An energy-balance analysis of deep convective self-aggregation
above uniform SST

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Abstract

The spatial organization of deep moist convection in radiative-convective equilibrium over a constant sea-
surface temperature is studied. A 50-day simulation is performed with a three-dimensional cloud-resolving
model over a \((576 \text{ km})^2\) domain with no ambient rotation and no mean wind. The convection self-aggregates
within ten days into quasi-stationary mesoscale patches of dry, subsiding and moist, rainy air columns. The
patches ultimately merge into a single intensely convecting moist patch surrounded by a broad region of
very dry subsiding air.

The self-aggregation is analyzed as an instability of a horizontally homogeneous convecting atmosphere
driven by convection-water vapor-radiation feedbacks which systematically dry the drier air columns and
moisten the moister air columns. Column-integrated heat, water and moist static energy budgets over \((72 \text{ km})^2\)
horizontal blocks show that this instability is primarily initiated by the reduced radiative cooling of
air columns in which there is extensive anvil cirrus, augmented by enhanced surface latent and sensible heat
fluxes under convectively active regions due to storm-induced gustiness. Mesoscale circulations intensify
the later stages of self-aggregation by fluxing moist static energy from the dry to the moist regions. A simple
mathematical model of the initial phase of self-aggregation is proposed based on the simulations.

Sensitivity studies show that the self-aggregation can be suppressed by horizontally homogenizing the
radiative cooling or surface fluxes. Lower tropospheric wind shear leads to slightly slower and less pro-
nounced self-aggregation into bands aligned along the shear vector. A change in ice microphysics compared
to the control simulation creates extensive thin cirrus above the dry region via a mechanism possibly relevant
to Eocene climate.
1 Introduction

Radiative-convective equilibrium (RCE) above a horizontally homogeneous surface is a time-honored idealization for understanding the tropical atmosphere and its sensitivity to perturbations in radiative or surface forcing, starting with single-column models (e.g. Manabe and Strickler 1964) and moving on to three-dimensional (3D) cloud-resolving models (CRMs) (e.g. Tompkins and Craig 1998).

It is natural to assume that RCE will be characterized by a quasi-homogeneous pattern of cumulus convection. However, several CRM studies of RCE have suggested otherwise. This is important because the spontaneous development of large scale convective organization (‘self-aggregation’) can profoundly affect the horizontal mean temperature and moisture profiles and the surface and top-of-atmosphere radiative fluxes. These CRM studies have generally assumed no ambient rotation or mean pressure gradients. Most, but not all, have used specified radiative cooling profiles. Held et al.’s (1993) seminal study of RCE in a broad two-dimensional (2D) domain showed that the 2D convection nevertheless created vigorous downward-propagating horizontal jets due to countergradient convective momentum transport. If the jets were suppressed by forcing the domain-mean horizontal wind profile to zero, the flow self-aggregated within 10 days into first two convective centers, then after 10 more days just a single narrow stationary region of persistent and vigorous deep convection surrounded by weak subsidence and very dry conditions over the remainder of the domain. Held et al. explained this as a positive feedback between convection and water vapor, in which deep convection can more easily develop where it is already moist in the mid-troposphere, and also tends to keep the middle and upper troposphere moist. They showed that addition of modest imposed vertical shear of the domain-averaged horizontal wind in the lower troposphere could prevent self-aggregation by shearing out incipient moisture anomalies.

Tompkins (2001) elegantly extended on the Held et al. study using a ‘bowling alley’ domain with a
short (32 gridpoint) transverse dimension to permit 3D convection and a smaller, more appropriate grid spacing. However, the added computational burden limited his simulations to 15 days. His results had both similarities and important differences to Held et al.’s purely 2D simulations. The 3D convection did not induce jets due to convective momentum transport, so did not require any domain-scale velocity nudging. He again found self-aggregation developing in approximately 10 days, though now in the form of moist precipitating and dry nonprecipitating patches that slowly moved. Unlike in the purely 2D case, the flow did not focus into a single vigorous convective center within the simulated period, but Tompkins also argued that his results were due to a water vapor-convection feedback.

Observations have also suggested the importance of mid-tropospheric water vapor in organizing marine tropical deep convection. Mid-tropospheric dry intrusions over the West Pacific ocean can suppress deep convection even in the presence of substantial conditional instability (e.g. Numaguti et al. 1995). Over the Indian Ocean and West Pacific, the convectively active phase of the Madden-Julian Oscillation is preceded by a several day period of lower-tropospheric moistening that has been suggested to precondition the atmosphere for deep convection (e.g. Maloney and Hartmann 1998, Wang and Schlesinger 1999, Wheeler et al. 2000). Statistically, a strong correlation is observed over the tropical oceans between convective precipitation and column water vapor on daily and longer timescales (Bretherton et al. 2004). RCE self-aggregation simulations can help us hone our understanding of water vapor-convection interactions and the role of radiative transfer and surface fluxes in mediating these interactions.

In this paper, we take advantage of continuing advances in computational power and present CRM simulations of self-aggregation in a large square domain using a fully interactive radiation parameterization. We then quantify the feedbacks that lead to self-aggregation in our simulations by considering the column-integrated moist static energy budget of mesoscale-size blocks, and present a simple mathematical model and some sensitivity studies that elucidate the self-aggregation process.
2 Model configuration and diagnosis

We use version 6.1 of the System for Atmospheric Modeling (SAM), which is an updated version of the Colorado State University Large Eddy Simulation / Cloud Resolving Model (Khairoutdinov and Randall 2003). The model uses the anelastic equations of motion with bulk microphysics. The prognostic thermodynamic variables are the mixing ratio of total non-precipitating water $q_l$ (composed of water vapor $q_v$ and cloud condensate $q_c$, which is partitioned based on temperature $T$ between cloud water $q_l$ and cloud ice $q_i$), total precipitating water $q_p$ ($T$-partitioned between snow, which has a $T$-dependent fallspeed, and rain, which has an intensity-dependent fallspeed), and liquid-ice static energy $s_{li} = c_p g z - L(q_l + q_i) - L_f q_i$, where $c_p$ is the isobaric specific heat of dry air, $g$ is the gravitational acceleration, $L$ is the latent heat of vaporization, and $L_f$ is the latent heat of freezing, all assumed constant.

A Kessler scheme for autoconversion of cloud liquid water to rain is used. Cloud ice is assumed to fall at 0.4 m s$^{-1}$ and autoconverts to snow at a $T$-dependent rate. In SAM 6.1, an autoconversion threshold of 0.1 g kg$^{-1}$ for cloud ice to snow is chosen—this is used in a sensitivity study MOREICE, but for all other simulations presented here this autoconversion threshold is set to zero to prevent excessive production of radiatively active thin cirrus.

We use a Smagorinsky-type parameterization for subgrid-scale turbulent fluxes. The surface turbulent fluxes are computed using Monin-Obukhov similarity theory. The longwave and shortwave radiation schemes are taken from the National Center for Atmospheric Research (NCAR) Community Climate Model (CCM3) (Kiehl et al., 1998). The effective radii of cloud water and cloud ice are assumed to be 10 $\mu$m and 10-30 $\mu$m, respectively, and precipitating water/ice is assumed to be radiatively negligible due to its large effective radius. Similar to Tompkins and Craig (1998), we remove the diurnal cycle by reducing the solar constant to 650.83 W m$^{-2}$ and fixing the solar zenith angle at 50.5°. The readers are referred to
Khairoutdinov and Randall (2003) for further details about the model.

We use 64 vertical grid levels with spacing of 75 m near the surface, smoothly increasing to 500 m above 2 km up to the domain top at approximately 28 km. The small vertical grid spacing in the boundary layer allows better resolution of evaporatively-driven cold pools generated by deep convection, and of the vertical structure of the boundary layer. In a sponge layer from 19.5-28 km, all prognostic variables are relaxed toward their domain-mean values with damping times ranging from 2 hours at the sponge bottom to 2 minutes at the domain top. We use a $576 \times 576$ km doubly periodic domain with horizontal grid spacing of 3 km. This configuration was chosen to allow long simulations on as large a domain as we could efficiently simulate on the Linux cluster used for these computations. A 50-day simulation takes 3 days of computation time on 8 dual-processor 2.3GHz Opteron nodes.

We specify a sea-surface temperature (SST) of 301 K, no ambient rotation, and no initial wind or large-scale pressure gradient. Before starting the full-domain simulation, we performed a 50-day small-domain (SD) radiative-convective equilibrium (RCE) simulation on a small horizontal domain (96 by 96 km). This domain is too small to permit noticeable self-aggregation. The domain-mean SD-RCE temperature and water vapor profiles averaged over days 30-50 were used as the initial profiles for the large-domain (LD) simulations.

The SD-RCE simulation was identically configured and forced to simulation MOREICE (which used the default SAM6.1 ice microphysics), except for domain size. Although we later decided to remove the autoconversion threshold for cloud ice for our other simulations, we continued to use the above SD-RCE simulation for the initial large-domain sounding for convenience. We have also performed SD-RCE simulations corresponding to all of the LD model configurations discussed in this paper. In all cases, their domain-mean equilibrium soundings are quite similar, varying up to a degree in temperature and ten percent in relative humidity.
To initiate convection, white noise is added to the initial $s_{ti}$ field at the five lowest grid levels, with an amplitude of 0.1 K at the lowest level, linearly decreasing to 0.02 K at the fifth level. Drifts of the mean profiles in the LD simulation away from their initial specifications are associated with the self-aggregation process and model physics differences; the former dominates when self-aggregation occurs. In both the SD and LD simulations, but in contrast to the two-dimensional simulations of Held et al. (1993), cumulus momentum transport does not spontaneously create significant mean vertical shear and the domain-mean wind remains close to zero at all levels at all times.

Simulations identical to the base run presented in this paper except with 2 km horizontal resolution on a $384 \times 384$ km domain gave similar results to the 3 km results presented here in this paper, though self-aggregation took slightly longer to initiate. They will not be discussed further.

2.1 Block-averaging

To focus on mesoscale organization, we will make extensive use of block-averaged daily-mean fields. We horizontally partition the computational domain into 64 (72 km)$^2$ blocks. We horizontally average selected 2D and 3D model outputs over each block. For the 2D output fields, we average the 24 saved hourly-average values for a given day to get a numerically exact daily average for each block. For the 3D output fields, we average the four stored 6-hourly instantaneous values per day to get an approximate daily average for each block.

2.2 Moist static energy and vertical averaging

Moist static energy is a useful thermodynamic variable for studying precipitating convection because it is approximately conserved in adiabatic displacements of fluid parcels, and only weakly affected by precipitation and evaporation of liquid water. To be consistent with the thermodynamic formulation of the CRM, we
work in terms of a 'frozen' moist static energy (FMSE) which is exactly conserved by the CRM governing equations following adiabatic fluid parcel displacements,

\[ h_f = s_{ti} + Lq_l = c_{p}T + gz + Lq_v - Lfq_i, \]

which differs from the conventional definition of moist static energy only in including the ice freezing term. From here on, when we refer to moist static energy we will mean FMSE.

We will consider budgets of FMSE and other quantities integrated over the atmospheric column. They will be phrased in terms of mass-weighted vertical integrals, denoted \( I_{hf}, I_{qt}, \ldots \) for FMSE, total water, etc., or mass-weighted vertical averages denoted with angle brackets, e.g. \( \langle h_f \rangle = g I_{hf}/p_{surf} \), where \( p_{surf} \) is the reference surface pressure.

3 Results

Fig. 1 shows horizontal maps of water vapor path (WVP), outgoing longwave radiation (OLR), precipitation \( P \) and total heat flux (THF), defined as the sum of sensible heat flux SHF and latent heat flux (LHF), with the surface wind vector superposed, averaged over day 10. Self-aggregation, in the form of a developing 'dry hole' centered at \( x = 180\,\text{km}, y = 350\,\text{km} \), is beginning. A slight minimum in WVP is collocated with suppressed precipitation and surface fluxes and larger OLR. By day 20 (Fig. 2), the dry hole greatly amplifies and expands northwestward. During this amplification phase, the WVP pattern mainly amplifies in place, with only slow spatial shifts in the locations of maxima and minima. By day 50 (Fig. 3), the moist convecting region focuses into a single, stationary, quasi-circular patch of intense precipitation and low OLR surrounded by very dry subsiding air with high OLR. Within the core of this region, winds are light and surface humidity very high, so there is a pronounced THF minimum. Strong surface winds associated with low-level convergence into this patch drives large latent heat flux around its periphery. In the subsiding
regions, THF is also elevated due to the low boundary layer humidity.

Fig. 4 shows the horizontally-averaged profiles of relative humidity (left) and moist static energy $h_f$ and saturation moist static energy $h_s$ (right) averaged over days 1 and 50. The relative humidity sounding highlights the intense drying by day 50 over most of the domain outside the small region of intense convection. The $h_s$ sounding shows considerable warming in response to the self-aggregation. This warming is a consequence of the elevated moist adiabat impressed on the entire domain by the very moist, high $h_f$ lower tropospheric air in the convective region. The mean $h_f$ profile decreases in the lower troposphere due to drying, and increases in the upper troposphere due to warming, but the mass-weighted vertically integrated moist static energy is not greatly changed during the simulation.

Fig. 5 shows the evolution toward lower domain-mean water vapor path and higher mass-weighted temperature during self-aggregation. The vertical scales have been chosen in the ratio $g L/(c_p \rho_{surf})$ so that a given variation in either quantity corresponds to the same change in column-integrated moist static energy.

### 3.1 Moisture-sorted time series

Because the dry holes move only slowly as they develop, one can quantify their development by sorting the $(72 \text{ km})^2$ blocks into quartiles by their daily block-average WVP, and plotting the time evolution of these quartiles (Fig. 6).

The WVP (Fig. 6a) of the driest quartile decreases almost four-fold in the first forty days. In the moistest quartile, WVP increases about 10% over this period. The WVP difference between these quartiles increases from less than 4 mm at day 10 to 25 mm at day 25, from which one can estimate an e-folding timescale of 8 days. Because the WVP decreases in the dry regions are much larger than the WVP increases in the moist regions, there is a 30% drop in mean WVP (thick black curve) by day 40. This is one example of how convective self-aggregation profoundly influences the horizontal-mean characteristics of the simulated...
atmosphere. After day 40, the moistest WVP quartile begins to dry because the moist area of active convection has focused into less than a quarter of the domain. More surprisingly, the driest WVP quartile begins to moisten slightly as the circulation comes into a steady state. This is in part because the surface winds and moisture fluxes are intensifying as the convection self-focuses (Fig. 6c).

The daily quartile-averaged precipitation (Fig. 6b) tracks the quartile-average WVP trends, but shows more daily variability about the trends. After day 20, there is no precipitation in the driest quartile, as the convection becomes increasingly focused into the moistest region. The strong relation between precipitation and WVP on daily and longer timescales is what gives the self-aggregation its ‘memory’ and will prove a key ingredient in mathematically modeling this phenomenon.

In quasi-steady RCE, there must be a domain-averaged balance between THF (Fig. 6c) and net column-integrated radiative cooling $\Delta R$ (Fig. 6d). In addition, there must be domain-averaged moisture balance between latent heat flux (which dominates the THF) and precipitation. Because this simulation is initialized from a small-domain simulation in RCE, precipitation, THF and radiative cooling all start out nearly in balance, near 100 W m$^{-2}$. As the simulation evolves due to self-aggregation, there is domain averaged warming and drying, so these balances are affected by storage. However, they remain approximately valid. The quartile-average THF shows that the moistest columns tend to have slightly enhanced THF in the early stages of self-aggregation up until day 20 due to convective gustiness. This is a positive feedback on self-aggregation, since it feeds energy into the moistest columns, which have the highest $\langle h_f \rangle$. After day 20, the reverse is true as large air-sea humidity differences start to amplify the THF into the driest columns, and winds from dry to moist regions increase the THF in the transitional regions. These effects also contribute to a gradual increase in domain-mean THF. The quartile-binned radiative feedbacks (Fig. 6d) show there is stronger atmospheric radiative flux divergence from the drier quartiles, reflecting anvil greenhouse warming of the moist, more convectively active columns. Like the surface flux, radiative cooling is a positive feedback.
on self-aggregation, since it preferentially removes energy from the driest columns. After day 35, all three dry quartiles are radiatively cooling at a similar rate, since the driest columns no longer have enough water vapor to cool more efficiently than somewhat moister columns.

### 3.2 Power spectra

Fig. 7 shows horizontal power spectra of daily-mean WVP and precipitation at the average scalar quantities from the full horizontal resolution data at days 10, 20 and 50. These were computed in the $x$ direction at each $y$ gridpoint, then averaged across all $y$ gridpoints. The WVP spectrum develops power at large scales between days 10 and 20, while at small scales it reduces between days 20 and 50 due to suppression of convection (a source of small-scale WVP variance) over much of the domain. The precipitation also develops more power at large scales by day 50, when domain-scale self-aggregation has matured.

### 3.3 Weak temperature gradient approximation

A key simplifying feature in understanding these simulations is that stratified adjustment efficiently removes mesoscale horizontal temperature (or more precisely, density) gradients. This is illustrated in Fig. 8, which shows day 16-20 mean quartile-binned profiles of the perturbation from the horizontal mean of density temperature $T_p = T(1 + 0.608q_v - q_c)$, where $q_c$ is the combined mixing ratio of all cloud condensate plus precipitating hydrometeors. Except within 1 km of the surface, these perturbations are extremely small, much less than 0.1 K. The condensate contribution to column integrated $T_p$ is very small compared to the contributions of temperature and water vapor. Hence, at any time the column-integrated difference (denoted by $\delta$) of frozen moist static energy between any two blocks must essentially reflect their difference in WVP:

$$\delta I_{hf} \approx (L - 0.608c_p(T)) \delta W$$  \hspace{1cm} (1)
In the lowest 1 km, the moistest quartile has the highest $T_p$. This helps hydrostatically induce relatively low surface pressure that drives boundary layer mass convergence into the moistest regions against the retarding effects of surface drag.

### 3.4 Precipitation and humidity

Another key feature of the simulated convection, already seen in Fig. 6b, is its strong correlation with humidity, as also clearly seen in satellite observations of tropical oceanic convection (Bretherton et al. 2004). Since this will prove central to developing a simple model of self-aggregation, we now quantify it. Following Raymond (1999) and Bretherton et al. (2004), we normalize WVP into a column relative humidity (CRH), defined as the ratio of WVP to the saturation water vapor path of the atmospheric column (i.e. the water vapor path were the entire air column moistened to 100% relative humidity with respect to water without change in temperature). Since deep convection involves microphysical and turbulent processes that tightly involve relative humidity, we expect CRH to be better tied to precipitation than is WVP. Bretherton et al. (2004) found that this is borne out by observations. Note that in our simulations, the saturation WVP (which depends only on temperature) is essentially horizontally uniform on mesoscale lengthscales and daily timescales, so horizontal variations of CRH are essentially equivalent to those of WVP. Bretherton et al. (2004) found an exponential increasing dependence of $P$ on CRH on daily timescales, with considerable scatter that could be considerably reduced by further time-averaging.

Fig. 9 shows a scatterplot of the five day means of block-averaged precipitation and CRH for the 64 blocks in the domain during three stages of the simulation. During the two later stages, some blocks had CRH < 0.4; all such blocks had no precipitation. In all three stages, precipitation increases strongly with CRH. While this relationship is somewhat stage-dependent, the exponential fit plotted on Fig. 9 matches all stages fairly well. This relation has the same form as the observationally-derived daily mean fit of Eq. (1)
of Bretherton et al. (2004), and matches that relationship for a CRH near 0.8, but has a somewhat stronger dependence of $P$ on CRH.

4 CRH-binned vertical motion, humidity, and cloud profiles

The self-aggregation also modulates the vertical structure of the vertical motion, relative humidity, condensate and hydrometeor profiles. Fig. 10 shows these profiles averaged over days 16-20, binned into the four quartiles of block-averaged CRH. The moistest quartiles have a ’top-heavy’ structure with accentuated upper-tropospheric upward motion and condensate. These quartiles have a higher relative humidity than the mean at all levels above the boundary layer. This variation in vertical structure can be interpreted in terms of the interplay of cumulus clouds with environmental humidity. For a given virtual temperature profile, cumuli are less susceptible to entrainment-induced drying and evaporative cooling when their environment is moister. Hence, the clouds will reach deeper where it is moist, and the associated block-averaged vertical motion will therefore also be more top-heavy. The deepest clouds will also develop larger stratiform anvils, further accentuating the top-heavy vertical motion, condensate and relative humidity profiles in the moist regions.

4.1 Budget analysis

Both diabatic and adiabatic processes contribute to the drying of dry air columns and moistening of moist air columns that occurs during self-aggregation. To quantify their roles, we consider the block-averaged heat ($e_{ti}$) and moisture ($q_t$) budgets partitioned by block-averaged WVP. Recalling that a mass-weighted vertical integral over the entire atmospheric column is denoted by $I$, these budgets have the following form.
in energy units of W m$^{-2}$:

$$\frac{dI_{si}}{dt} = LP + C_{si} + SHF + \Delta R,$$

(2)

$$L \frac{dI_{q}}{dt} = -LP + LC_{q} + LHF,$$

(3)

$$L \frac{dI_{hf}}{dt} = C_{hf} + THF + \Delta R.$$

(4)

A term $C_{0}$ indicates column-integrated net horizontal advective convergence, for instance $C_{s} = -\nabla \cdot (\mathbf{us})$, where $\mathbf{u}$ is the horizontal velocity vector. $\Delta R$ is the column-integrated radiative flux divergence. The $h_f$ budget is the sum of the $s_{li}$ (‘heat’) budget plus the $L_{q_i}$ (‘moisture’) budget. These budgets are exactly preserved by the CRM, except for nudging of $s_{li}$ in the sponge layer, which contributes negligibly to the column integrated heat source in this simulation. Thus we can derive the advective terms (which require infrequently stored 3D fields to compute directly) as budget residuals given the other terms, which we calculate continuously during the simulations.

Fig. 11 shows the heat and moisture budget terms averaged over days 6-10, and sorted by CRH. The column diabatic $s_{li}$ source $LP+SHF+\Delta R$ is negative in the dry columns, where it primarily reflects column-integrated radiative cooling, and positive in the moistest columns due to latent heating. Since there are many more moist than dry columns, there is little domain-averaged diabatic tendency of $s_{li}$. The advective tendency nearly exactly compensates the diabatic tendency to maintain all columns at the same temperature. The near-zero $s_{li}$ storage term for all blocks indicates that the domain-mean temperature is nearly steady during this time. The moisture budget shows diabatic moistening of the driest columns by surface fluxes and diabatic drying of the moist columns due to the additional effect of precipitation. The advective moisture convergence overcompensates these trends; the storage term shows the dry columns getting systematically drier.

Since both of these budgets include precipitation, which can be thought of as a consequence of the
convection and not a cause, it is also illuminating to consider the corresponding budget of moist static energy $h_f$ (Fig. 12a), which does not explicitly involve precipitation. The diabatic $h_f$ source removes energy from the dry columns, where radiative cooling exceeds surface total heat flux, and feeds it into the moist columns. The advective $h_f$ source also removes energy from the dry columns and the moistest columns, while feeding it into blocks with $0.62 < \text{CRH} < 0.72$, which have intermediate precipitation rates. We will shortly discuss this behavior, but the main result is to reinforce the preferential energy removal from the driest columns. Because all columns must maintain roughly the same temperature profile, this preferential energy removal must be manifest as drying of the driest columns, as we saw in Fig. 11b.

Figs. 12b-c show the moist static energy budget averaged over days 16-20 and 46-50, two representative later stages of self-aggregation. Note the expanded axis ranges in these panels. By days 16-20, the diabatic sink of $h_f$ in the drier columns is quite weak due to enhanced latent heat fluxes due to near-surface drying combined with inefficient radiative cooling due to the extreme dryness of the driest air columns. However, there is still substantial advective divergence of $h_f$ out of most dry blocks which sustains the net energy loss (drying) of the driest columns seen in the storage term. Thus, at this stage self-aggregation is dynamically driven even while it has become diabatically damped. In the final stage, days 46-50, the storage term has slightly reversed and the drier columns are no longer preferentially losing energy. The advective and diabatic $h_f$ tendencies have become weakly positive/negative respectively in the dry columns, while there is strong advective divergence of $h_f$ out of the moistest columns, balanced by diabatic $h_f$ increase in these columns. The convective organization has by this point evolved to compensate for the mechanisms that initiated self-aggregation.
4.2 Negative gross moist stability?

It initially surprised us that there is strong net advective divergence of $h_f$ out of dry regions during self-aggregation. Export of moist static energy out of regions of mean subsidence corresponds to negative ‘gross moist stability’ (Neelin and Held 1985). If one naively imagines air converging into these regions in the upper troposphere where $h_f$ is relatively high, and diverging out of dry regions in the lower troposphere where $h_f$ tends to be smaller, one would predict net convergence of $h_f$ into dry regions.

However, Fig. 13 shows that the mesoscale circulations that develop during self-aggregation differ somewhat from this preconception. The contours in this figure visualize the day 16-20 mean circulation pattern as an effective streamfunction $\Psi$, derived as follows. The 64 blocks are ordered from lowest to highest CRH, and given an index $i - 1/2, i = 1, 2, ..., 64$. Then, starting with $\Psi_0 = 0$, $\Psi$ is calculated as a horizontal integral over vertical velocity starting with the driest column,

$$\Psi_i(z) = \Psi_{i-1}(z) + \overline{\rho}(z)w_{i-1/2}(z),$$

where $w_{i-1/2}(z)$ is the block-average vertical velocity profile and $\overline{\rho}(z)$ is the reference density profile used in the anelastic governing equations. $\Psi_i(z)$ can be interpreted as the net upward mass flux at height $z$ accumulated over the $i$ driest blocks. The shading in Fig. 13 shows the moist static energy perturbation from the horizontal mean, which is essentially a measure of the humidity of each block at each level. Since we have destroyed the topology of the original circulation in the CRH-sorting of blocks, we should be careful about overinterpreting details of the apparent horizontal advective tendencies of $h_f$ implied by this streamfunction, but it does capture the general mechanisms of $h_f$ exchange between drier and moister columns.

The mesoscale inflow to the moist blocks (40-64) is almost entirely in the lowest 1 km, and the outflow is mainly between 10-12 km. In the subsidence region (the driest quartile of blocks), radiative cooling drives
the downward motion. The radiative cooling profile is controlled by water vapor and clouds, which are in turn affected by vertical and horizontal advection. The driest columns have little mid-tropospheric water vapor or cloud and relatively weak mid-tropospheric cooling, but strong cooling at the top of the moist near-surface boundary layer. Hence the subsidence maximizes at the boundary layer top (1 km elevation), where it is twice as strong as 500 m further aloft. A lateral inflow of air from above the PBL is needed to feed this subsidence maximum. This lateral inflow is dry and hence has low moist static energy. Coupled to PBL export of air whose moist static energy has been raised by surface heat fluxes, this circulation creates advective divergence of $h_f$ out of the driest columns. A similar mechanism may help force shallow cross-equatorial overturning circulations observed in the central Pacific ITCZ (e.g. Zhang and McGauley 2004), where the air just south of the equator is in a very dry trade-wind regime compared to the deep convection north of the equator.

4.3 Role of gravity waves in convective initiation

Mapes (2000) has emphasized the role of 'second-mode’ convectively generated gravity waves (i.e. with roughly two half-wavelengths spanning the troposphere) in initiating new deep convection. Tulich and Randall (2004) showed that in simulations of radiative-convective equilibrium in a very large (4000 km long) 2D domain, the dominant mode of convective organization was squall lines that were tied to these second-mode gravity waves in a form of wave-CISK.

In our 3D simulations and the bowling alley simulations of Tompkins (2001), water vapor/convection feedback dominates and creates a nearly stationary mode of self-aggregation. Presumably, the 3D geometry, which allows the second-mode gravity waves to propagate isotropically in all horizontal directions, is less favorable to development of a wave-CISK like instability than a 2D domain. However, we still discerned the second-mode mechanism of convective initiation coexisting in our simulations. We projected the simu-
lated temperature and moisture anomalies from the horizontal mean onto ‘first’ and ‘second’ vertical Fourier modes, with half-wavelengths equal to 15 and 7.5 km, and stored these projections every 15 minutes. By animating time-sequences of the second-mode projection of temperature, we saw a random ‘bath’ of waves initiated by prior convection propagating across the domain. We also saw new convection preferentially initiating when a lower-tropospheric cold anomaly propagated into a region, presumably as a result of reduced convective inhibition and favorable conditions for deepening of shallow convection.

5 A semi-empirical mathematical model of self-aggregation

In this section, we develop a simple semi-empirical mathematical model for the initiation of self-aggregation in the CRM, based on the above results. The model aims to predict the initial e-folding rate of self-aggregation, but not its scale or ultimate evolution. The column-integrated moist static energy equation is used to derive an ordinary differential equation (ODE) for the CRH \( r \) in each mesoscale gridbox, in which the equilibrium solution, corresponding to horizontally uniform radiative convective equilibrium, can be unstable.

The assumptions are:

1. The temperature profile doesn’t change in time in any block. As seen in Fig. 5, this is an acceptable assumption during the early stages of self-aggregation (days 5-15), even though this is followed by significant domain-mean warming.

2. Precipitation is an exponentially increasing function of CRH, following Fig. 9). The exponential fit can be recast as follows:

\[
P(r) = P_{RCE} \exp(a_m[r - \tau_{RCE}]}, \quad a_m = 16.6,
\]  

(5)
where $P_{RCE} = 3.5 \text{ mm d}^{-1}$ is the horizontal mean radiative-convective equilibrium rain rate, and $r_{RCE} = 0.72$ is the CRH corresponding to that rainrate.

3. The combined diabatic (surface flux and cloud-radiative) feedbacks are linear with the form

$$\text{THF} + \Delta R = c_{sr} L(P - P_{RCE}). \quad (6)$$

These feedbacks are phrased in terms of precipitation rather than WVP since on physical grounds, we anticipate that they should fundamentally depend on the amount of convection. In nonaggregated RCE ($P = P_{RCE}$), this forcing is the only forcing on the domain-mean moist static energy, and therefore it must be zero to maintain a steady state. Fig. 14 shows day 6-10 mean scatterplots of block-average THF and $\Delta R$ vs. $P$, both of which exhibit linear relationships with only slight scatter. The least-squares best-fit lines have slopes 0.12 and 0.17, respectively. On this basis we choose the nondimensional constant $c_{sr} = 0.29$. By days 16-20, THF systematically increases in the dry regions as boundary layer air dries and large-scale circulations organize in these regions, almost eliminating the surface flux feedback (not shown). However, the radiative feedback remains almost unchanged. The solid curve in Fig. 12a shows the fit (6), with (5) used to predict $P$ in terms of CRH, to the block-averaged day 6-10 diabatic forcing of column moist static energy. Again, the fit is quite good at this time, though it does not extend into later stages of self-aggregation. There is a small mean offset associated with domain-mean drying, because the initial sounding was derived from an SD-RCE simulation with slightly different physics than the control run.

4. The moist static energy convergence during the onset of self-aggregation (here taken as days 6-10 of the simulation) can be modelled using the form

$$C_h = \alpha_h L(P - P_{RCE})(r_h - r), \quad r_h = 0.62, \quad \alpha_h = 1.8 \quad (7)$$
This fit, shown as the dashed curve on Fig. 12a, is an idealized representation of the product of two factors, (a) the divergent mass circulation associated with diabatically-driven vertical motions in the column driven by the difference between precipitation heating and radiative cooling, here parameterized as \( L(P - P_{RCE}) \) and (b) cumuli in a moister environment with larger \( r \) having a more ‘top-heavy’ distribution of latent heating and associated vertical motion, creating more moist static energy divergence per unit of upward mass flux. For \( r < r_h \) (shallow convection), there is energy divergence out of subsiding regions (low-\( h_f \) inflow and high-\( h_f \) outflow), corresponding to negative gross moist stability.

Fig. 12a shows that the FMSE convergence in individual blocks scatters considerably about the curve fit, rendering this fit much more uncertain than the diabatic forcing. Given this uncertainty, we choose a value of \( \alpha_h \) in (7) that leads to self-aggregation in our idealized model with an e-folding time similar to that observed.

At later times during self-aggregation, \( C_h \) has a qualitatively similar structure in the range \( 0.65 < r < 0.8 \), but develops a positive offset from the above fit curve due to net import of moist static energy from the now well developed dry regions with \( r < 0.6 \) into the moister regions.

This convergence term is dependent on the column vertical motion profile, which is affected by its radiative cooling profile. Thus, the advective and diabatic parts of the moist static energy tendency are very tightly intertwined. Hence, if we artificially remove the true feedbacks between water vapor and cloud and radiative cooling, e. g. by specifying horizontally uniform radiative cooling, this will affect not only the diabatic but also the advective forcings that govern self-aggregation.

Furthermore, both the diabatic and advective forcings are not entirely locally determined. As mean circulations strengthen, the they increase the surface heat flux in the transitions between the dry and
moist regions. This effect depends on the circulation scale, which in turn depends on the domain size. The mean density temperature profile also adjusts to keep the domain-integrated convective mass fluxes in balance with radiatively driven compensating subsidence at all levels. Thus, the parameterizations we are using for the diabatic and advective forcings cannot be expected to remain valid throughout the entire self-aggregation process, or even if the domain size is changed. Instead, they are only fits to the initial stages of self-aggregation, useful mainly in rationalizing the stability of the system to self-aggregation.

Before combining these assumptions into the desired ODE, we note two consequences of the horizontal and temporal uniformity of the temperature profile that combine to simplify the storage term for column moist static energy. The first consequence is that the saturation WVP (which depends only on the temperature profile) is uniform and equal to its initial (RCE) value, \( W_* = W_{RCE}/\tau_{RCE} = 57 \text{ mm} \). To obtain the second consequence, we also note that the column-integrated condensate is comparatively small. Hence, the column-integrated liquid-ice static energy \( I_{sci} \), which depends on column-integrated temperature and column-integrated condensate, is approximately time-independent and horizontally uniform.

The column moist static energy storage can be approximated as

\[
\frac{dI_{hI}}{dt} = \frac{dI_{sci}}{dt} + L \frac{dI_{qt}}{dt}
\]

By the above assumptions, the first term on the right hand side is approximately zero. The second term on the right hand side can be rewritten by noting that the water vapor path \( W = \tau W_* \) makes up almost all of the column-integrated water \( I_{qt} \). Hence

\[
\frac{dI_{hI}}{dt} \approx L \frac{dW}{dt} = LW_* \frac{d\tau}{dt}
\]

Substituting the above approximations (5), (6), (7), and (8) for the precipitation, diabatic feedbacks, convergence feedbacks, and storage, respectively, into the column-integrated moist static energy budget (4),
we obtain:

\[
\frac{dr}{dt} = G(r) \equiv (c_{sr} - \alpha_h [r - r_h])(P(r) - P_{RCE})/W_s.
\]  

The equilibria of this ODE are the roots of \( G \). One root is at \( r_{RCE} = 0.72 \) and another is at \( r_{max} = r_h + c_{sr}/\alpha_h = 0.78 \), at which the equilibrium precipitation is \( P_{max} = 9.5 \text{ mm d}^{-1} \). Note that \( r_{max} \) is highly sensitive to \( \alpha_h \), which is poorly constrained by our budget diagnosis.

Each equilibrium is stable if and only if \( dG/dr < 0 \). From (9), for our choice of parameters,

\[
\frac{dG}{dr}(r_{RCE}) = (c_{sr} - \alpha_h [r_{RCE} - r_h])(\frac{dP}{dr})/W_s
\]

\[
= 0.11 \text{ d}^{-1},
\]

so \( P_{RCE} \) is an unstable equilibrium. Similarly, one can show that \( P_{max} \) is stable. Thus this model predicts that columns that start with \( r > r_{RCE} \) will moisten to an equilibrium CRH of \( r_{max} \), while columns with \( r < r_{RCE} \) will continue to dry indefinitely.

The e-folding time of the self-aggregation instability about radiative-convective equilibrium is

\[
\tau_{sa} = \left\{ \frac{dG}{dr}(r_{RCE}) \right\}^{-1}.
\]

For our parameters, \( \tau_{sa} = 9 \text{ days} \), in close agreement with Fig. 6a. If we removed either the cloud-radiation and surface flux feedbacks, the RCE equilibrium would become slightly stable, and this model would predict self-aggregation should no longer occur. This prediction is verified in Section 6.

This model has many caveats. As already noted, the representation of convergence, surface flux and radiation feedbacks are physically-motivated curve fits for the initial stages of self-aggregation, and are not accurate for other times or domain sizes. The convergence term, and the relationship between water vapor and precipitation both have a substantial random element. We have experimented with adding a noise term to the ODE to represent this randomness; it does not disrupt the basic self-aggregation mechanism or time scale.
6 Sensitivity studies

In this section, we present four simulations that illustrate important sensitivities of self-aggregation to different physical processes, and compare these to our control simulation BASE. The first, MOREICE, adds back the SAM6.1 default autoconversion threshold from cloud ice to snow; this has the effect of increasing the lifetime and radiative impacts of cirrus cloud. The second simulation, SHEAR, adds weak lower-tropospheric mean wind shear. In the third simulation, UNIRAD, local cloud-radiation feedbacks are removed by applying horizontal mean radiative heating rate profile to all grid columns. In the fourth simulation, UNISFLX, an analogous approach is used to remove the local feedbacks between surface turbulent fluxes and convection.

6.1 MOREICE

In simulation MOREICE, the threshold mixing ratio for autoconversion from cloud ice (which is radiatively active) to snow (which is not) is raised from zero to 0.1 g kg$^{-1}$. This single change from the control run, which enhances the radiative impact and longevity of simulated cirrus clouds, profoundly affects the self-aggregation simulation in the upper troposphere. Starting from the same initial state, self-aggregation of WVP and precipitation occurs in a qualitatively similar way as in the control simulation, but starts more slowly. Fig. 15 shows MOREICE WVP and OLR averaged over day 25, in a stage of self-aggregation roughly equivalent to day 20 of BASE. The OLR is very low over much of the prominent ‘dry hole’, reflecting the presence of widespread high thin cirrus over these regions. Fig. 16 shows the corresponding CRH-sorted streamfunction and $h_f$ profiles, to be compared to Fig. 13. In MOREICE, there is an intense reverse circulation near the tropopause, with rising motion and considerable cloud ice (i.e. radiatively active cirrus) in the dry regions and sinking motion over the moist, convectively active regions below. This
circulation appears to be radiatively driven by strong longwave heating at the base of the cirrus. Where the underlying air columns are dry, the upwelling longwave flux becomes very large, reflecting the high underlying SST. Thus the cirrus heating is strongest over the dry columns, inducing mean lifting near the tropopause in these regions, promoting further cirrus cloud development. This cirrus is actually separated from the cumulus anvils, though it does rely on the moisture detrained in those anvils. The cirrus formation slows down the self-aggregation by partially removing the anvil greenhouse feedback in which actively convective columns radiatively cool less rapidly.

As mentioned earlier, the ice microphysics in MOREICE are the default in the SAM model, and a nonzero autoconversion threshold for cloud ice to snow (though perhaps not as large as 0.1 g kg$^{-1}$) is plausible. However, this feedback of thin cirrus over dry regions is not generally observed over the Tropics. In addition to uncertainty about the optimal autoconversion formulation, another consideration is that the SST under dry regions of the Tropics is significantly less than under the moist region, reducing the maximum possible upwelling longwave radiation available for cirrus heating above the dry regions. However, MOREICE does emphasize the possible role for vertical velocity feedbacks in maintaining thin and subvisible tropopause cirrus that has drifted away from convective systems.

6.2 SHEAR

In simulation SHEAR, the model physics are identical to BASE, but the domain mean wind velocity is nudged with a relaxation time of 2 hours toward a profile with zero surface wind, constant 5 m s$^{-1}$ westerlies above a height of 5 km, and a $10^{-3}$ s$^{-1}$ westerly shear below 5 km. This shears out moisture anomalies in the $x$ direction, which might inhibit self-aggregation as found by Tompkins (2001) in a 'bowling alley' shaped domain. In our square domain, self-aggregation is slowed compared to the control run, but not halted, because the water vapor-convection feedback can still work in the $y$ direction. Fig. 17 shows the WVP
quartile time series for this simulation. After 50 days, the driest quartile is approximately two-thirds as moist as the moistest quartile. Fig. 18, the day-50 average structure, is characteristic of the entire latter half of the simulation, when self-aggregation has set in. It shows the expected zonal striping of convection, WVP and OLR by the westerly shear. An interesting feature of this simulation is the mean westerlies in the dry, subsiding branch of the circulation, brought down by the mean subsidence. These help increase the mean wind speed and latent heat flux in the dry band, nearly cancelling the convective gustiness feedbacks which promote self-aggregation in BASE. This is likely a major factor in the slower self-aggregation in SHEAR compared to BASE.

### 6.3 UNISFLX

In simulation UNISFLX, the surface turbulent flux is horizontally averaged and then applied to all model columns. This removes the gustiness feedback on initial self-aggregation, but does not interfere with the positive feedback between low-level radiative cooling and net moist static energy export in dry columns that helps amplify self-aggregation in BASE. However, this simulation does not self-aggregate, retaining nearly the initial mean sounding for 50 days except for slight drying from a mean WVP of 44.5 to 42 mm in the first 20 days due to the different cloud ice autoconversion threshold than in the small-domain RCE simulation.

### 6.4 UNIRAD

In simulation UNIRAD, the radiative heating profile is horizontally averaged and then applied to all model columns, thus removing horizontal radiation-cloud-water vapor feedbacks. The surface turbulent fluxes are not homogenized. Again no self-aggregation occurs.
7 Conclusions

Using CRM simulations in large 3D domains with no mean wind and no ambient rotation, we have analyzed the feedbacks that induce convective self-aggregation in radiative-convective equilibrium above constant SST. This self-aggregation is based on convection forming preferentially in regions with a moist mid-troposphere. Both cloud-radiation feedbacks and convective gustiness help initiate self-aggregation. As regions start to dry out, they start to radiatively cool more strongly in the lower troposphere than the upper troposphere. At the same time, stratified adjustment efficiently erases horizontal virtual temperature gradients. Hence, a bottom-heavy subsidence profile develops to compensate the bottom-heavy radiative cooling. To feed this subsidence, a lower to mid-tropospheric ‘return’ flow from moist to dry regions develops above the boundary layer flow from dry to moist, convecting regions. This low-level circulation, which resembles the cross-equatorial flow in the central and east Pacific, amplifies the self-aggregation by exporting moist static energy out of the dry regions, allowing them to dry further. The ultimate result after 50 days or so is to create a single small moist region of intense convection surrounded by subsidence. The domain-mean thermodynamic profiles are much warmer and drier than comparable small-domain RCE simulations.

The dynamical feedbacks on self-aggregation were different than we had expected, and inextricably intertwined with the diabatic forcings and their vertical profiles. From simple models of large-scale tropical circulation with a single vertical mode of variability (such as the QTCM and two-layer models), we had anticipated that convectively-induced large-scale circulations would have gross moist stability, exporting moist static energy from moist, precipitating regions and importing moist static energy into dry regions. In fact, the advective moist static energy fluxes were considerably more complex, and resembled the above preconception only in the most intensely convecting regions, with negative gross moist stability elsewhere. This illustrates the importance of the interconnected vertical profiles of convection, vertical motion, and
humidity in determining the exchange of moist static energy between moister and drier regions.

In the real atmosphere, tropical synoptic-scale and intraseasonal disturbances are ‘mixing the paint’ by shearing out water vapor anomalies on timescales of days to weeks, and SST gradients help pattern the large-scale circulation, so perhaps one should look only at the interplay of water vapor, convection, radiation in self-aggregating RCE as an singular and unrealizable metaphor. Perhaps the metaphor is most relevant over the Indian Ocean/West Pacific warm pool, where SST gradients and mean wind shear are weak. Here the MJO is a dominant form of disturbance. While the MJO clearly has a rotational component and has precursors in the wind field, the interplay of water vapor, convection, radiation in the developing MJO has intriguingly similar ingredients to our self-aggregation simulations, with a large region of shallow and then congestus convection developing over the central equatorial Indian Ocean, then becoming moister and heavily precipitating, with a more top-heavy vertical motion profile before propagating into the west Pacific due to beta-plane dynamics. We are investigating self-aggregation over a beta-plane to explore this analogy further. Another interesting extension of MJO relevance that we will report on in future is self-aggregation above a slab ocean with interactive SST.

A sensitivity study, MOREICE, in which the conversion from optically active cloud ice to larger and more rapidly falling snow was inhibited, has an intriguing possible connection to the hypothesis that optically thick upper atmospheric cloud may have helped moderate the high-latitude wintertime climate of the Eocene (Sloan and Pollard 1998, Kirk-Davidoff et al. 2002), allowing crocodiles and redwoods to thrive on such places as the north coast of Alaska and high-latitude continental interiors. Because of ice crystal sedimentation, it would be difficult to sustain such a cloud without sustained upward motion. However, even with high-latitude Eocene wintertime warming, there would be relatively low WVP in high latitudes during winter, allowing the surface-emitted upwelling longwave radiation to heat cirrus cloud layers in a manner perhaps analogous to simulation MOREICE. While there are many other difference (ambient rotation, dif-
different stratification and mechanisms of bringing moisture into the upper troposphere and stratosphere, etc.), associated large-scale lofting might help sustain the persistent optically thick high-latitude cirrus central to Sloan’s hypothesis.

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