Surface Air Temperature Jan 1999
Where is the sea ice edge?

Surface Air Temperature Jan 1999
Surface Air Temperature Jan 1999
Where is the coastline?

Surface Air Temperature Jul 1999
Where is the sea ice edge?

Surface Air Temperature Jul 1999
Surface Air Temperature Jul 1999
Oden ice breaker
Photo by Chris Linder, WHOI
Salt is in the brine pockets/channels
Alloy = material composed of two or more molecules. Often behaves unlike either element alone. It may be complete solid or partial solid/liquid.

Sea ice is a combination of NaCl and H2O. At normal temperatures on Earth it is a partial solid/liquid. ALL of the NaCl is in the liquid!!! Thus ice is an alloy of pure ice and brine.
Alloy = material composed of two or more molecules. Often behaves unlike either element alone. It may be complete solid or partial solid/liquid.

Sea ice is a combination of NaCl and H2O. At normal temperatures on Earth it is a partial solid/liquid. ALL of the NaCl is in the liquid!!! Thus ice is an alloy of pure ice and brine.

The concentration of salt affects the freezing point. Normally we assume the temperature of the ice/brine combination is at the BRINE FREEZING POINT, or LIQUIDUS. This assumption is known as local thermodynamic equilibrium (LTE). Eutectic materials (like sea ice) exhibit this behavior.

Sea ice is also known as a reactive porous medium because its ice/brine proportion changes with temperature. Brine pockets are pores.
Alloy = material composed of two or more molecules. Often behaves unlike either element alone. It may be complete solid or partial solid/liquid.

Sea ice is a combination of NaCl and H2O. At normal temperatures on Earth it is a partial solid/liquid. ALL of the NaCl is in the liquid!!! Thus ice is an alloy of pure ice and brine.

The concentration of salt affects the freezing point. Normally we assume the temperature of the ice/brine combination is at the Brine Freezing Point, or Liquidus. This assumption is known as local thermodynamic equilibrium (LTE). Eutectic materials (like sea ice) exhibit this behavior.

Sea ice is also known as a reactive porous medium because its ice/brine proportion changes with temperature. Brine pockets are pores.

Think of salting icy roads. The ice melts just enough to dilute the brine until it reaches the melting point, more salt makes more melt.

The local in LTE means the temperature need not be isothermal except just at the interface of ice and brine, which is at the liquidus temperature. The temperature also need not be in steady state, but changes slowly enough to maintain the liquidus condition. Again think of salting a road.
Size of channels, or porosity, grows with temperature

Because the brine is always at the freezing point, or liquidus temperature

\[ T = - \mu S_{\text{br}} \]

\( S_{\text{br}} = \text{Salinity in the brine channels} \)

\( T = \text{Temperature in °C} \)

Photos by B. Light
Idealized binary alloy eutectic diagram

T, Temperature

Freezing Point or Liquidus line

Ice + Brine

Pure Brine Phase

Brine + solid salt

Ice + solid salt

$S_{br}$, Brine Salinity
“Toward the end of July, pond coverage decreased markedly (Fig. 4) while radiation and air temperatures were still at their normal summer values. A possible explanation is that the ice had become sufficiently porous for the melt water ponds to drain by percolation.” Untersteiner on the NOAA Arctic Theme Page.
X-ray Computed Tomography Images by Dan Pringle
In Golden et al 2007

Permeability, $\Pi \sim \phi^n$
High permeability at base of young ice, interior is too cold

Permeability increases by 1-2 orders of magnitude in melt season

Malmgren (1927)
Brine pockets are internal melt. Their presence reduces energy needed to melt sea ice compared to fresh ice.

\[ q = \text{energy of melting (aka enthalpy)} \]
\[ L_0 = \text{latent heat of fusion for fresh ice} \]
Porous nature of sea ice causes:

Space for algae, chemistry, sea water cycling, gas to pass through, etc

We think sea ice plays a role in cycling carbon, sulfur, halogen (ozone), etc. Many open questions.

Sea ice rots from inside out

Pore structure affects deformation (virtually unstudied)
The graph shows the ice thickness over the day of the year with two sets of data:

- **Conserving** and **Non-conserving**
- **Semtner 3L**
- **Semtner 0L**

The bottom graph compares two models:

- **Full Toy Model**
- **Simplified Toy**

The y-axis represents ice thickness in centimeters, and the x-axis represents the day of the year.
What controls the equilibrium sea ice thickness?

A simple slab sea ice model for 80N, h=thickness

When perturbed a sea ice floe returns to its equilibrium thickness by adjusting its growth Untersteiner (1961)
Submarine Data of Ice Draft

Blue = Rothrock et al 1999
Red = Wadham and Davies 2000

Bitz and Roe (2004)
NH Sea Ice Volume in $10^{13}$ m$^3$

NH Sea Ice Area in $10^{12}$ m$^2$

CCSM3 climate model output

Which one has more memory?
Climate for approx. 80N

For simplicity figure ignores h-dependence of M so G(h) defines $h_{eq}$
Parallel with simple planetary energy balance

\[
\text{netSW} = \text{OLR}
\]

\[
G = M
\]
If $G \sim 1/h$ is the dominant $h$-dependence
Then $\partial G / \partial h \sim 1/h^2$ which controls sensitivity.
Basic results:

Thicker ice is more variable and has more low-frequency variability

Thicker ice is more sensitive to climate perturbations

The ice edge, which is more sensitive to the behavior of thin ice, will tend to have faster response time and higher frequency variability

BUT we have not really done justice to the ice edge where ice-albedo feedback is a big player

And we have ignored ice dynamics
CCSM3 Cross-section through Sea Ice

The maximum ice edge position hardly changes!
At the ice edge perhaps the situation is more like this?

Three equilibria

$\tau > 0$  $\tau < 0$  $\tau > 0$
When we add dynamics is the system more like this?

Outflow ~ ice divergence

Hibler and Hutchings (2002) using an idealized model with ice dynamics
Is there any evidence for “exotic phenomena” ie multiple equilibria?

Several papers ask if we are about to reach a “tipping point” in the Arctic, ie about to jump from perennial to seasonal ice?

Submarine data say Arctic lost 40% of its volume in 25 yrs. This reduces the thickness gradient and according to the Untersteiner picture, the ice can now retreat even faster.
IBCAO
Lindsay and Zhang 2005
Illustration by Jack Cook, Woods Hole Oceanographic Institution
Modern sinking locations and sea ice edge

model results

surface density (shading),
sea ice edge (black),
and deep mixing (green)

CCSM3 model results
Strengthened Thermohaline Circulation

Both redrawn by me
My own speculation

Redrawn by me

Weaker Thermohaline Circulation
Weaker Thermohaline Circulation (because the glacial ocean is more salt stratified)

Most agree

What did the Arctic Ocean look like?
Sinking locations and sea ice edge

Last Glacial Maximum (LGM)  Modern

surface density (shading),
sea ice edge (black),
and deep mixing (green)

CCSM3 model results
Net sea ice growth (shading), sea ice edge (black), and deep mixing (green)

CCSM3 model results
Ocean Heat Flux Convergence (in colors) 
Sets the Sea Ice Edge (in black) 

CCSM3 model results
"This is it, Jenkins ... indisputable proof that the Ice Age caught these people completely off guard."
Abrupt climate change in the past

Greenland temperature and global ice volume

Data from Lisiecki & Raymo 2005

figure by Camille Li
Modern view of Atlantic circulation

Freshwater added
Sea Ice Edge Expansion Causes Strong Local Cooling

pre freshening (solid)
after freshening (dashed, the one further south)

CCSM3 model results
Central England Surface Temperature Annual Cycle

Thick band is pre-freshening, width indicates natural variability
Black line is observations
Dashed is after AMOC shutdown

- 4XCO2
- Modern
- LGM

Month of year
deg C
an observed “dataset”
Surface Air Temp Trends 1982-1999 in various observed “datasets”
Bitz and Fu, 2009 comment in Nature on Graversen et al, 2008

DJF Trends
ERA hatched
MSU solid
Rigor and Wallace 2003
Next series of slides present the 4 governing equations for state of the art sea ice model used for climate studies (i.e., appropriate for basin scale or larger and for full seasonal cycle or longer)
1st Governing Equation

Ice thickness distribution $g(x,y,h,t)$ evolution equation from Thorndike et al. (1975)

$$\frac{Dg}{Dt} = -g \nabla \cdot \mathbf{u} + \Psi - \frac{\partial}{\partial h} (fg) + \mathcal{L}$$

A PDF of ice thickness $h$ in a region, such as a grid cell
\[ \frac{Dg}{Dt} = -g\nabla \cdot u + \Psi - \frac{\partial}{\partial h}(fg) + L \]

1. Lagrangian time derivative of \( g \) following “parcel”

2. Convergence of parcel

3. \( \Psi = \) Mechanical redistribution

4. Ice growth/melt results in “advection of \( g \) in thickness space”

5. \( L = \) Reduction of \( g \) from lateral melt

\[
\begin{align*}
  h & = \text{ice thickness} \\
  u & = \text{ice velocity} \\
  f & = \text{growth rate}
\end{align*}
\]
$$\Psi = \text{Mechanical redistribution}$$
Advection in thickness space from growth

$g(h)dh$
2nd Governing Equation

Conservation of momentum, see for example Hibler (1979)

\[ m \frac{Du}{Dt} = -mf k \times u + \tau_a + \tau_w - mg_r \nabla Y + \nabla \cdot \sigma \]
\[
m \frac{Du}{Dt} = -mfk \times u + \tau_a + \tau_w - mg_r \nabla Y + \nabla \cdot \sigma
\]

1. Lagrangian time derivative of \( u(x,y,t) \) following parcel
2. Coriolis force
3. \( \tau_a, \tau_w = \) air and water stresses
4. Ocean surface tilt
5. Ice interaction term

| m = mass per unit area |
| f = Coriolis parameter |
| \( g_r = \) gravity |
| Y = Sea surface height |
| \( \sigma = \) ice stress |
Conservation of Enthalpy $E(x,y,z,t)$, the heat required to melt a unit area of sea ice or snow, see for example Bitz et al (2001)

$$\frac{DE}{Dt} = -E \nabla \cdot u + \Pi + \mathcal{E}$$

Models that neglect the heat capacity of ice, do not have this equation because in their case $E$ is proportional to the ice volume
\[
\frac{DE}{Dt} = -E \nabla \cdot u + \Pi + \mathcal{E}
\]

1. Lagrangian time derivative of \( E(x,y,z,t) \) following parcel
2. Convergence of parcel
3. \( \Pi = \) Mechanical redistribution
4. \( \mathcal{E} = \) contribution by thermodynamic processes
4th Governing Equation

Heat equation of sea ice and snow, Maykut and Untersteiner (1971)

\[
\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + Q_{SW}(z)
\]

Used to estimate last term in previous slide
1. Thermal energy change at a point
2. Gradient of the conductive flux
3. $Q_{SW}(z)$ Absorption of solar radiation

\[ \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + Q_{SW}(z) \]

- $T =$ temperature
- $c =$ heat capacity
- $k =$ thermal conductivity
- $\rho =$ density
Caveats for this set of Governing Equation

1. No explicit equation for the ice volume (or mass, yet), because conservation of volume is contained in the equation for $g(h)$

2. I don’t have an equation for the salinity of the sea ice, which is time-independent in sea ice models used for climate (this may change soon). Must alter heat equation too for prognostic salinity.

3. Brine pockets were covered last week.

4. Radiative transfer - on your own.
Discretizations of $g(H)$ for thickness advection
Discretizations of $g(H)$ for thickness advection

- Assumed uniform $g$ within thickness bin, or category
- Mean thickness of ice in a category is midpoint of that category
- Eulerian
  Simple but diffusive

- Delta functions move when ice grows/melts
- Thickness is a prognostic variable
- Lagrangian
  Non-diffusive, bit less simple and categories empty abruptly
• g is a linear function of thickness in each bin
• Thickness is prognostic

Smooth and non-diffusive, but more complicated (though computationally efficient)

Area = sum of g for categories with finite thickness
How many people are actively developing sea ice models?
Sea ice models tend to be broken up numerically into three pieces

1. Dynamics including advection
2. Thermodynamics
3. Ice thickness distribution

Where more than one of these pieces influences a single equation, time splitting is employed, e.g.:

\[ A^{n+1/2} = A^n + \Delta t \text{ (term one)} \]
\[ A^{n+1} = A^{n+1/2} + \Delta t \text{ (term two)} \]

So the following slides break up governing equations
Sea Ice Dynamics in climate models

Past ad hoc method was to stop ice from moving at a critical thickness, sometimes called stopage.
Sea Ice Dynamics Constitutive Law

A constitutive law characterizes the relationship between stress $\sigma_{ij}$ and strain rate $\dot{\varepsilon}_{ij} = \partial u_i / \partial x_j$ defining the nature of the ice interaction.

The rheology used in most models is from Hibler 1979, where the the sea ice is treated as a continuum that is plastic at normal strain rates and viscous at very small strain rates.
Engineering Compressive Stress Test

At first

Ice floe side view

length $L$

After applying a compressive force.

Volume is conserved so the ice is thicker

$L + \delta L$

Strain $\epsilon = \frac{\delta L}{L}$

Strain rate $\dot{\epsilon} = \frac{\delta L}{L \delta t}$
VP Constitutive Law 1-D Representation

\[ \sigma \text{ (stress)} \]

\[ \varepsilon \text{ (strain rate)} \]

P = Ice compressive Strength

- when viscous, the stress state is maintained by a non-recoverable dissipation of energy
- when plastic, the ice yields and the strain energy goes into ridge building
VP Constitutive Law in 2-D

Invariants of stress ($\sigma_I$ and $\sigma_{II}$) and strain rate ($\varepsilon_I$ and $\varepsilon_{II}$) are related by

\[
\begin{align*}
\sigma_I &= \zeta \dot{\varepsilon}_I - P/2 \\
\sigma_{II} &= \eta \dot{\varepsilon}_{II}
\end{align*}
\]

$\zeta$, $\eta$ = bulk and shear viscosities:

\[
\begin{align*}
\zeta &= \frac{P}{2\Delta}, & \Delta &= \left(\dot{\varepsilon}_I^2 + \dot{\varepsilon}_{II}^2e^{-2}\right)^{1/2} \\
\eta &= \frac{\zeta}{e^2}
\end{align*}
\]

Are chosen so that the stress state for plastic flow lies on an elliptical yield curve with ratio of principal axes $e=2$
Some like to rotate

Same yield curve but plotted against principal stress states, rather than stress invariants

**Dynamics summary:**

Must solve momentum equation, $\mathbf{u}$ in terms of $\sigma$, simultaneous with constitutive law, $\sigma$ in terms of $\mathbf{u}$

EVP model uses explicit time stepping by adding elastic waves to constitutive law, see Hunke and Dukowicz (1997)