SENSITIVITY OF MONTHLY CONVECTIVE RAIN FRACTION
TO THE CHOICE OF Z–R RELATION

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1. INTRODUCTION

Radar reflectivity $Z$ is physically related to rainfall rate $R$. However, the rain detected by the radar is not identical to the rainfall reaching the earth’s surface, since the radar beam is generally well above the ground. Moreover, reflectivity measurements suffer from various beam geometry, range and attenuation effects. Raingauges provide a better (though still imperfect) measure of precipitation at the surface—but only at a few points. Radar, on the other hand, provides areal coverage and high spatial resolution. The only viable way to measure rain therefore is to combine the radar and raingauge information. The data from the raingauge calibrate the radar-rainfall estimates from reflectivity in the vicinity of the gauge. The more raingauges in a network, the more points can be calibrated in the radar area. This raingauge adjustment can be by means of matching mean accumulations of the radar rainfall estimates at the gauge sites with the gauge totals (Wilson and Brandes 1979; Klazura 1981; Steiner et al. 1995) or by matching percentiles of reflectivity and rain rate cumulative frequency distributions (Calheiros and Zawadzki 1987; Rosenfeld et al. 1994).

There is ongoing discussion regarding a potential improvement of the radar rainfall estimation by using multiple $Z–R$ relations (Joss and Waldvogel 1970; Austin 1987). For example, the use of two different $Z–R$ relationships for convective and stratiform precipitation has been suggested by Short et al. (1990). The present study investigates the sensitivity of the monthly convective rain fraction (i.e., the percentage of the monthly areal rainfall accumulation contributed by precipitation identified as convective) to the choice of $Z–R$ relation. For simplicity, we use only relations of the form

$$Z = A R^b$$

We show that for equal (gauge-adjusted) monthly areal mean rainfall accumulations the resulting convective rain fraction is dependent on whether one or two $Z–R$ relationships are used and what values are given to the multiplicative $A$ and power factors $b$ of the $Z–R$ relation(s). This analysis is done using data collected by the radar and raingauge network at Darwin, Australia, which is operated by the Australian Bureau of Meteorology Research Center.

This sensitivity analysis has direct implications for model-based estimation of the heating of the atmosphere (e.g., Tao et al. 1993). Since convective and stratiform rainfall are the two basic modes of precipitation relevant to the heating problem (Houze 1989; Mapes and Houze 1995), it is important to separate the two types of precipitation accurately, but also to use appropriate $Z–R$ relations to convert the radar reflectivity to rainfall.

2. DATA AND METHODOLOGY

Darwin, located within the Northern Territory of Australia, exhibits a monsoonal climate as described by Holland (1986) and Keenan et al. (1988). The rainy season normally begins during late December and extends through March. The rest of the year remains without significant rainfall. The precipitation seen on the Darwin radar is of three main types: 1) During monsoon periods, rainfall is mostly produced by extensive oceanic mesoscale convective systems. 2) During break periods, precipitating cloud systems are less frequent and are typical of a continental origin (e.g., squall lines). 3) Over the Bathurst and Melville Islands to the north of Darwin, pronounced diurnally forced thunderstorms occur.

For this study, we use Darwin radar and raingauge data collected during February 1988 and the four month period December 1993 through March 1994. The results of February 1988 (28 days) are based on four radar volume scans per day (see Steiner et al. 1995). The extensive dataset from the rainy season 1993/94 (volume scans every 10 min) are divided into two periods: The first time period consists of 25 days of radar information collected in December 1993 and January 1994, while the second has 20 days of data collected in February and March 1994. Only days with nearly complete records (i.e., less than 10% of radar volumes missing per day) are used for this analysis. Depending on the period of interest, the raingauge network consisted of 22 to 25 sites.

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The classification of the radar rainfall patterns into convective and stratiform elements is based on the objective separation technique described by Steiner et al. (1995). This classification scheme is independent of what \( Z - R \) relation is used to convert radar reflectivity to rain rate.

The radar rainfall estimates are modified on the basis of raingauge data such that

\[
R_c = A^{-\frac{1}{b}} \left( \frac{R_u}{R_s} \right) Z^{\frac{1}{b}} = \tilde{A}^{-\frac{1}{b}} Z^{\frac{1}{b}}
\]  

(2)

where \( R_u \) is the uncorrected mean rain rate estimated by the radar at the gauge sites and \( R_s \) the mean rate based on the raingauge measurements. \( R_c \) is the corrected radar-estimated rainfall rate and \( \tilde{A} \) the gauge-adjusted multiplicative factor. Using this adjusted \( Z - R \) relationship to convert radar reflectivity to rain rate produces rainfall estimates at the gauge locations that have the same mean rain accumulation as the raingauges. The adjustment factor is determined without considering the amounts of rain contributed by the different types of precipitation (i.e., convective and stratiform). If two different \( Z - R \) relationships are used for converting convective and stratiform reflectivity to rain rate, the same correction factor is assumed for both relations to preserve the initially applied ratio between the convective and stratiform multiplicative factors,

\[
\frac{\tilde{A}_{\text{conv}}}{\tilde{A}_{\text{stra}}} = \frac{A_{\text{conv}}}{A_{\text{stra}}}
\]  

(3)

For each choice of \( A_{\text{conv}}/A_{\text{stra}} \) and \( b \) a different convective rain fraction is obtained. The sensitivity of the convective rain fraction is explored over a wide range of \( A_{\text{conv}}/A_{\text{stra}} \) ratios (1/4, 1/2, 1/1, 2/1, and 4/1) and power factors \( b \) (1 \leq b \leq 3 \) in steps of 0.25; Smith and Krajewski 1993). We assume the same value of \( b \) for both the convective and stratiform \( Z - R \) relationship. A ratio of \( A_{\text{conv}}/A_{\text{stra}} = 1 \) implies the use of a single \( Z - R \) relation.

**TABLE 1.** Areal mean rainfall and convective rain fraction for three time periods at Darwin, Australia. The amounts are computed by a single, gauge-adjusted \( Z - R \) relation.

<table>
<thead>
<tr>
<th>Period</th>
<th>Days</th>
<th>Rain</th>
<th>Convective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb88</td>
<td>28</td>
<td>163 mm</td>
<td>59%</td>
</tr>
<tr>
<td>Dec93–Jan94</td>
<td>25</td>
<td>245 mm</td>
<td>51%</td>
</tr>
<tr>
<td>Feb94–Mar94</td>
<td>20</td>
<td>374 mm</td>
<td>48%</td>
</tr>
</tbody>
</table>

**FIGURE 1.** Sensitivity of the convective rain fraction to the choice of \( Z - R \) relation for data collected by the Darwin radar in February 1988.

**FIGURE 2.** Same as Fig. 1, except for period December 1993 – January 1994.

**FIGURE 3.** Same as Fig. 1, except for period February 1994 – March 1994.
3. RESULTS AND DISCUSSION

Using a single, gauge-adjusted $Z - R$ relation with $b = 1.25$, the rainfall amounts and corresponding convective rain fraction for the three time periods of interest are compiled in Table 1. The differences in the rainfall amounts and convective rain fractions among the three periods indicate differences in the frequency of occurrence of the various types of precipitation systems developing around Darwin rather than differences in how the data were processed.

Figures 1 to 3 show the sensitivity of the convective rain fraction to the choice of $Z - R$ relation. Because of the fact that for each $Z - R$ configuration a gauge adjustment is made, all the points in a particular Figure exhibit the same areal mean rainfall amount.

A significant sensitivity of the convective rain fraction to the choice of $Z - R$ relation is revealed by Figs. 1-3. All three time periods show similar behavior. For a power factor $b = 1.5$ (a widely used value), an uncertainty of $b \pm 0.1$ results in a $\pm 3.4\%$ uncertainty of the convective rain fraction. On the other hand, increasing the ratio of $A_{conv}/A_{tra}$ from 1/4 to 1/2, to 1/1, 2/1, or 4/1 results each time in a 10% reduction of the convective rain fraction.

4. SUMMARY AND CONCLUSIONS

The selection of an appropriate $Z - R$ relation for the radar rainfall estimation is an important task. Though an adjustment using rain gauges will ensure an accurate areal mean rainfall accumulation, the fraction of the rain contributed by precipitation identified as convective is very sensitive to whether one or two $Z - R$ relationships are used and what values are given to the multiplicative $A$ and power factors $b$.

The question is then, how to find the appropriate ratio of $A_{conv}/A_{tra}$ and the power factor(s) $b$? There are different ways to achieve this:

- Measurements of the raindrop size distribution can be explored to derive the power factor $b$ and ratio $A_{conv}/A_{tra}$ (i.e., Short et al. 1990).
- Least-square-fit (or probability-matching) methods can be applied to instantaneous pairs of radar reflectivity $Z$ and gauge-based rain rate $R$ that have been categorized according to the radar-based identification of the precipitation type (e.g., convective and stratiform) to obtain the $Z - R$ relation(s).
- Systematically varying the parameters $A$ and $b$ to find the maximum correlation (or minimum root mean square difference) between the site pairs of radar-estimated and gauge-based mean rainfall rate (or accumulation), the appropriate $Z - R$ relation(s) can be found. However, results obtained this way so far do not show enough sensitivity to be useful.

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REFERENCES


