output zero rates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding 365 daily compounding -1 continuous compounding
	OutputBasis	(Optional) Output day-count basis for annualizing the output zero rates. Allowed values are: 0 actual/actual (default) 1 30/360 2 actual/360

	3	actual/365
InputCompounding	(Optional)	A scalar that indicates the compounding frequency per year used for annualizing the input par rates. Allowed values are the same as for OutputCompounding. Default = OutputCompounding.
InputBasis	(Optional)	Input day-count basis used for annualizing the par rates. Allowed values are the same as for OutputBasis. Default = OutputBasis.
MaxIterations	(Optional)	Maximum number of iterations for deriving the zero rates in ZeroRates. A scalar. Default = 200. A larger value may slow processing.

Description

[ZeroRates, CurveDates] = pyld2zero(ParRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis, MaxIterations) returns a zero curve given a par yield curve and its maturity dates.

ZeroRates Column vector of decimal fractions. In aggregate, the rates in ZeroRates constitute a zero curve for the investment horizon represented by CurveDates.

CurveDates Column vector of maturity dates (as serial date numbers) corresponding to the zero rates. This vector is the same as the input vector CurveDates.

Examples

Given a par yield curve over a set of maturity dates and a settlement date:

```
ParRates = [0.0479
            0.0522
            0.0540
            0.0540
            0.0536
            0.0532
            0.0532
            0.0539
            0.0558
            0.0543];
```

```
CurveDates = [datetime('06-Nov-2000')
              datetime('11-Dec-2000')
              datetime('15-Jan-2001')
              datetime('05-Feb-2001')
              datetime('04-Mar-2001')
              datetime('02-Apr-2001')
              datetime('30-Apr-2001')
              datetime('25-Jun-2001')
              datetime('04-Sep-2001')
              datetime('12-Nov-2001')];
```

```
Settle = datetime('03-Nov-2000');
```

Set monthly compounding for the zero curve, on an actual/365 basis. The par yield curve was compounded annually on an actual/actual basis. Derive the zero curve within 50 iterations.

```
OutputCompounding = 12;
OutputBasis = 3;
InputCompounding = 1;
InputBasis = 0;
MaxIterations = 50;
```

Execute the function

```
[ZeroRates, CurveDates] = pyld2zero(ParRates, CurveDates,...
Settle, OutputCompounding, OutputBasis, InputCompounding,...
InputBasis, MaxIterations)
```

which returns the zero curve ZeroRates at the maturity dates CurveDates.

```
ZeroRates =
    0.0461
    0.0498
    0.0519
    0.0520
    0.0510
    0.0510
    0.0508
    0.0520
    0.0543
    0.0530
```

```
CurveDates =
```

```
730796  
730831  
730866  
730887  
730914  
730943  
730971  
731027  
731098  
731167
```

For readability, `ParRates` and `ZeroRates` are shown only to the basis point. However, MATLAB computes them at full precision. If you enter `ParRates` as shown, `ZeroRates` may differ due to rounding.

See Also

`zero2pyld` and other functions for Term Structure of Interest Rates

Purpose	Price tick series from incremental returns and initial price
Syntax	<code>[TickSeries, TickTimes] = ret2tick(RetSeries, StartPrice, RetIntervals, StartTime)</code>
Arguments	RetSeries Number of observations (NUMOBS) by number of assets (NASSETS) matrix of incremental return observations. The <i>i</i>'th return is quoted for the period <code>TickTimes(i)</code> to <code>TickTimes(i+1)</code> and is not scaled to a yearly return. StartPrice (Optional) 1-by-NASSETS vector of initial asset prices. Prices start at 1 if <code>StartPrice</code> is not specified. RetIntervals (Optional) Scalar or NUMOBS-by-1 vector of interval times between observations. If not specified, all intervals are assumed to have length 1. StartTime (Optional) Starting time for first observation.
Description	<code>ret2tick</code> generates price values from the starting prices of NASSETS investments and NUMOBS incremental return observations. <code>TickSeries</code> is a NUMOBS+1-by-NASSETS matrix of prices of equity assets. The first row is the starting price of the assets. <code>TickTimes</code> is a NUMOBS+1-by-1 vector of tick observation times. The initial time is zero unless specified in <code>StartTime</code>.
Examples	Compute the price increase of two stocks over a year's time based on three incremental return observations. <pre>RetSeries = [0.10 0.12 0.05 0.04 -0.05 0.05]; RetIntervals = [182 91 92]; StartTime = datenum('18-Dec-2000');</pre>

ret2tick

```
[TickSeries, TickTimes] = ret2tick(RetSeries, [], RetIntervals, ...  
StartTime)
```

```
TickSeries =  
    1.0000    1.0000  
    1.1000    1.1200  
    1.1550    1.1648  
    1.0973    1.2230
```

```
TickTimes =  
  
    730838  
    731020  
    731111  
    731203
```

```
datestr(TickTimes)
```

```
ans =  
18-Dec-2000  
18-Jun-2001  
17-Sep-2001  
18-Dec-2001
```

See Also

portsim, tick2ret

Purpose	Seconds of date or time
Syntax	<code>Seconds = second(Date)</code>
Description	<code>Seconds = second(Date)</code> returns the seconds given a serial date number or a date string.
Examples	<pre>Seconds = second(738647.558427893) or Seconds = second('06-May-2022, 13:24:08.17')</pre> returns <pre>Seconds = 8.1700</pre>
See Also	<code>datevec</code> , <code>hour</code> , <code>minute</code>

taxedrr

Purpose	After-tax rate of return
Syntax	<code>Return = taxedrr(PreTaxReturn, TaxRate)</code>
Arguments	PreTaxReturn Nominal rate of return. Enter as a decimal fraction. TaxRate Tax rate. Enter as a decimal fraction.
Description	<code>Return = taxedrr(PreTaxReturn, TaxRate)</code> calculates the after-tax rate of return.
Examples	An investment has a 12% nominal rate of return and is taxed at a 30% rate. The after-tax rate of return is <code>Return = taxedrr(0.12, 0.30)</code> <code>Return =</code> <code>0.0840</code> or 8.4%
See Also	<code>effrr, irr, mirr, nomrr, xirr</code>

Purpose	Treasury bond parameters given Treasury bill parameters
Syntax	[TBondMatrix, Settle] = tbl2bond(TBillMatrix)
Arguments	TBillMatrix Treasury bill parameters. An N-by-5 matrix where each row describes a Treasury bill. N is the number of Treasury bills. Columns are [Maturity DaysMaturity Bid Asked AskYield] where: Maturity Maturity date, as a serial date number. Use datenum to convert date strings to serial date numbers. DaysMaturity Days to maturity, as an integer. Days to maturity is quoted on a skip-day basis; the actual number of days from settlement to maturity is DaysMaturity + 1. Bid Bid bank-discount rate: the percentage discount from face value at which the bill could be bought, annualized on a simple-interest basis. A decimal fraction. Asked Asked bank-discount rate, as a decimal fraction. AskYield Asked yield: the bond-equivalent yield from holding the bill to maturity, annualized on a simple-interest basis and assuming a 365-day year. A decimal fraction.
Description	[TBondMatrix, Settle] = tbl2bond(TBillMatrix) restates U.S. Treasury bill market parameters in U.S. Treasury bond form as zero-coupon bonds. This function makes Treasury bills directly comparable to Treasury bonds and notes.

tbl2bond

TBondMatrix	Treasury bond parameters. An N-by-5 matrix where each row describes an equivalent Treasury (zero-coupon) bond. Columns are [CouponRate Maturity Bid Asked AskYield] where
CouponRate	Coupon rate, which is always 0.
Maturity	Maturity date, as a serial date number. This date is the same as the Treasury bill Maturity date.
Bid	Bid price based on \$100 face value.
Asked	Asked price based on \$100 face value.
AskYield	Asked yield to maturity: the effective return from holding the bond to maturity, annualized on a compound-interest basis.

Examples

Given published Treasury bill market parameters for December 22, 1997:

```
TBill = [datenum('jan 02 1998') 10 0.0526 0.0522 0.0530
         datenum('feb 05 1998') 44 0.0537 0.0533 0.0544
         datenum('mar 05 1998') 72 0.0529 0.0527 0.0540];
```

Execute the function.

```
TBond = tbl2bond(TBill)

TBond =
    0 729760    99.854    99.855    0.053
    0 729790    99.344    99.349    0.0544
    0 729820    98.942    98.946    0.054
```

(Example output has been formatted for readability.)

See Also

tr2bonds and other functions for Term Structure of Interest Rates

Purpose	Incremental return series from a tick price series
Syntax	<code>[RetSeries, RetIntervals] = tick2ret(TickSeries, TickTimes)</code>
Arguments	TickSeries Number of observations (NUMOBS) by number of assets (NASSETS) matrix of prices of equity assets. First row is oldest observation. Last row is most recent. TickTimes (Optional) NUMOBS-by-1 increasing vector of observation times. Times are taken as serial date numbers (day units) or as decimal numbers in arbitrary units (e.g., yearly).
Description	<code>tick2ret</code> computes the asset returns realized between NUMOBS observations of prices of NASSETS assets. <code>RetSeries</code> is a (NUMOBS-1)-by-NASSETS matrix of incremental return observations. The i'th return is quoted for the period <code>TickTimes(i)</code> to <code>TickTimes(i+1)</code> and is not scaled to a yearly return. $\text{RetSeries}(i) = \text{TickSeries}(i+1)/\text{TickSeries}(i) - 1$ <code>RetIntervals</code> is a (NUMOBS-1)-by-1 vector of interval times between observations. If <code>TickTimes</code> is not specified, all intervals are assumed to have length 1.
Examples	Compute the periodic returns of two stocks observed in the first, second, third, and fourth quarters. <pre> TickSeries = [100 80 110 90 115 88 110 91]; TickTimes = [0 6 9 12]; </pre>

tick2ret

```
[RetSeries, RetIntervals] = tick2ret(TickSeries, TickTimes)
```

```
RetSeries =  
    0.1000    0.1250  
    0.0455   -0.0222  
   -0.0435    0.0341
```

```
RetIntervals =  
    6  
    3  
    3
```

See Also

ewstats, ret2tick

Purpose	Current date
Syntax	Datenum = today
Description	Datenum = today returns the current date as a serial date number.
Examples	<pre>Datenum = today returns Datenum = 730695 on July 28, 2000.</pre>
See Also	datenum, datestr, now

tr2bonds

Purpose	Term-structure parameters given Treasury bond parameters
Syntax	[Bonds, Prices, Yields] = tr2bonds(TreasuryMatrix, Settle)
Arguments	TreasuryMatrix Treasury bond parameters. An n-by-5 matrix, where each row describes a Treasury bond. Columns are [CouponRate Maturity Bid Asked AskYield] where CouponRate Coupon rate, as a decimal fraction. Maturity Maturity date, as a serial date number. Use datenum to convert date strings to serial date numbers. Bid Bid price based on \$100 face value. Asked Asked price based on \$100 face value. AskYield Asked yield to maturity, as a decimal fraction. Settle (Optional) Date string or serial date number of the settlement date for the analysis.
Description	[Bonds, Prices, Yields] = tr2bonds(TreasuryMatrix, Settle)) returns term-structure parameters (bond information, prices, and yields) sorted by ascending maturity date, given Treasury bond parameters. The formats of the output matrix and vectors meet requirements for input to the zbtprice and zbtyield zero-curve bootstrapping functions.

Bonds	Coupon bond information. An n-by-6 matrix where each row describes a bond. Columns are [Maturity CouponRate Face Period Basis EndMonthRule] where:												
	<table> <tr> <td>Maturity</td><td>Maturity date of the bond, as a serial date number. Use <code>datestr</code> to convert serial date numbers to date strings.</td></tr> <tr> <td>CouponRate</td><td>Coupon rate of the bond, as a decimal fraction.</td></tr> <tr> <td>Face</td><td>Redemption or face value of the bond, always 100.</td></tr> <tr> <td>Period</td><td>Coupons per year of the bond, always 2.</td></tr> <tr> <td>Basis</td><td>Day-count basis of the bond, always 0 (actual/actual).</td></tr> <tr> <td>EndMonthRule</td><td>End-of-month flag, always 1, meaning that a bond's coupon payment date is always the last day of the month.</td></tr> </table>	Maturity	Maturity date of the bond, as a serial date number. Use <code>datestr</code> to convert serial date numbers to date strings.	CouponRate	Coupon rate of the bond, as a decimal fraction.	Face	Redemption or face value of the bond, always 100.	Period	Coupons per year of the bond, always 2.	Basis	Day-count basis of the bond, always 0 (actual/actual).	EndMonthRule	End-of-month flag, always 1, meaning that a bond's coupon payment date is always the last day of the month.
Maturity	Maturity date of the bond, as a serial date number. Use <code>datestr</code> to convert serial date numbers to date strings.												
CouponRate	Coupon rate of the bond, as a decimal fraction.												
Face	Redemption or face value of the bond, always 100.												
Period	Coupons per year of the bond, always 2.												
Basis	Day-count basis of the bond, always 0 (actual/actual).												
EndMonthRule	End-of-month flag, always 1, meaning that a bond's coupon payment date is always the last day of the month.												
Prices	Prices. A column vector containing the price of each bond in <code>bonds</code> , respectively. The number of rows (n) matches the number of rows in <code>bonds</code> .												
Yields	Yields. A column vector containing the yield to maturity of each bond in <code>bonds</code> , respectively. The number of rows (n) matches the number of rows in <code>bonds</code> . If <code>Settle</code> is input, <code>Yields</code> is computed as a semi-annual yield to maturity. If <code>Settle</code> is not input, the quoted input yields will be used.												

Examples

Given published Treasury bond market parameters for December 22, 1997:

```
Matrix = [0.0650 datenum('15-apr-1999') 101.03125 101.09375 0.0564
          0.05125 datenum('17-dec-1998') 99.4375 99.5 0.0563
          0.0625 datenum('30-jul-1998') 100.3125 100.375 0.0560
          0.06125 datenum('26-mar-1998') 100.09375 100.15625 0.0546];
```

Execute the function.

```
[Bonds, Prices, Yields] = tr2bonds(Matrix)
```

tr2bonds

```
Bonds =  
  
    729840    0.06125    100    2    0    1  
    729966    0.0625     100    2    0    1  
    730106    0.05125    100    2    0    1  
    730225    0.065      100    2    0    1  
  
Prices =  
  
    100.1563  
    100.3750  
     99.5000  
    101.0938  
  
Yields =  
  
     0.0546  
     0.056  
     0.0563  
     0.0564
```

(Example output has been formatted for readability.)

See Also tbl2bond, zbtprice, zbtyield, and other functions for Term Structure of Interest Rates

Purpose Univariate GARCH(P,Q) parameter estimation with Gaussian innovations

Syntax [Kappa, Alpha, Beta] = ugarch(U, P, Q)

Arguments

- U Single column vector of random disturbances, i.e., the residuals or innovations (ϵ_t), of an econometric model representing a mean-zero, discrete-time stochastic process. The innovations time series U is assumed to follow a GARCH(P,Q) process.
- P Non-negative, scalar integer representing a model order of the GARCH process. P is the number of lags of the conditional variance. P can be zero; when $P = 0$, a GARCH(0,Q) process is actually an ARCH(Q) process.
- Q Positive, scalar integer representing a model order of the GARCH process. Q is the number of lags of the squared innovations.

Description [Kappa, Alpha, Beta] = ugarch(U, P, Q) computes estimated univariate GARCH(P,Q) parameters with Gaussian innovations.

Kappa is the estimated scalar constant term (κ) of the GARCH process.

Alpha is a P-by-1 vector of estimated coefficients, where P is the number of lags of the conditional variance included in the GARCH process.

Beta is a Q-by-1 vector of estimated coefficients, where Q is the number of lags of the squared innovations included in the GARCH process.

The time-conditional variance, σ_t^2 , of a GARCH(P,Q) process is modeled as

$$\sigma_t^2 = \kappa + \sum_{i=1}^P \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^Q \beta_j \epsilon_{t-j}^2$$

where α represents the argument Alpha, β represents Beta, and the GARCH(P, Q) coefficients $\{\kappa, \alpha, \beta\}$ are subject to the following constraints.

$$\sum_{i=1}^P a_i + \sum_{j=1}^Q \beta_j < 1$$

$$\kappa > 0$$

$$a_i \geq 0 \quad i = 1, 2, \dots, P$$

$$\beta_j \geq 0 \quad j = 1, 2, \dots, Q$$

Note that $\mathbf{U}$ is a vector of residuals or innovations (ε_t) of an econometric model, representing a mean-zero, discrete-time stochastic process.

Although σ_t^2 is generated using the equation above, ε_t and σ_t^2 are related as

$$\varepsilon_t = \sigma_t v_t$$

where $\{v_t\}$ is an independent, identically distributed (i.i.d.) sequence $\sim N(0,1)$.

Note `ugarch` corresponds generally to the GARCH Toolbox function `garchfit`. The GARCH Toolbox provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information, see the *GARCH Toolbox User's Guide* or the financial products Web page at <http://www.mathworks.com/products/finprod/>.

Examples

See `ugarchsim` for an example of a GARCH(P,Q) process.

See Also

`ugarchpred`, `ugarchsim`, and the GARCH Toolbox function `garchfit`

References

James D. Hamilton, *Time Series Analysis*, Princeton University Press, 1994

Purpose	Log-likelihood objective function of univariate GARCH(P,Q) processes with Gaussian innovations
Syntax	LogLikelihood = ugarchllf(Parameters, U, P, Q)
Arguments	Parameters (1 + P + Q)- by-1 column vector of GARCH(P,Q) process parameters. The first element is the scalar constant term K of the GARCH process; the next P elements are coefficients associated with the P lags of the conditional variance terms; the next Q elements are coefficients associated with the Q lags of the squared innovations terms. U Single column vector of random disturbances, i.e., the residuals or innovations (ϵ_t), of an econometric model representing a mean-zero, discrete-time stochastic process. The innovations time series U is assumed to follow a GARCH(P,Q) process. P Nonnegative, scalar integer representing a model order of the GARCH process. P is the number of lags of the conditional variance. P can be zero; when $P = 0$, a GARCH(0,Q) process is actually an ARCH(Q) process. Q Positive, scalar integer representing a model order of the GARCH process. Q is the number of lags of the squared innovations.
Description	LogLikelihood = ugarchllf(Parameters, U, P, Q) computes the log-likelihood objective function of univariate GARCH(P,Q) processes with Gaussian innovations. LogLikelihood is a scalar value of the GARCH(P,Q) log-likelihood objective function given the input arguments. This function is meant to be optimized via the fmincon function of the MATLAB Optimization Toolbox. fmincon is a minimization routine. To maximize the log-likelihood function, the LogLikelihood output parameter is actually the negative of what is formally presented in most time series or econometrics references. The time-conditional variance, σ_t^2, of a GARCH(P,Q) process is modeled as

$$\sigma_t^2 = \kappa + \sum_{i=1}^P \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^Q \beta_j \varepsilon_{t-j}^2$$

where α represents the argument Alpha, and β represents Beta.

$\mathbf{U}$ is a vector of residuals or innovations (ε_t) representing a mean-zero, discrete time stochastic process. Although σ_t^2 is generated via the equation above, ε_t and σ_t^2 are related as

$$\varepsilon_t = \sigma_t v_t$$

where $\{v_t\}$ is an independent, identically distributed (i.i.d.) sequence $\sim N(0,1)$.

Since `ugarchllf` is really just a helper function, no argument checking is performed. This function is not meant to be called directly from the command line.

Note `ugarchllf` corresponds generally to the GARCH Toolbox function `garchllfn`. The GARCH Toolbox provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information see the *GARCH Toolbox User's Guide* or the financial products Web page at <http://www.mathworks.com/products/finprod/>.

See Also

`ugarch`, `ugarchpred`, `ugarchsim`, and the GARCH Toolbox function `garchllfn`

Purpose Forecast conditional variance of univariate GARCH(P,Q) processes

Syntax [VarianceForecast, H] = ugarchpred(U, Kappa, Alpha, Beta, NumPeriods)

Arguments

U	Single column vector of random disturbances, i.e., the residuals or innovations (ϵ_t), of an econometric model representing a mean-zero, discrete-time stochastic process. The innovations time series U is assumed to follow a GARCH(P,Q) process.
Kappa	Scalar constant term κ of the GARCH process.
Alpha	P-by-1 vector of coefficients, where P is the number of lags of the conditional variance included in the GARCH process. Alpha can be an empty matrix, in which case P is assumed 0; when $P = 0$, a GARCH(0,Q) process is actually an ARCH(Q) process.
Beta	Q-by-1 vector of coefficients, where Q is the number of lags of the squared innovations included in the GARCH process.
NumPeriods	Positive, scalar integer representing the forecast horizon of interest, expressed in periods compatible with the sampling frequency of the input innovations column vector U.

Description [VarianceForecast, H] = ugarchpred(U, Kappa, Alpha, Beta, NumPeriods) forecasts the conditional variance of univariate GARCH(P,Q) processes.

VarianceForecast is a number of periods (NUMPERIODS)-by-1 vector of the minimum mean-square error forecast of the conditional variance of the innovations time series vector U (i.e., ϵ_t). The first element contains the 1-period-ahead forecast, the second element contains the 2-period-ahead forecast, and so on. Thus, if a forecast horizon greater than 1 is specified (NUMPERIODS > 1), the forecasts of all intermediate horizons are returned as well. In this case, the last element contains the variance forecast of the specified horizon, NumPeriods from the most recent observation in U.

H is a vector of the conditional variances (σ_t^2) corresponding to the innovations vector U. It is inferred from the innovations U, and is a reconstruction of the

“past” conditional variances, whereas the `VarianceForecast` output represents the projection of conditional variances into the “future.” This sequence is based on setting pre-sample values of σ_t^2 to the unconditional variance of the $\{\epsilon_t\}$ process. H is a single column vector of the same length as the input innovations vector U .

The time-conditional variance, σ_t^2 , of a GARCH(P,Q) process is modeled as

$$\sigma_t^2 = \kappa + \sum_{i=1}^P \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^Q \beta_j \epsilon_{t-j}^2$$

where α represents the argument Alpha, β represents Beta, and the GARCH(P,Q) coefficients $\{\kappa, \alpha, \beta\}$ are subject to the following constraints.

$$\begin{aligned} \sum_{i=1}^P a_i + \sum_{j=1}^Q \beta_j &< 1 \\ \kappa &> 0 \\ a_i &\geq 0 \quad i = 1, 2, \dots, P \\ \beta_j &\geq 0 \quad j = 1, 2, \dots, Q \end{aligned}$$

Note that U is a vector of residuals or innovations (ϵ_t) of an econometric model, representing a mean-zero, discrete-time stochastic process.

Although σ_t^2 is generated using the equation above, ϵ_t and σ_t^2 are related as

$$\epsilon_t = \sigma_t v_t$$

where $\{v_t\}$ is an independent, identically distributed (i.i.d.) sequence $\sim N(0,1)$.

Note `ugarchpred` corresponds generally to the GARCH Toolbox function `garchpred`. The GARCH Toolbox provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information see the *GARCH Toolbox User's Guide* or the financial products Web page at <http://www.mathworks.com/products/finprod/>.

Examples See `ugarchsim` for an example of forecasting the conditional variance of a univariate GARCH(P,Q) process.

See Also `ugarch`, `ugarchsim`, and the GARCH Toolbox function `garchpred`

ugarchsim

Purpose	Simulate a univariate GARCH(P,Q) process with Gaussian innovations		
Syntax	[U, H] = ugarchsim(Kappa, Alpha, Beta, NumSamples)		
Arguments	Kappa	Scalar constant term κ of the GARCH process.	
	Alpha	P-by-1 vector of coefficients, where P is the number of lags of the conditional variance included in the GARCH process. Alpha can be an empty matrix, in which case P is assumed 0; when P = 0, a GARCH(0,Q) process is actually an ARCH(Q) process.	
	Beta	Q-by-1 vector of coefficients, where Q is the number of lags of the squared innovations included in the GARCH process.	
	NumSamples	Positive, scalar integer indicating the number of samples of the innovations U and conditional variance H (see below) to simulate.	

Description

[U, H] = ugarchsim(Kappa, Alpha, Beta, NumSamples) simulates a univariate GARCH(P,Q) process with Gaussian innovations.

U is a number of samples (NUMSAMPLES)-by-1 vector of innovations (ϵ_t), representing a mean-zero, discrete-time stochastic process. The innovations time series U is designed to follow the GARCH(P,Q) process specified by the inputs Kappa, Alpha, and Beta.

H is a NUMSAMPLES-by-1 vector of the conditional variances (σ_t^2) corresponding to the innovations vector U. Note that U and H are the same length, and form a “matching” pair of vectors. As shown in the following equation, σ_t^2 (i.e., H(t)) represents the time series inferred from the innovations time series $\{\epsilon_t\}$ (i.e., U).

The time-conditional variance, σ_t^2 , of a GARCH(P,Q) process is modeled as

$$\sigma_t^2 = \kappa + \sum_{i=1}^P \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^Q \beta_j \epsilon_{t-j}^2$$

where α represents the argument Alpha, β represents Beta, and the GARCH(P,Q) coefficients $\{\kappa, \alpha, \beta\}$ are subject to the following constraints.

$$\sum_{i=1}^P a_i + \sum_{j=1}^Q \beta_j < 1$$

$$\kappa > 0$$

$$a_i \geq 0 \quad i = 1, 2, \dots, P$$

$$\beta_j \geq 0 \quad j = 1, 2, \dots, Q$$

Note that $\mathbf{U}$ is a vector of residuals or innovations (ε_t) of an econometric model, representing a mean-zero, discrete-time stochastic process.

Although σ_t^2 is generated using the equation above, ε_t and σ_t^2 are related as

$$\varepsilon_t = \sigma_t v_t$$

where $\{v_t\}$ is an independent, identically distributed (i.i.d.) sequence $\sim N(0,1)$.

The output vectors $\mathbf{U}$ and $\mathbf{H}$ are designed to be steady-state sequences in which transients have arbitrarily small effect. The (arbitrary) metric used by `ugarchsim` strips the first N samples of $\mathbf{U}$ and $\mathbf{H}$ such that the sum of the GARCH coefficients, excluding κ , raised to the N th power, does not exceed 0.01.

$$0.01 = (\text{sum}(\text{Alpha}) + \text{sum}(\text{Beta}))^N$$

Thus

$$N = \log(0.01)/\log((\text{sum}(\text{Alpha}) + \text{sum}(\text{Beta})))$$

Note `ugarchsim` corresponds generally to the GARCH Toolbox function `garchsim`. The GARCH Toolbox provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information see the *GARCH Toolbox User's Guide* or the financial products Web page at <http://www.mathworks.com/products/finprod/>.

Examples

This example simulates a GARCH(P,Q) process with $P = 2$ and $Q = 1$.

```
% Set the random number generator seed for reproducibility.

randn('seed', 10)

% Set the simulation parameters of GARCH(P,Q) = GARCH(2,1) process.

Kappa = 0.25;          %a positive scalar.
Alpha = [0.2 0.1]';   %a column vector of nonnegative numbers (P = 2).
Beta = 0.4;           % Q = 1.
NumSamples = 500;    % number of samples to simulate.

% Now simulate the process.

[U , H] = ugarchsim(Kappa, Alpha, Beta, NumSamples);

% Estimate the process parameters.

P = 2;    % Model order P (P = length of Alpha).
Q = 1;    % Model order Q (Q = length of Beta).
[k, a, b] = ugarch(U , P , Q);
disp(' ')
disp(' Estimated Coefficients:')
disp(' -----')
disp([k; a; b])
disp(' ')

% Forecast the conditional variance using the estimated
% coefficients.

NumPeriods = 10;    % Forecast out to 10 periods.
[VarianceForecast, H1] = ugarchpred(U, k, a, b, NumPeriods);
disp(' Variance Forecasts:')
disp(' -----')
disp(VarianceForecast)
disp(' ')
```

When the above code is executed, the screen output looks like the display shown.

%%%%%%%%%%
%%%%%%%%%

Diagnostic Information

Number of variables: 4

Functions
Objective: ugarchllf
Gradient: finite-differencing
Hessian: finite-differencing (or Quasi-Newton)

Constraints
Nonlinear constraints: do not exist
Number of linear inequality constraints: 1
Number of linear equality constraints: 0
Number of lower bound constraints: 4
Number of upper bound constraints: 0
Algorithm selected
medium-scale

%%%%%%%%%%
%%%%%%%%%

End diagnostic information

Iter	F-count	f(x)	max constraint	Step-size	Directional derivative	Procedure
1	5	699.185	-0.125	1	-2.97e+006	
2	22	658.224	-0.1249	0.000488	-64.6	
3	28	610.181	0	1	-49.4	
4	35	590.888	0	0.5	-38.9	
5	42	583.961	-0.03317	0.5	-29.8	
6	49	583.224	-0.02756	0.5	-31.8	
7	57	582.947	-0.02067	0.25	-7.28	
8	63	578.182	0	1	-2.43	
9	71	578.138	-0.09145	0.25	-0.55	
10	77	577.898	-0.04452	1	-0.148	
11	84	577.882	-0.06128	0.5	-0.0488	
12	90	577.859	-0.07117	1	-0.000758	
13	96	577.858	-0.07033	1	-0.000305	Hessian modified
14	102	577.858	-0.07042	1	-3.32e-005	Hessian modified
15	108	577.858	-0.0707	1	-1.29e-006	Hessian modified

ugarchsim

16	114	577.858	-0.07077	1	-1.29e-007	Hessian modified
17	120	577.858	-0.07081	1	-1.97e-007	Hessian modified

Optimization Converged Successfully
Magnitude of directional derivative in search direction
less than 2*options.TolFun and maximum constraint violation
is less than options.TolCon
No Active Constraints

Estimated Coefficients:

0.2520
0.0708
0.1623
0.4000

Variance Forecasts:

1.3243
0.9594
0.9186
0.8402
0.7966
0.7634
0.7407
0.7246
0.7133
0.7054

See Also ugarch, ugarchpred, and the GARCH Toolbox function garchsim

References James D. Hamilton, *Time Series Analysis*, Princeton University Press, 1994

Purpose Day of the week

Syntax [DayNum, DayString] = weekday(Date)

Description [DayNum, DayString] = weekday(Date) returns the day of the week in numeric and string form given the date as a serial date number or date string. The days of the week have these values:

DayNum	DayString
1	Sun
2	Mon
3	Tue
4	Wed
5	Thu
6	Fri
7	Sat

Note This function now ships with basic MATLAB. It originally shipped only with the Financial Toolbox. This description remains here for your convenience.

weekday

Examples

```
[DayNum, DayString] = weekday(730845)
```

or

```
[DayNum, DayString] = weekday('25-Dec-2000')
```

returns

```
DayNum =
```

```
2
```

```
DayString =
```

```
Mon
```

See Also

[datenum](#), [datestr](#), [datevec](#), [day](#)

Purpose	Number of working days between dates
Syntax	<code>Days = wrkdydif(StartDate, EndDate, Holidays)</code>
Description	<code>Days = wrkdydif(StartDate, EndDate, Holidays)</code> returns the number of working days between dates <code>StartDate</code> and <code>EndDate</code> . <code>Holidays</code> is the number of holidays between the given dates, an integer. Enter dates as serial date numbers or date strings.
Examples	<pre>Days = wrkdydif('9/1/2000', '9/11/2000', 1) or Days = wrkdydif(730730, 730740, 1) returns Days = 6</pre>
See Also	<code>busdate</code> , <code>datewrkdy</code> , <code>days360</code> , <code>days365</code> , <code>daysact</code> , <code>daysdif</code> , <code>holidays</code> , <code>yearfrac</code>

x2mdate

Purpose Excel serial date number to MATLAB serial date number

Syntax MATLABDate = x2mdate(ExcelDateNumber, Convention)

Arguments

ExcelDateNumber	A vector or scalar of Excel serial date numbers.
Convention	(Optional) Excel date system. A vector or scalar. When Convention = 0 (default), the Excel 1900 date system is in effect. When Convention = 1, the Excel 1904 date system is used. In the Excel 1900 date system, the Excel serial date number 1 corresponds to January 1, 1900 A.D. In the Excel 1904 date system, date number 0 is January 1, 1904 A.D.

Vector arguments must have consistent dimensions.

Description DateNumber = x2mdate(ExcelDateNumber, Convention) converts Excel serial date numbers to MATLAB serial date numbers. MATLAB date numbers start with 1 = January 1, 0000 A.D., hence there is a difference of 693961 relative to the 1900 date system, or 695422 relative to the 1904 date system. This function is useful with MATLAB Excel Link.

Examples Given Excel date numbers in the 1904 system

```
ExDates = [35423 35788 36153];
```

convert them to MATLAB date numbers

```
MATLABDate = x2mdate(ExDates, 1)
```

```
MATLABDate =
```

```
730845      731210      731575
```

and then to date strings:


```
datestr(MATLABDate)
```

```
ans =
```

```
25-Dec-2000
```

```
25-Dec-2001
```

```
25-Dec-2002
```

See Also

[datetime](#), [datestr](#), [m2xdate](#)

xirr

Purpose Internal rate of return for nonperiodic cash flow

Syntax `Return = xirr(CashFlow, CashFlowDates, Guess, MaxIterations)`

Arguments

CashFlow	A vector of nonperiodic cash flows. Include the initial investment as the initial cash flow value (a negative number).
CashFlowDates	A vector of dates on which the cash flows occur. Enter dates as serial date numbers or date strings.
Guess	(Optional) Initial estimate of the expected return. Default = 0.1 (10%).
MaxIterations	(Optional) Number of iterations used by Newton's method to solve for Return. Default = 50.

Description `Return = xirr(CashFlow, CashFlowDates, Guess, MaxIterations)` returns the internal rate of return for a schedule of nonperiodic cash flows.

Examples An investment of \$10,000 returns this nonperiodic cash flow. The original investment and its date are included.

Cash flow	Dates
(\$10000)	January 12, 2000
\$2500	February 14, 2001
\$2000	March 3, 2001
\$3000	June 14, 2001
\$4000	December 1, 2001

To calculate the internal rate of return for this nonperiodic cash flow

```
CashFlow = [-10000, 2500, 2000, 3000, 4000];  
CashFlowDates = ['01/12/2000'  
                 '02/14/2001'  
                 '03/03/2001'  
                 '06/14/2001'  
                 '12/01/2001'];
```

```
Return = xirr(CashFlow, CashFlowDates)

returns

Return =
    0.1009 (or 10.09%)
```

See Also `fvvar`, `irr`, `mirr`, `pvvar`

References Sharpe and Alexander, *Investments*, 4th edition, page 463.

year

Purpose	Year of date
Syntax	<code>Year = year(Date)</code>
Description	<code>Year = year(Date)</code> returns the year of a serial date number or a date string.
Examples	<pre>Year = year(731798.776) or Year = year('05-Aug-2003') returns Year = 2003</pre>
See Also	<code>datevec</code> , <code>day</code> , <code>month</code> , <code>yeardays</code>

Purpose Number of days in year

Syntax Days = yeardays(Year)

Description Days = yeardays(Year) returns the actual number of days in the given year.
Enter Year as a four-digit integer.

Examples Days = yeardays(2000)

 Days =

 366

See Also days360, days365, daysact, year, yearfrac

yearfrac

Purpose Fraction of year between dates

Syntax `Fraction = yearfrac(StartDate, EndDate, Basis)`

Arguments

<code>StartDate</code>	Enter as serial date numbers or date strings.
<code>EndDate</code>	Enter as serial date numbers or date strings.
<code>Basis</code>	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

Description `Fraction = yearfrac(StartDate, EndDate, Basis)` returns a fraction based on the number of days between dates `StartDate` and `EndDate` using the given day-count basis. If `EndDate` is earlier than `StartDate`, `YearFraction` is negative.

Examples

```
Fraction = yearfrac('14 mar 01', '14 sep 01', 0)

Fraction =

    0.5041

Fraction = yearfrac('14 mar 01', '14 sep 01', 1)

Fraction =

    0.5000
```

See Also `days360`, `days365`, `daysact`, `daysdif`, `months`, `wrkdydif`, `yeardays`

Purpose	Yield of discounted security
Syntax	<code>Yield = ylddisc(Settle, Maturity, Face, Price, Basis)</code>
Arguments	Settle Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity. Maturity Maturity date. Enter as serial date number or date string. Face Redemption (par, face) value. Price Discounted price of the security. Basis (Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
Description	<code>Yield = ylddisc(Settle, Maturity, Face, Price, Basis)</code> finds the yield of a discounted security.
Examples	Using the data <pre>Settle = '10/14/2000'; Maturity = '03/17/2001'; Face = 100; Price = 96.28; Basis = 2;</pre> <code>Yield = ylddisc(Settle, Maturity, Face, Price, Basis)</code> returns <pre>Yield =</pre> 0.0903 (or 9.03%)
See Also	<code>acrudisc</code> , <code>bndprice</code> , <code>bndyield</code> , <code>prdisc</code> , <code>yldmat</code> , <code>yldtbill</code>
References	Mayle, <i>Standard Securities Calculation Methods</i> , Volumes I-II, 3rd edition. Formula 1.

yldmat

Purpose	Yield with interest at maturity														
Syntax	<code>Yield = yldmat(Settle, Maturity, Issue, Face, Price, CouponRate, Basis)</code>														
Arguments	<table><tr><td>Settle</td><td>Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.</td></tr><tr><td>Maturity</td><td>Maturity date. Enter as serial date number or date string.</td></tr><tr><td>Issue</td><td>Issue date. Enter as serial date number or date string.</td></tr><tr><td>Face</td><td>Redemption (par, face) value.</td></tr><tr><td>Price</td><td>Price of the security.</td></tr><tr><td>CouponRate</td><td>Coupon rate. Enter as decimal fraction.</td></tr><tr><td>Basis</td><td>(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.</td></tr></table>	Settle	Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.	Maturity	Maturity date. Enter as serial date number or date string.	Issue	Issue date. Enter as serial date number or date string.	Face	Redemption (par, face) value.	Price	Price of the security.	CouponRate	Coupon rate. Enter as decimal fraction.	Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
Settle	Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.														
Maturity	Maturity date. Enter as serial date number or date string.														
Issue	Issue date. Enter as serial date number or date string.														
Face	Redemption (par, face) value.														
Price	Price of the security.														
CouponRate	Coupon rate. Enter as decimal fraction.														
Basis	(Optional) Day-count basis: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.														
Description	<code>Yield = yldmat(Settle, Maturity, Issue, Face, Price, CouponRate, Basis)</code> returns the yield of a security paying interest at maturity.														
Examples	Using the data <pre>Settle = '02/07/2000'; Maturity = '04/13/2000'; Issue = '10/11/1999'; Face = 100; Price = 99.98; CouponRate = 0.0608; Basis = 1;</pre> <code>Yield = yldmat(Settle, Maturity, Issue, Face, Price, ... CouponRate, Basis)</code> returns <pre>Yield = 0.0607 (or 6.07%)</pre>														
See Also	<code>acrubond, bndprice, bndyield, prmat, ylddisc, yldtbill</code>														

References

Mayle, *Standard Securities Calculation Methods*, Volumes I-II, 3rd edition.
Formula 3.

yldtbill

Purpose Yield of Treasury bill

Syntax `Yield = yldtbill(Settle, Maturity, Face, Price)`

Arguments

Settle	Settlement date. Enter as serial date number or date string. Settle must be earlier than or equal to Maturity.
Maturity	Maturity date. Enter as serial date number or date string.
Face	Redemption (par, face) value.
Price	Price of the Treasury bill.

Description `Yield = yldtbill(Settle, Maturity, Face, Price)` returns the yield for a Treasury bill.

Examples The settlement date of a Treasury bill is February 10, 2000, the maturity date is August 6, 2000, the par value is \$1000, and the price is \$981.36. Using this data

```
Yield = yldtbill('2/10/2000', '8/6/2000', 1000, 981.36)
```

returns

```
Yield =
```

```
0.0384 (or 3.84%)
```

See Also `beytbill`, `bndyield`, `prtbill`, `yldmat`

References Bodie, Kane, and Marcus, *Investments*, pages 41-43.

Purpose	Zero curve bootstrapping from coupon bond data given price	
Syntax	[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle, OutputCompounding, OutputBasis, MaxIterations)	
Arguments	Bonds	Coupon bond information used to generate the zero curve. An n-by-2 to n-by-6 matrix where each row describes a bond. The first two columns are required; the rest are optional but must be added in order. All rows in Bonds must have the same number of columns. Columns are [Maturity CouponRate Face Period Basis EndMonthRule] where Maturity Maturity date of the bond, as a serial date number. Use datenum to convert date strings to serial date numbers. CouponRate Coupon rate of the bond, as a decimal fraction. Face (Optional) Redemption or face value of the bond. Default = 100. Period (Optional) Coupons per year of the bond, as an integer. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2. Basis (Optional) Day-count basis of the bond: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365. EndMonthRule (Optional) End-of-month flag. This flag applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore flag, meaning that a bond's coupon payment date is always the same day of the month. 1 = set flag (default), meaning that a bond's coupon payment date is always the last day of the month.

Prices	A column vector containing the clean price (price without accrued interest) of each bond in Bonds, respectively. The number of rows (n) must match the number of rows in Bonds.
Settle	Settlement date, as a scalar serial date number. This represents time zero for deriving the zero curve, and it is normally the common settlement date for all the bonds.
OutputCompounding	(Optional) A scalar that sets the compounding frequency per year for the output zero rates in ZeroRates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding
OutputBasis	(Optional) Output day-count basis for mapping cash-flow dates to years, in generating the output zero rates in ZeroRates. A scalar. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
MaxIterations	(Optional) Maximum number of iterations for deriving the zero rates in ZeroRates. A scalar. Default = 50. A value greater than 50 may slow processing.

Description

[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle, OutputCompounding, OutputBasis, MaxIterations) uses the bootstrap method to return a zero curve given a portfolio of coupon bonds and their prices. A zero curve consists of the yields to maturity for a portfolio of theoretical zero-coupon bonds that are derived from the input Bonds portfolio. The bootstrap method that this function uses does *not* require alignment among the cash-flow dates of the bonds in the input portfolio. It uses theoretical par bond

arbitrage and yield interpolation to derive all zero rates. For best results, use a portfolio of at least 30 bonds evenly spaced across the investment horizon.

ZeroRates An m-by-1 vector of decimal fractions that are the implied zero rates for each point along the investment horizon represented by **CurveDates**; m is the number of bonds of unique maturity dates. In aggregate, the rates in **ZeroRates** constitute a zero curve.

If more than one bond has the same maturity date, **zbtprice** returns the mean zero rate for that maturity.

CurveDates An m-by-1 vector of unique maturity dates (as serial date numbers) that correspond to the zero rates in **ZeroRates**; m is the number of bonds of different maturity dates. These dates begin with the earliest maturity date and end with the latest maturity date **Maturity** in the **Bonds** matrix.

Examples

Given data and prices for 12 coupon bonds, two with the same maturity date; and given the common settlement date:

```
Bonds = [datenum('6/1/1998')    0.0475    100    2    0    0;
          datenum('7/1/2000')    0.06      100    2    0    0;
          datenum('7/1/2000')    0.09375   100    6    1    0;
          datenum('6/30/2001')    0.05125   100    1    3    1;
          datenum('4/15/2002')    0.07125   100    4    1    0;
          datenum('1/15/2000')    0.065     100    2    0    0;
          datenum('9/1/1999')     0.08      100    3    3    0;
          datenum('4/30/2001')    0.05875   100    2    0    0;
          datenum('11/15/1999')   0.07125   100    2    0    0;
          datenum('6/30/2000')    0.07      100    2    3    1;
          datenum('7/1/2001')     0.0525    100    2    3    0;
          datenum('4/30/2002')    0.07      100    2    0    0];
```

```
Prices = [99.375;
          99.875;
          105.75 ;
          96.875;
          103.625;
          101.125;
          103.125;
```

zbtprice

```
    99.375;  
    101.0  ;  
    101.25 ;  
    96.375;  
    102.75 ];
```

```
Settle = datenum('12/18/1997');
```

Set semi-annual compounding for the zero curve, on an actual/365 basis.
Derive the zero curve within 50 iterations.

```
OutputCompounding = 2;  
OutputBasis = 3;  
MaxIterations = 50;
```

Execute the function

```
[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle,...  
    OutputCompounding, OutputBasis, MaxIterations)
```

which returns the zero curve at the maturity dates. Note the mean zero rate for the two bonds with the same maturity date*.

```
ZeroRates =
```

```
    0.0616  
    0.0609  
    0.0658  
    0.0590  
    0.0648  
    0.0655*  
    0.0606  
    0.0601  
    0.0642  
    0.0621  
    0.0627
```

CurveDates =

729907	(serial date number for 01-Jun-1998)
730364	(01-Sep-1999)
730439	(15-Nov-1999)
730500	(15-Jan-2000)
730667	(30-Jun-2000)
730668	(01-Jul-2000)*
730971	(30-Apr-2001)
731032	(30-Jun-2001)
731033	(01-Jul-2001)
731321	(15-Apr-2002)
731336	(30-Apr-2002)

See Also

zbtyield and other functions for Term Structure of Interest Rates

References

Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dossa Fabozzi, eds. *The Handbook of Fixed Income Securities*. 4th ed. New York: Irwin Professional Publishing. 1995.

McEnally, Richard W. and James V. Jordan. "The Term Structure of Interest Rates." Ch. 37 in Fabozzi and Fabozzi, *ibid*.

Das, Satyajit. "Calculating Zero Coupon Rates." *Swap and Derivative Financing*. Appendix to Ch. 8, pp. 219-225. New York: Irwin Professional Publishing. 1994.

zbtyield

Purpose	Zero curve bootstrapping from coupon bond data given yield		
Syntax	[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle, OutputCompounding, OutputBasis, MaxIterations)		
Arguments	Bonds	Coupon bond information used to generate the zero curve. An n-by-2 to n-by-6 matrix where each row describes a bond. The first two columns are required; the rest are optional but must be added in order. All rows in Bonds must have the same number of columns. Columns are [Maturity CouponRate Face Period Basis EndMonthRule] where	
	Maturity	Maturity date of the bond, as a serial date number. Use datenum to convert date strings to serial date numbers.	
	CouponRate	Coupon rate of the bond, as a decimal fraction.	
	Face	(Optional) Redemption or face value of the bond. Default = 100.	
	Period	(Optional) Coupons per year of the bond, as an integer. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2.	
	Basis	(Optional) Day-count basis of the bond: 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.	

	EndMonthRule (Optional) End-of-month flag. This flag applies only when Maturity is an end-of-month date for a month having 30 or fewer days. 0 = ignore flag, meaning that a bond's coupon payment date is always the same day of the month. 1 = set flag (default), meaning that a bond's coupon payment date is always the last day of the month.
Yields	A column vector containing the yield to maturity of each bond in Bonds, respectively. The number of rows (n) must match the number of rows in Bonds.
Settle	Settlement date, as a scalar serial date number. This represents time zero for deriving the zero curve, and it is normally the common settlement date for all the bonds.
OutputCompounding	(Optional) A scalar that sets the compounding frequency per year for the output zero rates in ZeroRates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding
OutputBasis	(Optional) Output day-count basis for mapping cash-flow dates to years, in generating the output zero rates in ZeroRates. A scalar. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
MaxIterations	(Optional) Maximum number of iterations for deriving the zero rates in ZeroRates. A scalar. Default = 50. A value greater than 50 may slow processing.

Description

[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle, OutputCompounding, OutputBasis, MaxIterations) uses the bootstrap method to return a zero curve given a portfolio of coupon bonds and their yields. A zero curve consists of the yields to maturity for a portfolio of theoretical zero-coupon bonds that are derived from the input Bonds portfolio. The bootstrap method that this function uses does *not* require alignment among the cash-flow dates of the bonds in the input portfolio. It uses theoretical par bond arbitrage and yield interpolation to derive all zero rates. For best results, use a portfolio of at least 30 bonds evenly spaced across the investment horizon.

ZeroRates An m-by-1 vector of decimal fractions that are the implied zero rates for each point along the investment horizon represented by CurveDates; m is the number of bonds of different maturity dates. In aggregate, the rates in ZeroRates constitute a zero curve.

If more than one bond has the same maturity date, zbtyield returns the mean zero rate for that maturity.

CurveDates An m-by-1 vector of unique maturity dates (as serial date numbers) that correspond to the zero rates in ZeroRates; m is the number of bonds of different maturity dates. These dates begin with the earliest maturity date and end with the latest maturity date Maturity in the Bonds matrix. Use datestr to convert serial date numbers to date strings.

Examples

Given data and yields to maturity for 12 coupon bonds, two with the same maturity date; and given the common settlement date:

```
Bonds = [datenum('6/1/1998')    0.0475    100    2    0    0;
          datenum('7/1/2000')    0.06      100    2    0    0;
          datenum('7/1/2000')    0.09375   100    6    1    0;
          datenum('6/30/2001')   0.05125   100    1    3    1;
          datenum('4/15/2002')   0.07125   100    4    1    0;
          datenum('1/15/2000')   0.065     100    2    0    0;
          datenum('9/1/1999')    0.08      100    3    3    0;
          datenum('4/30/2001')   0.05875   100    2    0    0;
          datenum('11/15/1999')  0.07125   100    2    0    0;
          datenum('6/30/2000')   0.07      100    2    3    1;
          datenum('7/1/2001')    0.0525    100    2    3    0;
```

```

        datenum('4/30/2002') 0.07      100 2 0 0];

Yields = [0.048;
          0.06 ;
          0.089;
          0.053;
          0.069;
          0.064;
          0.078;
          0.059;
          0.071;
          0.069;
          0.057;
          0.068];

```

```
Settle = datenum('12/18/1997');
```

Set semi-annual compounding for the zero curve, on an actual/365 basis.
Derive the zero curve within 50 iterations.

```

OutputCompounding = 2;
OutputBasis = 3;
MaxIterations = 50;

```

Execute the function

```

[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle,...
OutputCompounding, OutputBasis, MaxIterations)

```

which returns the zero curve at the maturity dates. Note the mean zero rate for the two bonds with the same maturity date*.

```

ZeroRates =
    0.0480
    0.0577
    0.0909
    0.0529
    0.0699
    0.0724*
    0.0584
    0.0716
    0.0696

```

0.0526

0.0687

CurveDates =

729907	(serial date number for 01-Jun-1998)
730364	(01-Sep-1999)
730439	(15-Nov-1999)
730500	(15-Jan-2000)
730667	(30-Jun-2000)
730668	(01-Jul-2000)*
730971	(30-Apr-2001)
731032	(30-Jun-2001)
731033	(01-Jul-2001)
731321	(15-Apr-2002)
731336	(30-Apr-2002)

See Also

zbtprice and other functions for Term Structure of Interest Rates

References

Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dossa Fabozzi, eds. *The Handbook of Fixed Income Securities*. 4th ed. New York: Irwin Professional Publishing. 1995.

McEnally, Richard W. and James V. Jordan. "The Term Structure of Interest Rates." Ch. 37 in Fabozzi and Fabozzi, *ibid*.

Das, Satyajit. "Calculating Zero Coupon Rates." *Swap and Derivative Financing*. Appendix to Ch. 8, pp. 219-225. New York: Irwin Professional Publishing. 1994.

Purpose	Discount curve given a zero curve	
Syntax	[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates, Settle, InputCompounding, InputBasis)	
Arguments	ZeroRates	A number of bonds (NUMBONDS) by 1 vector of annualized zero rates, as decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates.
	CurveDates	A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the zero rates.
	Settle	A serial date number that is the common settlement date for the zero rates; i.e., the settlement date for the bonds from which the zero curve was bootstrapped.
	InputCompounding	(Optional) A scalar that indicates the compounding frequency per year used for annualizing the input zero rates in ZeroRates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding 365 daily compounding -1 continuous compounding
	InputBasis	(Optional) Input day-count basis used for annualizing the input zero rates. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.
Description	[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates, Settle, InputCompounding, InputBasis) returns a discount curve given a zero curve and its maturity dates.	

DiscRates A NUMBONDS-by-1 vector of discount factors, as decimal fractions. In aggregate, the factors in constitute a discount curve for the investment horizon represented by **CurveDates**.

CurveDates A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the discount rates. This vector is the same as the input vector **CurveDates**.

Examples

Given a zero curve over a set of maturity dates and a settlement date:

```
ZeroRates = [0.0464
              0.0509
              0.0524
              0.0525
              0.0531
              0.0525
              0.0530
              0.0531
              0.0549
              0.0536];

CurveDates = [datenum('06-Nov-2000')
              datenum('11-Dec-2000')
              datenum('15-Jan-2001')
              datenum('05-Feb-2001')
              datenum('04-Mar-2001')
              datenum('02-Apr-2001')
              datenum('30-Apr-2001')
              datenum('25-Jun-2001')
              datenum('04-Sep-2001')
              datenum('12-Nov-2001')];

Settle = datenum('03-Nov-2000');
```

The zero curve was compounded daily on an actual/365 basis.

```
InputCompounding = 365;
InputBasis = 3;
```

Execute the function

```
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,...  
Settle, InputCompounding, InputBasis)
```

which returns the discount curve DiscRates at the maturity dates CurveDates.

```
DiscRates =  
    0.9996  
    0.9947  
    0.9896  
    0.9866  
    0.9826  
    0.9787  
    0.9745  
    0.9665  
    0.9552  
    0.9466
```

```
CurveDates =  
    730796  
    730831  
    730866  
    730887  
    730914  
    730943  
    730971  
    731027  
    731098  
    731167
```

For readability, ZeroRates and DiscRates are shown here only to the basis point. However, MATLAB computed them at full precision. If you enter ZeroRates as shown, DiscRates may differ due to rounding.

See Also

disc2zero and other functions for Term Structure of Interest Rates

zero2fwd

Purpose Forward curve given a zero curve

Syntax `[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis)`

Arguments

ZeroRates	A number of bonds (NUMBONDS) by 1 vector of annualized zero rates, as decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates.
CurveDates	A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the zero rates.
Settle	A serial date number that is the common settlement date for the zero rates.
OutputCompounding	(Optional) Output compounding. A scalar that sets the compounding frequency per year for annualizing the output forward rates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding 365 daily compounding -1 continuous compounding
OutputBasis	(Optional) Output day-count basis for annualizing the forward rates. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

InputCompounding (Optional) A scalar that indicates the compounding frequency per year used for annualizing the input zero rates. Allowed values are the same as for **OutputCompounding**. Default = **OutputCompounding**.

InputBasis (Optional) Input day-count basis used for annualizing the input zero rates. Allowed values are the same as for **OutputBasis**. Default = **OutputBasis**.

Description

`[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis)` returns an implied forward rate curve given a zero curve and its maturity dates.

ForwardRates A **NUMBONDS**-by-1 vector of decimal fractions. In aggregate, the rates in **ForwardRates** constitute a forward curve over the dates in **CurveDates**.

CurveDates A **NUMBONDS**-by-1 vector of maturity dates (as serial date numbers) that correspond to the forward rates in. This vector is the same as the input vector **CurveDates**.

Examples

Given a zero curve over a set of maturity dates and a settlement date:

```
ZeroRates = [0.0458
             0.0502
             0.0518
             0.0519
             0.0524
             0.0519
             0.0523
             0.0525
             0.0541
             0.0529];

CurveDates = [datenum('06-Nov-2000')
              datenum('11-Dec-2000')
              datenum('15-Jan-2001')
              datenum('05-Feb-2001')
              datenum('04-Mar-2001')]
```

```
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
```

```
Settle = datenum('03-Nov-2000');
```

Set annual compounding for the forward curve, on an actual/actual basis. The zero curve was compounded daily on an actual/365 basis.

```
OutputCompounding = 1;
OutputBasis = 0;
InputCompounding = 365;
InputBasis = 3;
```

Execute the function

```
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates,...
    Settle, OutputCompounding, OutputBasis, InputCompounding,...
    InputBasis)
```

which returns the forward rate curve `ForwardRates` at the maturity dates `CurveDates`.

```
ForwardRates =
```

```
    0.0469
    0.0519
    0.0550
    0.0536
    0.0556
    0.0511
    0.0559
    0.0546
    0.0612
    0.0487
```

```
CurveDates =
```

```
730796  
730831  
730866  
730887  
730914  
730943  
730971  
731027  
731098  
731167
```

For readability, ZeroRates and ForwardRates are shown here only to the basis point. However, MATLAB computed them at full precision. If you enter ZeroRates as shown, ForwardRates may differ due to rounding.

See Also

fwd2zero and other functions for Term Structure of Interest Rates

zero2pyld

Purpose Par yield curve given a zero curve

Syntax `[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis)`

Arguments

ZeroRates	A number of bonds (NUMBONDS) by 1 vector of annualized zero rates, as decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates.
CurveDates	A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the zero rates.
Settle	A serial date number that is the common settlement date for the zero rates.
OutputCompounding	(Optional) Output compounding. A scalar that sets the compounding frequency per year for annualizing the output forward rates. Allowed values are: 1 annual compounding 2 semi-annual compounding (default) 3 compounding three times per year 4 quarterly compounding 6 bimonthly compounding 12 monthly compounding 365 daily compounding -1 continuous compounding
OutputBasis	(Optional) Output day-count basis for annualizing the forward rates. 0 = actual/actual (default), 1 = 30/360, 2 = actual/360, 3 = actual/365.

InputCompounding	(Optional) A scalar that indicates the compounding frequency per year used for annualizing the input zero rates. Allowed values are the same as for OutputCompounding. Default = OutputCompounding.
InputBasis	(Optional) Input day-count basis used for annualizing the input zero rates. Allowed values are the same as for OutputBasis. Default = OutputBasis.

Description

[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates, Settle, OutputCompounding, OutputBasis, InputCompounding, InputBasis) returns a par yield curve given a zero curve and its maturity dates.

ParRates	A NUMBONDS-by-1 vector of annualized par yields, as decimal fractions. (Par yields = coupon rates.) In aggregate, the yield rates in ParRates constitute a par yield curve for the investment horizon represented by CurveDates.
CurveDates	A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the par yield rates. This vector is the same as the input vector CurveDates.

Examples

Given a zero curve over a set of maturity dates and a settlement date:

```
ZeroRates = [0.0457
             0.0487
             0.0506
             0.0507
             0.0505
             0.0504
             0.0506
             0.0516
             0.0539
             0.0530];
```

```
CurveDates = [datetime('06-Nov-2000')
               datetime('11-Dec-2000')
               datetime('15-Jan-2001')
               datetime('05-Feb-2001')
               datetime('04-Mar-2001')
               datetime('02-Apr-2001')
               datetime('30-Apr-2001')
               datetime('25-Jun-2001')
               datetime('04-Sep-2001')
               datetime('12-Nov-2001')];
```

```
Settle = datetime('03-Nov-2000');
```

Set annual compounding for the par yield curve, on an actual/actual basis. The zero curve was compounded monthly, on an actual/365 basis.

```
OutputCompounding = 1;
OutputBasis = 0;
InputCompounding = 12;
InputBasis = 3;
```

Execute the function

```
[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...
    Settle, OutputCompounding, OutputBasis, InputCompounding,...
    InputBasis)
```

which returns the par yield curve at the maturity dates.

```
ParRates =
```

```
0.0479
0.0511
0.0530
0.0531
0.0526
0.0524
0.0525
0.0534
0.0555
0.0543
```

```
CurveDates =
```

```
730796  
730831  
730866  
730887  
730914  
730943  
730971  
731027  
731098  
731167
```

For readability, ZeroRates and ParRates are shown only to the basis point. However, MATLAB computed them at full precision. If you enter ZeroRates as shown, ParRates may differ due to rounding.

See Also

pyld2zero and other functions for Term Structure of Interest Rates

Glossary

American option - An option that can be exercised any time until its expiration date. Contrast with European option.

Amortization - Reduction in value of an asset over some period for accounting purposes. Generally used with intangible assets. Depreciation is the term used with fixed or tangible assets.

Annuity - A series of payments over a period of time. The payments are usually in equal amounts and usually at regular intervals such as quarterly, semi-annually, or annually.

Arbitrage - The purchase of securities on one market for immediate resale on another market in order to profit from a price or currency discrepancy.

Basis point - One hundredth of one percentage point, or 0.0001.

Beta - The price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is most commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument.

Binomial model - A method of pricing options or other equity derivatives in which the probability over time of each possible price follows a binomial distribution. The basic assumption is that prices can move to only two values (one higher and one lower) over any short time period.

Black-Scholes model - The first complete mathematical model for pricing options, developed by Fischer Black and Myron Scholes. It examines market price, strike price, volatility, time to expiration, and interest rates. It is limited to only certain kinds of options.

Bollinger band chart - A financial chart that plots actual asset data along with three other bands of data: the upper band is two standard deviations above a user-specified moving average; the lower band is two standard deviations below that moving average; and the middle band is the moving average itself.

Bootstrapping, bootstrap method - An arithmetic method for backing an implied zero curve out of the par yield curve.

Building a binomial tree - For a binomial option model: plotting the two possible short-term price-changes values, and then the subsequent two values each, and then the subsequent two values each, and so on over time, is known as “building a binomial tree.” See Binomial model.

Call - a. An option to buy a certain quantity of a stock or commodity for a specified price within a specified time. See Put. **b.** A demand to submit bonds to the issuer for redemption before the maturity date. **c.** A demand for payment of a debt. **d.** A demand for payment due on stock bought on margin.

Callable bond - A bond that allows the issuer to buy back the bond at a predetermined price at specified future dates. The bond contains an embedded call option; i.e., the holder has sold a call option to the issuer. See Puttable bond.

Candlestick chart - A financial chart usually used to plot the high, low, open, and close price of a security over time. The body of the “candle” is the region between the open and close price of the security. Thin vertical lines extend up to the high and down to the low, respectively. If the open price is greater than the close price, the body is empty. If the close price is greater than the open price, the body is filled. See also High-low-close chart.

Cap - Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain level.

Cash flow - Cash received and paid over time.

Collar - Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain upper level nor fall below a lower level. It is designed to protect an investor against wide fluctuations in interest rates.

Convexity - A measure of the rate of change in duration; measured in time. The greater the rate of change, the more the duration changes as yield changes.

Correlation - The simultaneous change in value of two random numeric variables.

Correlation coefficient - A statistic in which the covariance is scaled to a value between minus one (perfect negative correlation) and plus one (perfect positive correlation).

Coupon - Detachable certificate attached to a bond that shows the amount of interest payable at regular intervals, usually semi-annually. Originally coupons were actually attached to the bonds and had to be cut off or “clipped” to redeem them and receive the interest payment.

Coupon dates - The dates when the coupons are paid. Typically a bond pays coupons annually or semi-annually.

Coupon rate - The nominal interest rate that the issuer promises to pay the buyer of a bond.

Covariance - A measure of the degree to which returns on two assets move in tandem. A positive covariance means that asset returns move together; a negative covariance means they vary inversely.

Delta - The rate of change of the price of a derivative security relative to the price of the underlying asset; i.e., the first derivative of the curve that relates the price of the derivative to the price of the underlying security.

Depreciation - Reduction in value of fixed or tangible assets over some period for accounting purposes. See Amortization.

Derivative - A financial instrument that is based on some underlying asset. For example, an option is a derivative instrument based on the right to buy or sell an underlying instrument.

Discount curve - The curve of discount rates vs. maturity dates for bonds.

Duration - The expected life of a fixed-income security considering its coupon yield, interest payments, maturity, and call features. As market interest rates rise, the duration of a financial instrument decreases. See Macaulay duration.

Efficient frontier - A graph representing a set of portfolios that maximizes expected return at each level of portfolio risk. See Markowitz model.

Elasticity - See Lambda.

European option - An option that can be exercised only on its expiration date. Contrast with American option.

Exercise price - The price set for buying an asset (call) or selling an asset (put). The strike price.

Face value - The maturity value of a security. Also known as par value, principal value, or redemption value.

Fixed-income security - A security that pays a specified cash flow over a specific period. Bonds are typical fixed-income securities.

Floor - Interest-rate option that guarantees that the rate on a floating-rate loan will not fall below a certain level.

Forward curve - The curve of forward interest rates vs. maturity dates for bonds.

Forward rate - The future interest rate of a bond inferred from the term structure, especially from the yield curve of zero-coupon bonds, calculated from the growth factor of an investment in a zero held until maturity.

Future value - The value that a sum of money (the present value) earning compound interest will have in the future.

Gamma - The rate of change of delta for a derivative security relative to the price of the underlying asset; i.e., the second derivative of the option price relative to the security price.

Greeks - Collectively, “greeks” refer to the financial measures delta, gamma, lambda, rho, theta, and vega, which are sensitivity measures used in evaluating derivatives.

Hedge - A securities transaction that reduces or offsets the risk on an existing investment position.

High-low-close chart - A financial chart usually used to plot the high, low, open, and close price of a security over time. Plots are vertical lines whose top is the high, bottom is the low, open is a short horizontal tick to the left, and close is a short horizontal tick to the right.

Implied volatility - For an option, the variance that makes a call option price equal to the market price. Given the option price, strike price, and other factors, the Black-Scholes model computes implied volatility.

Internal rate of return - **a.** The average annual yield earned by an investment during the period held. **b.** The effective rate of interest on a loan. **c.** The discount rate in discounted cash flow analysis. **d.** The rate that adjusts the value of future cash receipts earned by an investment so that interest earned equals the original cost. See Yield to maturity.

Issue date - The date a security is first offered for sale. That date usually determines when interest payments, known as coupons, are made.

Ito process - Statistical assumptions about the behavior of security prices. For details, see the book by Hull listed in the “Bibliography”.

Lambda - The percentage change in the price of an option relative to a 1% change in the price of the underlying security. Also known as Elasticity.

Long position - Outright ownership of a security or financial instrument. The owner expects the price to rise in order to make a profit on some future sale.

Long rate - The yield on a zero-coupon Treasury bond.

Macaulay duration - A widely used measure of price sensitivity to yield changes developed by Frederick Macaulay in 1938. It is measured in years and is a weighted average-time-to-maturity of an instrument. The Macaulay duration of an income stream, such as a coupon bond, measures how long, on average, the owner waits before receiving a payment. It is the weighted average of the times payments are made, with the weights at time T equal to the present value of the money received at time T.

Markowitz model - A model for selecting an optimum investment portfolio, devised by H. M. Markowitz. It uses a discrete-time, continuous-outcome approach for modeling investment problems, often called the mean-variance paradigm. See Efficient frontier.

Maturity date - The date when the issuer returns the final face value of a bond to the buyer.

Mean - **a.** A number that typifies a set of numbers, such as a geometric mean or an arithmetic mean. **b.** The average value of a set of numbers.

Modified duration - The Macaulay duration discounted by the per-period interest rate; i.e., divided by $(1 + \text{rate}/\text{frequency})$.

Monte-Carlo simulation - A mathematical modeling process. For a model that has several parameters with statistical properties, pick a set of random values for the parameters and run a simulation. Then pick another set of values, and run it again. Run it many times (often 10,000 times) and build up a statistical distribution of outcomes of the simulation. This distribution of outcomes is then used to answer whatever question you are asking.

Moving average - A price average that is adjusted by adding other parametrically determined prices over some time period.

Moving-averages chart - A financial chart that plots leading and lagging moving averages for prices or values of an asset.

Normal (bell-shaped) distribution - In statistics, a theoretical frequency distribution for a set of variable data, usually represented by a bell-shaped curve symmetrical about the mean.

Odd first or last period - Fixed-income securities may be purchased on dates that do not coincide with coupon or payment dates. The length of the first and last periods may differ from the regular period between coupons, and thus the

bond owner is not entitled to the full value of the coupon for that period. Instead, the coupon is pro-rated according to how long the bond is held during that period.

Option - A right to buy or sell specific securities or commodities at a stated price (exercise or strike price) within a specified time. An option is a type of derivative.

Par value - The maturity or face value of a security or other financial instrument.

Par yield curve - The yield curve of bonds selling at par, or face, value.

Point and figure chart - A financial chart usually used to plot asset price data. Upward price movements are plotted as X's and downward price movements are plotted as O's.

Present value - Today's value of an investment that yields some future value when invested to earn compounded interest at a known interest rate.; i.e., the future value at a known period in time discounted by the interest rate over that time period.

Principal value - See Par value.

Purchase price - Price actually paid for a security. Typically the purchase price of a bond is not the same as the redemption value.

Put - An option to sell a stipulated amount of stock or securities within a specified time and at a fixed exercise price. See Call.

Puttable bond - A bond that allows the holder to redeem the bond at a predetermined price at specified future dates. The bond contains an embedded put option; i.e., the holder has bought a put option. See Callable bond.

Quant - A quantitative analyst; someone who does numerical analysis of financial information in order to detect relationships, disparities, or patterns that can lead to making money.

Redemption value - See Par value.

Regression analysis - Statistical analysis techniques that quantify the relationship between two or more variables. The intent is quantitative prediction or forecasting, particularly using a small population to forecast the behavior of a large population.

Rho - The rate of change in a derivative's price relative to the underlying security's risk-free interest rate.

Sensitivity - The "what if" relationship between variables; the degree to which changes in one variable cause changes in another variable. A specific synonym is volatility.

Settlement date - The date when money first changes hands; i.e., when a buyer actually pays for a security. It need not coincide with the issue date.

Short rate - The annualized one-period interest rate.

Short sale, short position - The sale of a security or financial instrument not owned, in anticipation of a price decline and making a profit by purchasing the instrument later at a lower price, and then delivering the instrument to complete the sale. See Long position.

Spot curve, spot yield curve - See Zero curve.

Spot rate - The current interest rate appropriate for discounting a cash flow of some given maturity.

Spread - For options, a combination of call or put options on the same stock with differing exercise prices or maturity dates.

Standard deviation - A measure of the variation in a distribution, equal to the square root of the arithmetic mean of the squares of the deviations from the arithmetic mean; the square root of the variance.

Stochastic - Involving or containing a random variable or variables; involving chance or probability.

Straddle - A strategy used in trading options or futures. It involves simultaneously purchasing put and call options with the same exercise price and expiration date, and it is most profitable when the price of the underlying security is very volatile.

Strike - Exercise a put or call option.

Strike price - See Exercise price.

Swap - A contract between two parties to exchange cash flows in the future according to some formula.

Swaption - A swap option; an option on an interest-rate swap. The option gives the holder the right to enter into a contracted interest-rate swap at a specified future date. See Swap.

Term structure - The relationship between the yields on fixed-interest securities and their maturity dates. Expectation of changes in interest rates affects term structure, as do liquidity preferences and hedging pressure. A yield curve is one representation in the term structure.

Theta - The rate of change in the price of a derivative security relative to time. Theta is usually very small or negative since the value of an option tends to drop as it approaches maturity.

Treasury bill - Short-term U.S. government security issued at a discount from the face value and paying the face value at maturity.

Treasury bond - Long-term debt obligation of the U.S. government that makes coupon payments semi-annually and is sold at or near par value in \$1000 denominations or higher. Face value is paid at maturity.

Variance - The dispersion of a variable. The square of the standard deviation.

Vega - The rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large the security is sensitive to small changes in volatility.

Volatility - **a.** Another general term for sensitivity. **b.** The standard deviation of the annualized continuously compounded rate of return of an asset. **c.** A measure of uncertainty or risk.

Yield - **a.** Measure of return on an investment, stated as a percentage of price. Yield can be computed by dividing return by purchase price, current market value, or other measure of value. **b.** Income from a bond expressed as an annualized percentage rate. **c.** The nominal annual interest rate that gives a future value of the purchase price equal to the redemption value of the security. Any coupon payments determine part of that yield.

Yield curve - Graph of yields (vertical axis) of a particular type of security versus the time to maturity (horizontal axis). This curve usually slopes upward, indicating that investors usually expect to receive a premium for securities that have a longer time to maturity. The benchmark yield curve is for U.S. Treasury securities with maturities ranging from three months to 30 years. See Term structure.

Yield to maturity - A measure of the average rate of return that will be earned on a bond if held to maturity.

Zero curve, zero-coupon yield curve - A yield curve for zero-coupon bonds; zero rates versus maturity dates. Since the maturity and duration (Macaulay duration) are identical for zeros, the zero curve is a pure depiction of supply/demand conditions for loanable funds across a continuum of durations and maturities. Also known as spot curve or spot yield curve.

Zero-coupon bond, or Zero - A bond that, instead of carrying a coupon, is sold at a discount from its face value, pays no interest during its life, and pays the principal only at maturity.

Bibliography

Bond Pricing and Yields	B-2
Term Structure of Interest Rates	B-2
Derivatives Pricing and Yields	B-2
Portfolio Analysis	B-3
Other References	B-3

For the well-known algorithms and formulas used in the Financial Toolbox (such as how to compute a loan payment given principal, interest rate, and length of the loan), no references are given here. The references here pertain to less common formulas.

Bond Pricing and Yields

The pricing and yield formulas for fixed-income securities come from:

Mayle, Jan. *Standard Securities Calculation Methods*. New York: Securities Industry Association, Inc. Vol. 1, 3rd ed., 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.

In many cases these formulas compute the price of a security given yield, dates, rates, and other data. These formulas are nonlinear, however; so when solving for an independent variable within a formula, the Financial Toolbox uses Newton's method. See any elementary numerical methods textbook for the mathematics underlying Newton's method.

Term Structure of Interest Rates

The formulas and methodology for term structure functions come from:

Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dossa Fabozzi, eds. *The Handbook of Fixed Income Securities*. 4th ed. New York: Irwin Professional Publishing. 1995. ISBN 0-7863-0001-9.

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Derivatives Pricing and Yields

The pricing and yield formulas for derivative securities come from:

Cox, J.; S. Ross; and M. Rubenstein, "Option Pricing: A Simplified Approach", *Journal of Financial Economics* 7, Sept. 1979, pp. 229 - 263

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Portfolio Analysis

The Markowitz model is used for portfolio analysis computations. For a discussion of this model see Chapter 7 of:

Bodie, Zvi, Alex Kane, and Alan J. Marcus. *Investments*. Burr Ridge, IL: Irwin. 2nd. ed., 1993, ISBN 0-256-08342-8.

To solve the quadratic minimization problem associated with finding the efficient frontier, the toolbox uses the `constr` function (finds the constrained minimum of a function of several variables) in the MATLAB Optimization Toolbox. See that toolbox documentation for more details.

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Numerics

1900 date system 4-172, 4-258

1904 date system 4-172, 4-258

360-day year 4-132

365-day year 4-133

A

accrued interest 2-21, 4-19, 4-20

 computing fractional period 4-17

acrubond **4-19**

acrudisc **4-20**

actual days

 between dates 4-134

adding a scalar and a matrix 1-7

adding matrices 1-6

advance payments, periodic payment given 4-184

after-tax rate of return 4-234

algebra, linear 1-7, 1-12

American options 2-3, 2-35

amortization 1-20, 2-18, 2-19, 4-21

amortize 1-20, **4-21**

analysis models for equity derivatives 2-33

analyzing

 and computing cash flows 2-16

 equity derivatives 2-32

 portfolios 2-37

annuity 2-18

 payment of with odd first period 4-185

 periodic interest rate of 4-23

 periodic payment of loan or 4-186

annurate **4-23**

annuterm **4-24**

apostrophe or prime character (') 1-5

arguments

 function return 1-19

 interest rate 1-20

 matrices as, limitations 1-20

 vectors as, limitations 1-20

array operations 1-15

ASCII character 1-18

asset covariance matrix with exponential weighting 4-148

asset life 1-20

axis labels, converting 4-116

B

bank format 4-115

base date 4-122

basis 2-21

basis, day-count 4-135

bdtbond **4-25**

bdttrans **4-33**

beytbill **4-36**

binomial

 functions 2-3

 model 2-34

 put and call pricing 4-37

 tree, building 2-35

binprice 2-35, **4-37**

Black's option pricing 4-39

Black-Scholes

 elasticity 4-44

 functions 2-3

 implied volatility 4-43

 model 2-33

 options 3-21, 3-23

 put and call pricing 4-45

 sensitivity to

 interest rate change 4-47

 time-until-maturity change 4-48

 underlying delta change 4-42

- underlying price change 4-41
 - underlying price volatility 4-49
- blkprice **4-39**
- blsdelta 2-34, 3-22, 3-24, **4-41**
- blsgamma 2-34, 3-22, 3-24, **4-42**
- blsimpv **4-43**
- blslambda 2-34, **4-44**
- blsprice 2-34, **4-45**
- blsrho **4-47**
- blstheta **4-48**
- blsvega 2-34, **4-49**
- bndconvp **4-50**
- bndconvy **4-53**
- bnddurp **4-56**
- bnddury **4-59**
- bndprice **4-62**
- bndyield **4-65**
- bolling 2-13, **4-68**
- Bollinger band chart 2-14, 4-68
- bond
 - convexity 3-3, 3-6
 - duration 3-3, 3-6
 - equivalent yield for Treasury bill 4-36
 - portfolio
 - constructing to hedge against duration and convexity 3-6
 - visualizing sensitivity of price to parallel shifts in the yield curve 3-8
 - sensitivity of prices to changes in interest rates 3-3
- bonds
 - zero-coupon 4-270
- bootstrapping 2-30, 4-240, 4-269, 4-274
- building a binomial tree 2-35
- busdate 2-10, **4-69**
- business date
 - last of month 4-168

- business day
 - next 2-10, 4-69
 - previous 4-69
- business days 4-167

C

- call and put pricing
 - Black-Scholes 4-45
- candle 2-13, **4-70**
- candlestick chart 4-70
- capital allocation line 2-37
- cash flow
 - analyzing and computing 2-16
 - convexity 4-76
 - dates 2-10, 4-77
 - duration 4-80
 - future value of varying 4-157
 - internal rate of return 4-166
 - internal rate of return for nonperiodic 4-260
 - irregular 4-157
 - modified internal rate of return 4-175
 - negative 2-16
 - portfolio form of amounts 4-81
 - present value of varying 4-225
 - sensitivity of 2-17
 - uniform payment equal to varying 4-187
- cell array 3-16
- cfamounts **4-71**
- cfconv **4-76**
- cfdates 2-10, **4-77**
- cfdur **4-80**
- cfport **4-81**
- cftimes **4-84**
- character array
 - strings stored as 1-18
- character, ASCII 1-18

- chart
 - Bollinger band 2-14, 4-68
 - candlestick 4-70
 - high, low, open, close 4-163
 - leading and lagging moving averages 4-178
 - point and figure 4-198
- charting financial data 2-12
- colon (:) 1-4
- commutative law 1-7, 1-12
- computing
 - cash flows 2-16
 - dot products of vectors 1-8
 - yields for fixed-income securities 2-20
- configuration information xvi
- constraint functions 2-48
- constraint matrix 2-50
- constructing
 - a bond portfolio to hedge against duration and convexity 3-6
 - greek-neutral portfolios of European stock options 3-12
- conventions
 - SIA 2-20
- conventions in our documentation (table) xviii
- conversions
 - currency 2-12
 - date input 2-5
 - date output 2-7
- converting
 - and handling dates 2-4
 - axis labels 4-116
- convexity 3-3
 - cash flow 4-76
 - constructing a bond portfolio to hedge against 3-6
 - portfolio 3-4, 3-6
- corr2cov **4-86**
- coupon bond
 - prices to zero curve 4-269
 - yields to zero curve 4-274
- coupon date
 - after settlement date 4-91
 - days between 4-105, 4-108
- coupon dates 2-26
- coupon payments remaining until maturity 4-88
- coupon period
 - containing settlement date 4-111
 - fraction of 4-16
- coupons payable between dates 4-88
- cov2corr **4-87**
- covariance matrix 2-39
- covariance matrix with exponential weighting 4-148
- cpncount **4-88**
- cpndaten **4-91**
- cpndatenq **4-94**
- cpndatep **4-98**
- cpndatepq **4-101**
- cpndaysn **4-105**
- cpndaysp **4-108**
- cpnpersz **4-111**
- cur2frac 2-12, **4-114**
- cur2str 2-12, **4-115**
- currency
 - converting 2-12
 - decimal 4-151
 - formatting 2-12
 - fractional 4-114, 4-151
 - values 4-114
- current date 4-239
 - and time 2-8, 4-180

D

date

- base 4-122
- components 4-128
- conversions 2-5
- current 2-8, 4-180, 4-239
- end of month 4-146
- first business, of month 4-150
- formats 2-4
- hour of 4-165
- input conversions 2-5
- last date of month 4-146
- last weekday in month 4-170
- maturity 2-21
- minute of 4-174
- number 2-4, 4-122
 - displaying as string 4-118
 - Excel to MATLAB 4-258
 - indices of in matrix 4-119
 - MATLAB to Excel 4-172
- of day in future or past month 4-120
- of future or past workday 4-130
- output conversions 2-7
- seconds of 4-233
- starting, add month to 4-120
- string 2-4, 4-125
- vector 4-128
- year of 4-262

date 2-8

date of specific weekday in month 4-181

date system

1900 4-172, 4-258

1904 4-172, 4-258

dateaxis 2-14, **4-116**datedisp 2-5, **4-118**datefind **4-119**datemnth **4-120**datenum 2-5, **4-122**

dates

- actual days between 4-134
- business days 4-167
- cash-flow 2-10, 4-77
- coupon 2-26
- days between 4-132, 4-133, 4-134, 4-135
- determining 2-8
- first coupon 2-20
- fraction of year between 4-264
- handling and converting 2-4
- investment horizon 2-30
- issue 2-20
- last coupon 2-20
- number of months between 4-177
- quasi-coupon 2-20
- settlement 2-20
- vector of 1-19
- working days between 4-257

datestr 2-5, 2-7, **4-125**datevec 2-5, **4-128**datewrkdy **4-130**

day

- date of specific weekday in month 4-181
- of month 4-131
- of month, last 4-147
- of the week 4-255

day **4-131**

day-count basis 4-135

day-count convention 2-21

days

between

coupon date and settlement date 4-108

dates 4-132, 4-133, 4-134, 4-135, 4-257

settlement date and next coupon date 4-105

business 4-167

holidays 4-164

- in coupon period containing settlement date 4-111
 - last business date of month 4-168
 - last weekday in month 4-170
 - non-trading 4-164
 - number of, in year 4-263
- days360 **4-132**
- days365 **4-133**
- daysact **4-134**
- daysdif **4-135**
- decimal currency 4-151
 - to fractional currency 4-114
- declining-balance depreciation
 - fixed 2-17, 4-136
 - general 2-17, 4-137
- definitions 1-3
- delta 2-32, 3-12, 3-21
 - change, Black-Scholes sensitivity to underlying 4-42
- depfixdb 1-20, **4-136**
- depgendb 1-20, 2-18, **4-137**
- deprdv **4-138**
- depreciable value, remaining 4-138
- depreciation 1-20, 2-17
 - fixed declining-balance 2-17, 4-136
 - general declining-balance 2-17, 4-137
 - straight-line 2-17, 4-140
 - sum of years' digits 2-17, 4-139
- depsoyd 1-20, **4-139**
- depstln **4-140**
- derivatives
 - equity, pricing and analyzing 2-32
 - sensitivity measures for 2-32
- determining dates 2-8
- disc2zero **4-141**
- discount curve
 - from zero curve 4-279
 - to zero curve 4-141
- discount rate of a security 4-144
- discount security 4-20
 - future value of 4-155
 - price of 4-220
 - yield of 4-265
- discrate **4-144**
- dividing matrices 1-12
- dot products of vectors 1-8
- duration
 - cash-flow and modified 4-80
 - constructing a bond portfolio to hedge against 3-6
 - for fixed-income securities 2-28
 - Macaulay 2-28
 - modified 2-28
 - portfolio 3-4, 3-6
- E**
- effective rate of return 2-17, 4-145
- efficient frontier 2-37, 2-39
 - plotting an 3-19
- effrr 2-17, **4-145**
- elasticity
 - Black-Scholes 4-44
- element-by-element 1-6
 - operating 1-15
- elements, referencing matrix 1-3
- e-mail to financial products xvii
- end-of-month rule 2-22
- enlarging matrices 1-4
- eomdate **4-146**
- eomday **4-147**
- equations
 - solving simultaneous linear 1-12
- equity derivatives 2-32

- analysis models for 2-33
- European options 2-3
 - constructing greek-neutral portfolios of 3-12
- ewstats 2-39, **4-148**
- Excel date number
 - from MATLAB date number 4-172
 - to MATLAB date number 4-258
- exponential weighting of covariance matrix 4-148

F

- fbusdate **4-150**
- financial data
 - charting 2-12
- financial products information
 - e-mail address xvii
 - Web site xvii
- first business date of month 4-150
- first coupon date 2-20
- fixed declining-balance depreciation 2-17, 4-136
- fixed periodic payments
 - future value with 4-156
- fixed-income securities
 - cash-flow dates 4-77
 - Macauley and modified durations for 2-28
 - pricing 2-27
 - pricing and computing yields for 2-20
 - terminology 2-20
 - yield functions for 2-27
- fixed-income sensitivities 2-28
- formats
 - bank 4-115
 - date 2-4
- formatting currency and charting financial data 2-12
- forward curve

- from zero curve 4-282
- to zero curve 4-159
- forward price 4-39
- frac2cur 2-12, **4-151**
- fraction of
 - coupon period 4-16
 - year between dates 4-264
- fractional currency 4-114, 4-151
- frontcon 2-39, **4-152**
- frontier
 - plotting an efficient 3-19
- frontier, efficient 2-37, 2-39
- function
 - return arguments 1-19
- future month, date of day in 4-120
- future value 2-17, 4-24
 - of discounted security 4-155
 - of varying cash flow 4-157
 - with fixed periodic payments 4-156
- fvdisc **4-155**
- fvfix **4-156**
- fvvar **4-157**
- fwd2zero **4-159**

G

- gamma 2-32, 3-12, 3-21, 3-23
- general declining-balance depreciation 2-17, 4-137
- generating and referencing matrix elements 1-4
- graphics
 - producing 3-19
 - three-dimensional 3-12
- greek-neutral portfolios, constructing 3-12
- greeks 2-32, 3-12

H

handling and converting dates 2-4
 hedging 3-3
 a bond portfolio against duration and convexity 3-6
 high, low, open, close chart 4-163
 highlow 2-13, **4-163**
 holidays 2-9
 holidays 2-9, **4-164**
 holidays and non-trading days 4-164
 hour **4-165**
 hour of date or time 4-165

I

identity matrix 1-12
 implied volatility 2-33
 Black-Scholes 4-43
 indices
 of date numbers in matrix 4-119
 of nonrepeating integers in matrix 4-119
 indifference curve 2-41
 inner dimension rule 1-7
 input
 conversions 2-5
 string 1-18
 installing the Financial Toolbox xvi
 interest 4-21
 accrued 4-19, 4-20
 on loan 2-18
 interest rate swap 3-16
 interest rates
 arguments 1-20
 Black-Scholes sensitivity to change 4-47
 of annuity, periodic 4-23
 rate of return 2-16
 risk-free 3-24

sensitivity of bond prices to changes in 3-3
 term structure 2-2, 2-29

internal rate of return 4-166
 for nonperiodic cash flow 4-260
 modified 4-175
 inversion, matrix 1-12
 investment horizon 2-30
 irr **4-166**
 isbusday **4-167**
 issue date 2-20
 Ito process 2-33

L

lagging and leading moving averages chart 4-178
 lambda 2-32
 last
 business date of month 4-168
 date of month 4-146
 day of month 4-147
 weekday in month 4-170
 last coupon date 2-20
 lbusdate **4-168**
 leading and lagging moving averages chart 4-178
 left division 1-15
 leverage of an option 4-44
 linear algebra 1-7, 1-12
 linear equations 3-7
 solving simultaneous 1-12
 system of 1-12
 loan 4-21, 4-23, 4-24, 4-184
 interest on 2-18
 payment with odd first period 4-185
 periodic payment of 4-186
 lweekdate 2-9, **4-170**

M

m2xdate 2-5, **4-172**

Macauley duration 3-3

for fixed-income securities 2-28

MATLAB

date number

from Excel date number 4-258

to Excel date number 4-172

matrices

adding and subtracting 1-6

as arguments, limitations 1-20

dividing 1-12

enlarging 1-4

multiplying 1-7, 1-10

multiplying vectors and 1-9

of string input 1-18

singular 1-12

square 1-12

transposing 1-5

matrix 1-3

adding or subtracting a scalar 1-7

algebra refresher 1-6

covariance 4-148

elements

generating 1-4

referencing 1-3

identity 1-12

indices of date numbers 4-119

indices of integers in 4-119

inversion 1-12

multiplying by a scalar 1-11

numbers and strings in a 1-19

maturity

price with interest at 4-221

yield of a security paying interest at 4-266

maturity date 2-21

minute **4-174**

minute of date or time 4-174

mirr **4-175**

modified duration 3-3, 4-80

for fixed-income securities 2-28

modified internal rate of return 4-175

month

add, to starting date 4-120

date of specific weekday 4-181

day of 4-131

first business date of 4-150

last business date 4-168

last date of 4-146

last day of 4-147

month **4-176**

months

last weekday in 4-170

number of months between dates 4-177

monthsbd 4-177

movavg 2-13, **4-178**

moving averages chart 4-178

multiplying

a matrix by a scalar 1-11

matrices 1-7

two matrices 1-10

vectors 1-8

vectors and matrices 1-9

N

names

variable 1-6

NaN 2-24

negative cash flows 2-16

Newton's method 2-27, 4-43

next

business day 2-10

coupon date after settlement date 4-91

- or previous business day 4-69
- nominal rate of return 4-179
- nomrr 2-17, **4-179**
- non-trading days 4-164
- nontrading days 2-9
- normcdf 4-39, 4-41, 4-43–4-48
- normpdf 4-42, 4-43, 4-48, 4-49
- notation 1-3
 - row, column 1-3
- now 2-8, **4-180**
- number of
 - days in year 4-263
 - periods to obtain value 4-24
 - whole months between dates 4-177
- numbers
 - and strings in a matrix 1-19
 - date 2-4
- nweekdate 2-9, **4-181**

O

- odd first period
 - payment of loan or annuity with 4-185
- operating element-by-element 1-15
- operations, array 1-15
- opprofit **4-183**
- optimal portfolio 2-37
- option
 - leverage of 4-44
 - plotting sensitivities of 3-21
 - plotting sensitivities of a portfolio of 3-23
 - pricing
 - Black's model 4-39
 - profit 4-183
- output conversions, date 2-7

P

- par value 2-21
- par yield curve
 - from zero curve 4-286
 - to zero curve 4-227
- past month, date of day in 4-120
- payadv **4-184**
- payment
 - of loan or annuity with odd first period 4-185
 - periodic, given number of advance payments 4-184
 - periodic, of loan or annuity 4-186
 - uniform, equal to varying cash flow 4-187
- payodd **4-185**
- payper **4-186**
- payuni **4-187**
- pcalims **4-188**
- pcgcomp **4-191**
- pcglims **4-193**
- pcpval **4-196**
- period 2-21
- periodic interest rate of annuity 4-23
- periodic payment
 - future value with fixed 4-156
 - given advance payments 4-184
 - of loan or annuity 4-186
 - present value with fixed 4-224
- pivot year 4-122, 4-125, 4-128
- plotting
 - efficient frontier 3-19
 - sensitivities of a portfolio of options 3-23
 - sensitivities of an option 3-21
- point and figure chart 4-198
- pointfig 2-13, **4-198**
- portalloc 2-42, 2-43, **4-199**
- portcons 2-48, **4-202**
- portfolio

- convexity 3-4, 3-6
- duration 3-4, 3-6
- expected rate of return 4-212
- of options, plotting sensitivities of 3-23
- optimal 2-37
- risks, returns, and weights
 - randomized 4-209
- portfolio optimization 2-37
- portfolio selection 2-41
- portfolios
 - analyzing 2-37
 - of European stock options
 - constructing greek-neutral 3-12
- portopt 2-43, 2-47, **4-206**
- portrand **4-209**
- portsim **4-210**
- portstats **4-212**
- portvrisk **4-214**
- prbyzero **4-216**
- prdisc **4-220**
- present value 2-17
 - of varying cash flow 4-225
 - with fixed periodic payments 4-224
- previous quasi coupon date 4-102
- price
 - change, Black-Scholes sensitivity to underlying 4-41
 - forward 4-39
 - of discounted security 4-220
 - of Treasury bill 4-223
 - volatility, Black-Scholes sensitivity to underlying 4-49
 - with interest at maturity 4-221
- pricing
 - and analyzing equity derivatives 2-32
 - and computing yields for fixed-income securities 2-20

- fixed-income securities 2-27
- principal 4-21
- prmat **4-221**
- profit, option 4-183
- prtbill **4-223**
- purchase price 2-21
- put and call pricing
 - binomial 4-37
 - Black-Scholes 4-45
- pvinfos **4-224**
- pvinfos **4-225**
- pyld2zero **4-227**

Q

- quasi coupon date
 - previous 4-102
- quasi-coupon dates 2-20

R

- randomized portfolio risks, returns, and weights
 - 4-209
- rate of a security, discount 4-144
- rate of return 2-16
 - after-tax 4-234
 - effective 2-17, 4-145
 - internal 4-166
 - internal for nonperiodic cash flow 4-260
 - modified internal 4-175
 - nominal 4-179
 - portfolio expected 4-212
- redemption value 2-21
- reference date 2-26
- referencing matrix elements 1-3, 1-4
- remaining depreciable value 2-18, 4-138
- ret2tick **4-231**

return arguments, function 1-19
rho 2-32

risk aversion 2-41

risk-free interest rates 3-24

risks

returns, and weights

randomized portfolio 4-209

row, column notation 1-3

row-by-column 1-3

S

savings account 4-24, 4-156, 4-224

scalar 1-3

adding or subtracting 1-7

multiplying a matrix by 1-11

second **4-233**

seconds of date or time 4-233

securities industry association 2-20

sensitivity

fixed-income 2-28

measures for derivatives 2-32

of a portfolio of options, plotting 3-23

of an option, plotting 3-21

of bond prices to changes in interest rates 3-3

of cash flow 2-17

to

interest rate change, Black-Scholes 4-47

to time-until-maturity change, Black-Scholes
4-48

to underlying delta change, Black-Scholes
4-42

to underlying price change, Black-Scholes
4-41

to underlying price volatility, Black-Scholes
4-49

visualizing to parallel shifts in the yield curve
3-8

settlement date 2-20

coupon period containing 4-111

days between previous coupon date and 4-108

days between, and coupon date 4-105

next coupon date after 4-91

SIA 2-20

compatibility 2-20

default parameter values 2-23

framework 2-22

order of precedence 2-26

use of nonlinear formulas 2-27, B-2

SIA conventions 2-20

single quotes 1-18

singular matrices 1-12

solving

sample problems with the toolbox 3-2

spreadsheets 1-3

square matrices 1-12

straight-line depreciation 2-17, 4-140

strings

and numbers in a matrix 1-19

date 2-4, 4-125

input, matrices of 1-18

stored as character array 1-18

subtracting

a scalar and a matrix 1-7

matrices 1-6

sum of years' digits depreciation 2-17, 4-139

swap 3-16

synch date 2-26

synchronization date 2-26

system of linear equations 1-12

T

taxedrr **4-234**
tbl2bond **4-235**
term structure 2-2, 2-29, 3-3, 4-141, 4-159, 4-227,
4-235, 4-269, 4-274, 4-279, 4-282, 4-286
parameters from Treasury bond parameters
4-240
terminology, fixed-income securities 2-20
theta 2-33
three-dimensional graphics 3-12
tick labels 4-116
tick2ret **4-237**
time
current 2-8, 4-180
hour of 4-165
minute of 4-174
seconds of 4-233
time-until-maturity change
Black-Scholes sensitivity to 4-48
today 2-8, **4-239**
tr2bonds **4-240**
transposing matrices 1-5
Treasury bill 2-29
bond equivalent yield for 4-36
parameters to Treasury bond parameters
4-235
price of 4-223
yield of 4-268
Treasury bond 2-29
parameters
from Treasury bill parameters 4-235
to term-structure parameters 4-240

U

ugarch **4-243**
ugarchllf **4-245**

ugarchpred **4-247**

ugarchsim **4-250**

uniform payment equal to varying cash flow 4-187

V

variable names 1-6
vector 1-3
date 4-128
of dates 1-19
vectors
as arguments, limitations 1-20
computing dot products of 1-8
multiplying 1-8
multiplying matrices and 1-9
vega 2-33, 3-12
visualizing the sensitivity of a bond portfolio's price
to parallel shifts in the yield curve 3-8
volatility
Black-Scholes implied 4-43
implied 2-33

W

Web site for financial products xvii
week, day of 4-255
weekday
date of specific, in month 4-181
weekday **4-255**
workday, date of future or past 4-130
working days between dates 4-257
wrkdydif **4-257**

X

x2mdate 2-5, **4-258**
xirr **4-260**

Y

year

fraction of between dates 4-264

number of days in 4-263

of date 4-262

year **4-262**yeardays **4-263**yearfrac **4-264**

yield

curve 3-3, 3-6

visualizing sensitivity of bond portfolio's

price to parallel shifts in 3-8

for Treasury bill, bond equivalent 4-36

functions for fixed-income securities 2-27

of discounted security 4-265

of security paying interest at maturity 4-266

of Treasury bill 4-268

yields

for fixed-income securities, pricing and computing 2-20

yield-to-maturity 2-21

ylddisc **4-265**yldmat **4-266**yltdbill **4-268**

to par yield curve 4-286

zero2disc 2-31, **4-279**zero2fwd 2-31, **4-282**zero2pyld 2-31, **4-286**

zero-coupon bonds 4-142, 4-270, 4-276

Zzbtprice 2-30, **4-269**zbtyield 2-30, **4-274**

zero curve 4-240, 4-270, 4-276

from coupon bond prices 4-269

from coupon bond yields 4-274

from discount curve 4-141

from forward curve 4-159

from par yield curve 4-227

to discount curve 4-279

to forward curve 4-282