Dual-Aircraft Investigation of the Inner Core of Hurricane Norbert.
Part III: Water Budget

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ABSTRACT

The hydrometeor water budget of Hurricane Norbert on 24 September 1984 is computed using two microphysical retrieval techniques. Three-dimensional distributions of condensation, evaporation, precipitation, and advection of cloud and precipitation are computed, and a bulk water budget is computed as the volume integral of these distributions.

The role of the microphysical retrievals is to provide the three-dimensional distribution of cloud water content, since it cannot be determined with the equipment available. Both retrieval methods use the steady-state continuity equation for water. The first method determines precipitation formation mechanisms from the radar-reflectivity and Doppler wind fields. The cloud water content is determined, through microphysical modeling, to be the amount necessary to explain the rate of precipitation formation. The second method (that of Hauser et al.) solves the water continuity equations as a boundary value problem, while also employing microphysical modeling. This method is applied in three dimensions for the first time.

Asymmetries in the water budget of Hurricane Norbert were important, apparently accounting for nearly half the net condensation. The most condensation and heaviest precipitation was to the left of the storm track, while the strongest evaporation was to the rear of the storm. Many of the downdrafts were unsaturated because they were downwind of the precipitation maximum where little water was available for evaporation. Since the evaporation in the downdrafts was significantly less than the condensation in their counterpart updrafts, net condensation (bulk condensation – bulk evaporation) was significantly greater than would be implied by the net upward mass flux. Much of the vapor required to account for the greater bulk condensation appears to have come from enhanced sea surface evaporation under the dry downdraft air to the right of the storm track.

The net outflow of condensate from the storm inner core was quite small, although there were appreciable outward and inward horizontal fluxes at certain locations. A maximum of ice outflow to the left of the storm track in the front of the storm corresponded well to the ice particle trajectories that Houze et al. suggested were feeding the stratiform precipitation found farther outward from the storm center.

1. Introduction

The release of latent heat in clouds can provide the energy for cyclone development and maintenance (Kutzbach 1979). Espy (1841) measured the effect of adiabatic expansion on saturated air and concluded that the release of latent heat could produce a warm core in a storm, which causes inflow into the storm center. This idea is essentially correct for a hurricane, and an understanding of the distribution of latent heating therefore seems essential for understanding the intensification and maintenance of hurricanes. An accurate determination of the distribution and intensity of condensation and evaporation (i.e., a water budget) is required.

The intensity of the storm (defined either by minimum pressure or maximum wind) is not directly proportional to the total latent energy release. Malkus and Riehl (1960) noted that the intensity change of a storm should be related more directly to the equivalent potential temperature of the air that rises near the center. The very low pressures and high equivalent potential temperatures are only possible because of vigorous sensible and latent heat fluxes from the sea surface. Emanuel (1986) and Rotunno and Emanuel (1987) found that the hurricane can be viewed as a type of Carnot heat engine. They found that the intensity and structure of an axisymmetric hurricane is controlled

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essentially by the sea surface and hurricane-outflow temperatures and by the initial structure of the vortex. The mechanisms that determine how and whether this maximum intensity is realized are still not fully understood.

Hurricane intensification cannot be explained fully without considering the dynamics in the eye and the relationship between the latent heating distribution and the angular momentum field. The intensity is related to subsidence in the eye. Willoughby (1979) showed that in a hurricane in gradient-wind balance, the transverse (secondary) circulation of the storm is related to the radial gradient of heating. The heating gradient appears to be related to the indirect circulation in the eye. The associated subsidence warming in the eye results in a lowering of surface pressure. Shapiro and Willoughby (1982) found that the intensification rate in an axisymmetric hurricane depended upon the location of heat sources relative to the radius of maximum wind. Heat sources centered near the radius of maximum wind produced the most rapid storm intensification.

One important factor in evaluating the pattern of heating in a storm is local efficiency of the clouds in producing precipitation from condensation. If the cloud is perfectly efficient then condensation is equal to the sum of precipitation that falls to the surface as rain, and the latent heat release is proportional to the condensation. Since precipitation is not produced with perfect efficiency, accurate formulation of precipitation mechanisms is required to estimate condensation and heating. Better understanding of microphysical processes involved in producing hurricane precipitation is therefore one of the major purposes of this research.

Hurricane microphysical processes hence strongly affect the distribution of heating and cooling in a hurricane, and hence the storm dynamics. Precipitation particles that do not fall out directly under the clouds are advected elsewhere, where their melting, evaporation, and freezing influence the dynamical evolution of the storm. Marks (1985) noted that the horizontal vapor convergence into the eyewall may be twice the observed eyewall precipitation. Although such conclusions remain obscured by uncertainties in the observations, they nevertheless suggest that a significant portion of the condensate produced in the hurricane eyewall may be advected outward in the form of slowly falling ice particles that fall in the surrounding stratiform precipitation regions, affecting the dynamics of the regions surrounding the eyewall.

Lord et al. (1984) used a nonhydrostatic axisymmetric model to examine the role of the ice particle microphysics in the development of hurricanes. They found that the cooling associated with the melting of frozen precipitation in the stratiform regions outside the eyewall produced mesoscale downdrafts. These downdrafts slow the intensification of a hurricane.

This paper is the third in a series examining the inner core structure of Hurricane Norbert (1984), which was explored by a coordinated dual-aircraft mission. One aircraft sampled ice particle microphysics while the other obtained detailed Doppler radar measurements of the air motions. The goal of this series of papers is to use these aircraft data to describe the kinematic structure, microphysics, and water budget of the central region of Norbert in a way that will elucidate the processes influencing the structure and intensity of tropical cyclones.

Marks et al. (1992; hereafter referred to as Part I of this series) discussed the three-dimensional Doppler wind field in the inner core of Hurricane Norbert. Their Doppler winds are the basis of calculations of the water budget presented in the present paper.

Part I showed the distinct asymmetries in the wind field of Hurricane Norbert. Upward motion predominated to the left of the track, while downward motion predominated to the right of the track. Peak updrafts were located mostly to the left of the storm track, while peak downdrafts were located mostly to the right. The strongest downdrafts were slightly radially inward from, and also downwind of, the peak updrafts and peak reflectivity. Below the 2-km level, the radial flow was inward in front of the storm, and outward to the rear. Above the 3-km level the radial wind flow switched. There was a deep layer of inflow into the rear of the storm, as well as deep outflow in front. This was indicative of strong shear in the flow in which the vortex was embedded.

In Part II, Houze et al. (1992) analyzed the microphysical structure of Hurricane Norbert. Graupel particles were found to fall out quickly from eyewall convective updrafts. Lighter ice particles generated in the eyewall updrafts remained suspended above the melting level much longer and were transported around the storm and outward. Outside the eyewall region, they underwent aggregation and fell through the melting level in a stratiform rainband.

In this study, the water budget of Hurricane Norbert is assessed by using the Doppler radar–observed kinematic structure of the storm analyzed in Part I. The detailed three-dimensional wind field is used not only to calculate the advection of moisture but also as a basis for retrieving the thermodynamic and microphysical structure of the storm. In the context of radar meteorology, retrieval is a method that uses physical relationships such as the conservation of momentum, water, and heat to derive the fields of thermodynamic, and/or microphysical variables that are consistent with a Doppler radar–observed wind field. Two methods for retrieving cloud water content are discussed in section 5. Retrievals permit more detailed water budgets than that of Marks and Houze (1987), who used airborne Doppler data and simple assumptions (no retrievals) concerning cloud content. The relationship of the computed water budget to storm inner core structure and dynamics is evaluated. The results are com-
pared with the earlier water budget studies of Hawkins and Rubsam (1968) and Hawkins and Imbembo (1976) for moderate (Hilda) and intense (Inez) hurricanes, respectively. These earlier studies used only flight-level data and were thus very limited in spatial resolution. The retrieved microphysical characteristics and transport are compared with the analyses of the cloud microphysical data in Part II. The role of asymmetric contributions to the water budget is evaluated.

2. Description of water budget

The bulk water budget of a system is an accounting of the total condensation, deposition, evaporation, sublimation, storage (net time change in the total mass of suspended condensate), and precipitation within the volume being studied, as well as the advective and diffusive transports of water and ice through the outer boundaries of that volume. This budget is obtained from the continuity equation of condensed water and ice:

$$\rho(c - e) = \frac{\partial}{\partial t} (\rho q_w) + \nabla \cdot \rho V q_w - K_H \rho \nabla^2_q q_w$$

$$- K_z \frac{\partial}{\partial z} (\rho \frac{\partial q_w}{\partial z}) - \frac{\partial}{\partial z} (\rho V_T q_p),$$  \hspace{1cm} (1)

where $c$ and $e$ are the local values of condensation (and deposition) and evaporation (and sublimation), respectively, $q_w$ is the sum of cloud and precipitation water mixing ratio, $q_p$ is the precipitation mixing ratio, $\rho$ is the density of air, $K_H$ is the horizontal eddy diffusivity, $K_z$ is the vertical eddy diffusivity, $V$ is the three-dimensional wind, and $V_T$ is the terminal fall speed of the precipitation (positive downward). The first term on the right-hand side of (1) is the storage term, the second is the divergence of the flux of cloud and precipitation by the resolvable wind, the third and fourth terms are the divergences of the horizontal and vertical diffusional fluxes, respectively, of condensate, and the fifth is the precipitation flux divergence (sedimentation). The left side of (1), $\rho(c - e)$, is the net condensation (and deposition) or evaporation (and sublimation) in the region represented by a given grid point.

In this study it is assumed that either condensation (or deposition) or evaporation (or sublimation), but not both, are occurring within a single grid element. It is not possible to determine both individually from (1). The term bulk condensation (evaporation) will refer to the total water condensed or deposited (evaporated or sublimated) in all grid cells in the volume where the net value of $\rho(c - e)$ is positive (negative). Net condensation will refer to the difference, condensation (or deposition) minus evaporation (or sublimation), whether for a single grid element or for the sum of all the grid cells in a given volume.

The bulk water budget is the volume integral of (1). In this study it is computed for a cylindrical volume surrounding the storm center. Bulk condensation is then the volume integral of the local condensation (or deposition) as in the following equation:

$$C = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho(c - e)rdrd\theta dz,$$  \hspace{1cm} (2)

while bulk evaporation (or sublimation) is

$$E = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\text{max}}} (\delta - 1)\rho(c - e)rdrd\theta dz.$$  \hspace{1cm} (3)

The value of $\delta$ is 1 when $c - e$ is positive and 0 when $c - e$ is negative. The heights of the top and bottom of the budget volume are $z_t$ and $z_b$, respectively. The radial distance from the storm center is $r$ and the outer boundary of a cylindrical budget volume is $r_{\text{max}}$, while $\theta$ is the heading of a given location relative to the storm center (this will also be referred to as azimuth in the rest of the paper). Since the system is assumed to be in a steady state, the storage term in (1) is set to zero. The volume integral of the horizontal divergence portion of the second term on the rhs of (1) is the outward (all terms except rainfall at the top of the budget volume will be defined positive outward) horizontal transport of water by the resolvable wind:

$$T_H = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\text{max}}} \nabla \cdot \rho V_H (q_c + q_p) rdrd\theta dz,$$  \hspace{1cm} (4)

where $q_c$ is cloud mixing ratio, $q_w = q_c + q_p$, and $V_H$ is the horizontal wind. When the divergence theorem is applied, then

$$T_H = r_{\text{max}} \int_{z_b}^{z_t} \int_0^{2\pi} \rho(r_{\text{max}}, \theta, z) V_{r}(r_{\text{max}}, \theta, z)$$

$$\times (q_c(r_{\text{max}}, \theta, z) + q_p(r_{\text{max}}, \theta, z))d\theta dz.$$  \hspace{1cm} (5)

When $T_H$ is positive (negative), the bulk horizontal transport is outward (inward), and $V_r$ is the radial wind reckoned outward from the storm center. By the divergence theorem, the volume integral of the vertical component of the cloud water portion of the second term on the rhs of (1) is equal to the outward advection of cloud water through the bottom and top boundaries.

$$T_{zc} = \int_{z_b}^{z_t} \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho w q_w rdrd\theta dz$$

$$= \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho(w(r, \theta, z_T)q_w(r, \theta, z_T)) rdrd\theta$$

$$- \int_0^{2\pi} \int_0^{r_{\text{max}}} \rho(w(r, \theta, z_B)q_w(r, \theta, z_B)) rdrd\theta$$

$$= T_{zcT} + T_{zcB},$$  \hspace{1cm} (6)

where $w$ is the vertical component of the air motion or vertical wind. The first and second terms on the rhs of (6) are the transports of cloud through the top (pos-
The vertical outward transports of precipitation are determined by combining the precipitation portion of the second term with the fifth term on the rhs of (1). They are integrated over the volume as follows:

$$R_{\text{net}} = \int_{z_t}^{z_k} \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\partial}{\partial z} [\rho (w - V_T) q_p] r dr d\theta dz$$

$$= \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \rho (r, \theta, z_t) [w (r, \theta, z_t) - V_T (r, \theta, z_t)] q_p (r, \theta, z_t) r dr d\theta$$

$$- \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \rho (r, \theta, z_b) [w (r, \theta, z_b) - V_T (r, \theta, z_b)] q_p (r, \theta, z_b) r dr d\theta = -R_T + R_B,$$  \hspace{1cm} (7)

where $R_{\text{net}}$ is the mass of precipitation exiting the budget volume through the top and bottom boundaries, $R_T$ is the mass of precipitation falling into the top of the budget volume per unit time, and $R_B$ is the rain falling out the bottom (if the bottom is near the surface $R_B$ is approximately the area-integrated rainfall rate). Here $R_T$ and $R_B$ are both positive for rain moving downward, and therefore a positive $R_T$ indicates precipitation moving into the volume.

The third term of (1) is the horizontal diffusion. Integrated over volume it becomes

$$D_H = -\int_{z_t}^{z_k} \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \rho K_H \nabla \cdot \nabla_H (q_c + q_p) r dr d\theta dz$$

$$= -K_H r_{\text{max}} \int_{z_t}^{z_b} \int_0^{2\pi} \rho (r_{\text{max}}, \theta, z) \frac{\partial}{\partial r} [q_c (r_{\text{max}}, \theta, z) + q_p (r_{\text{max}}, \theta, z)] r dr d\theta dz.$$  \hspace{1cm} (8)

Here $D_H$ is positive when the net diffusion is outward from the volume. The bulk diffusion through the vertical boundaries $D_z$ is the integral of the fourth term of (1). Integrated over the volume, it becomes

$$D_z = -\int_{z_b}^{z_k} \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\partial}{\partial z} \left( \rho K_z \frac{\partial}{\partial z} [q_c + q_p] \right) r dr d\theta dz$$

$$= -\rho (z_t) K_z \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\partial}{\partial z} [q_c (r, \theta, z_t) + q_p (r, \theta, z_t)] r dr d\theta$$

$$+ \rho (z_b) K_z \int_0^{2\pi} \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\partial}{\partial z} [q_c (r, \theta, z_b) + q_p (r, \theta, z_b)] r dr d\theta = D_T + D_B,$$  \hspace{1cm} (9)

where $D_T$ (positive upward) is diffusion out the top and $D_B$ (positive downward) is diffusion out the bottom. In this study, $K_H$ and $K_z$ are assumed to be independent of height. Once all terms of (1) have been integrated, the steady-state bulk water budget may be expressed as

$$C + R_T = E + T_H + T_{\text{EC}}T + T_{\text{EC}B}$$

$$+ R_B + D_H + D_T + D_B,$$ \hspace{1cm} (10)

where $C$ and $R_T$ are the sources of condensed water and ice, while all other terms are sinks. These processes are displayed graphically in Fig. 1.

The absence of a storage term from (10) does not imply a constant suspended water content, but rather that the storage term cannot be estimated from data spanning a period of 2 to 4 h. Averaged over the whole eyewall, the maximum rate of change of suspended liquid water and ice might be $\sim 0.1$ g kg$^{-1}$ h$^{-1}$, and if the change is negative it would represent a rainfall rate (neglecting the effects of advection) of $\sim 1$ mm h$^{-1}$, or about 10% of the mean rainfall rate. Therefore, meaningful results can still be achieved with the neglect of storage.

FIG. 1. Schematic of hurricane bulk water budget. The budget volume is a cylinder. One sector is cut away to show the regions in which the various processes occur. Terms are defined in section 2.
3. Storm description

Hurricane Norbert formed in the eastern Pacific Ocean on 16 September 1984. It made landfall on 26 September along the northern Baja Peninsula. Research missions were flown, which began on 22, 23, and 24 September. The data used in this paper, as in Parts I and II, were obtained in the middle of the last flight, over the period 0018–0215 UTC 25 September 1984. At this time, the storm was filling at the rate of $\approx 1$ hPa h$^{-1}$. Immediately after this period, the storm filled at a rate of nearly 2 hPa h$^{-1}$. The wind maximum at the 3-km flight level decreased in 4.5 h by 6 m s$^{-1}$. A more detailed description of the storm is given in Part I.

The synoptic development of Hurricane Norbert is described by Gunther and Cross (1985). They mention that after 23 September 1984 Hurricane Norbert was under the influence of an upper-level trough, which steered the storm northeastward. The associated vertical wind shear evidently was weakening the storm by the time of our observations, since the storm was still over water with temperatures $\sim 27^\circ$C. This shear is discussed in detail in Part I.

4. Data

The inputs to the retrieval methods are airborne Doppler wind data, radar-reflectivity data, and in situ flight-level data. Doppler winds and reflectivity data were described in Part I, so they are only briefly reviewed here.

The Doppler radar was located on the aircraft that flew at an altitude of $\approx 3$ km. The Doppler radar scanning axis was along the aircraft fuselage, and radial precipitation velocities were obtained in a scan perpendicular to this axis. Data for the two horizontal wind components were obtained by flying in two roughly perpendicular directions. Vertical wind was obtained by integrating horizontal divergence and then used iteratively to correct the horizontal winds (since most of the Doppler radials are not purely horizontal), as described in Jorgensen et al. (1983). The specific flight plan for Norbert was described in Part I.

The final wind field was filtered to alleviate noise in the thermodynamic and microphysical retrievals. A Gaussian filter was applied to radial and tangential velocities. The $e$-folding distance for the filter was 1 km in the radial direction and 10 km in the tangential direction. Filtering the Cartesian wind components with a Cartesian weighting function would have distorted the swirling wind field.

Horizontal divergence was integrated upward from the surface. Vertical wind was set to zero at the surface and at 12.5 km. The divergence was adjusted to obtain zero vertical motion at 12.5 km, and the horizontal winds were modified to account for the change in divergence. Thus, the adjusted wind field still satisfied mass continuity. Since the winds from 0 to 1 km were the most affected by sea surface contamination, half of the total divergence correction was applied to the layer from 0 to 2 km in height. The remainder was applied from 2 to 12.5 km. An irrotational wind field with a vector mean of zero, but which had a divergence field equal to the necessary divergence correction field, was added to the originally analyzed horizontal wind field.

Before the divergence adjustment was made, the integrated divergence of the analyzed wind field produced a mean vertical wind of $-0.25$ m s$^{-1}$ at 12.5 km. The mean error in the vertical wind in the budget volume was therefore probably $\sim 0.25$ m s$^{-1}$. Since the maximum mean upward vertical wind was $\sim 0.2$ m s$^{-1}$, such a vertical wind error could produce a factor of 2 error in the condensation produced by the axisymmetric part of the wind field. Systematic errors associated with aircraft navigation and attitude, and sea surface contamination are probably responsible for this mean error. The ramifications will be discussed briefly in section 6b.

Radar reflectivity data obtained from the same vertically scanning radar as the Doppler data were damaged, so data obtained with a non-Doppler radar aboard the aircraft flying at $\approx 6$ km were used to map the three-dimensional reflectivity structure. These data were filtered in the same manner as the tangential and radial Doppler winds.

The thermodynamic retrieval described in section 5 derives the deviation of temperature and pressure from an unknown constant temperature at a constant height. The thermodynamic equation is used to determine the vertical variation of the horizontally averaged potential temperature and pressure. Aircraft in situ data were used to anchor the pressure and temperature retrieval. The mean difference of the in situ data from the retrieved temperature and pressure along the flight track was added to the retrievals. This procedure led to the best match of the in situ and retrieved data. Three-dimensional distributions of temperature and pressure obtained by retrieval were used in this study only to determine the value of saturation humidity.
5. Retrieval methods

To evaluate the terms on the rhs of (1), and hence (10), we need \( p, V_H, w, q_c, q_p, \) and \( V_T \). Terms \( c \) and \( e \) are calculated as residuals of terms on the rhs of (1). A base-state density is assumed and used throughout the domain, and \( V_H \) and \( w \) are determined as described in Part I and section 4. The fallspeed \( V_T \) is parameterized as a function of \( q_p \). The remaining variables \( (q_c \) and \( q_e \) are determined by two different retrieval methods, which will be described in detail in the following.

The first method, developed by the authors, uses the Doppler winds and radar-reflectivity data as the primary sources of information. Precipitation content, \( M \), and precipitation flux divergence (sedimentation) are computed from radar reflectivity. The rate of precipitation production is determined from the steady-state continuity equation of precipitation. Inverse micrometeorological modeling is employed to obtain cloud content \( (M_c) \) from \( M \) and the precipitation production rate.

The second method (Hauser et al. 1988; hereafter referred to as HRA) uses the Doppler wind field and micrometeorological modeling. Reflectivity is used only to establish boundary values of precipitation content. The distributions of temperature \( (T) \) and pressure \( (p) \) are dynamically retrieved (Roux 1988) from the momentum and thermodynamic equations with the Doppler winds as input. Saturation specific humidity \( (q_s) \) is determined from \( T \) and \( p \). The three-dimensional continuity equations for \( q_p \) and total water \( (q_T = q_e + q_p + q_c) \), where \( q_e \) is water vapor mixing ratio, are solved simultaneously using the technique of Hauser and Amayenc (1986, hereafter HA) and HRA. The value of \( q_c \) is determined as a residual from \( q_T, q_p, \) and \( q_s \). Parameterized micrometeorological equations are required to determine the sources and sinks in the continuity equations. One of the sinks, precipitation sedimentation, is determined from the derived \( q_p \) and \( a q_p V_T \) relationship. The method of HRA is applied here for the first time in three dimensions.

In method 1 observed reflectivity is used to determine precipitation content. It is assumed that, below the \( 0^\circ \text{C} \) level, all precipitation is rain and all cloud is water. Above the \(-20^\circ \text{C} \) level, all precipitation and cloud are assumed to be ice. The fraction of precipitation that is assumed to be ice varies linearly from 0 at the \( 0^\circ \text{C} \) level to 1 at the \(-20^\circ \text{C} \) level. Requirements to compute frozen and liquid precipitation at the same grid point by method 2 were too great, so the precipitation was either all ice (temperatures < \( 0^\circ \text{C} \)) or all water (temperatures $\geq 0^\circ \text{C}$).

Method 1 requires much less computation. The budget is determined more directly from the data. The value of specific humidity is not required, and hence it need not be modeled. The value of \( q_p \) is determined directly from the reflectivity data, and only some inverse micrometeorological modeling is required to obtain \( q_c \). Large errors in precipitation amount may arise, however, from the attenuation of the X-band radar signal by intervening precipitation. Errors are especially high in the melting band, where the relative quantities of ice and water are unknown, and where fallspeeds of particles change rapidly with height.

Method 2 requires much more computation. The budget is determined less directly from the data and more from micrometeorological modeling. Dynamic retrieval of temperature and pressure is required to obtain saturation vapor content. The nature of the solution method requires that vapor diffusion be included, although it can only be modeled crudely. The value of \( q_p \) is determined from micrometeorological modeling, and \( q_e \) is a residual. Errors across the melting band, however, are smaller, since reflectivity is not a major input and continuity of precipitation is required by the method. Continuity of total water content is directly specified by one of the two continuity equations, and this constraint is a point in favor of method 2.

One of the improvements of the Norbert dataset over those of earlier budget studies is that it is three dimensional. The water budget is thus determined without assuming axisymmetry. To estimate the improvement in water budget computation attained with the three-dimensional data, the method 2 three-dimensional budget is compared with a water budget computed by applying method 2 to the axisymmetric wind field, computed as described in section 5c.

a. Method 1

The first method is similar to that of Churchill and Houze (1984). Precipitation content and fallspeed are expressed as functions of reflectivity. The rate of precipitation production is determined from the radar reflectivity and Doppler wind. The cloud content necessary to produce the estimated production is determined by inverting the micrometeorological parameterizations for autoconversion and collection, which appear in the continuity equations for precipitation and cloud water [see Kessler (1969) for the definition and physical interpretation of these terms]. Once the cloud content is determined everywhere from the inverted equations, then the total hydrometeor content is known everywhere. The rates of condensation, deposition, evaporation or sublimation, and the outward transport of condensate may then be determined everywhere from the wind field, precipitation fallspeed, precipitation content, and cloud content.

Precipitation content is determined from a reflectivity composite through the use of reflectivity mass relationships of the form

\[
Z = aM^b, \tag{11}
\]

where \( Z \) is the radar reflectivity expressed in units of \( 10^{-18} \text{ m}^3 \) (traditionally \( \text{mm}^6 \text{ m}^{-3} \)), \( a \) and \( b \) are empirical constants, and \( M \) is the precipitation content in
grams per cubic meter. In this study, \(a = 14,630\) and \(b = 1.4482\) in the rain. These values were determined in the study of Jorgensen and Willis (1982) but not published (P. Willis, personal communication). The values used in the case of ice are \(a = 670\) and \(b = 1.79\) (Black 1990). These relationships were derived from observations obtained within hurricanes. The rainfall rate in hurricanes is given by Jorgensen and Willis (1982):

\[
Z = 300 R^{1.35},
\]

where \(R\) is rainfall in millimeters per hour (or kg h\(^{-1}\) m\(^{-2}\)). Once \(R\) and \(M\) are determined, then the mass-weighted terminal fallspeed of rain, \(V_T\) (in m s\(^{-1}\)), is computed from

\[
V_T = R \frac{(\rho_0/\rho)_0^0.4}{(3.6 M)},
\]

where \(\rho_0\) is the air density at the surface, and the numerator on the rhs of (13) is a correction for air density (Foote and du Toit 1969). Equation (13) is based on the continuity of precipitation if \(R\) and \(M\) are known and the mean \(w\) in deriving (12) is assumed to be zero. The fallspeed of Atlas et al. (1973) was used for ice precipitation.

Precipitation production \(P\) is determined from \(M\), the Doppler winds, and from \(V_T\) through the following relation:

\[
P = \nabla \cdot VM - \frac{\partial}{\partial z} (V_T M).
\]

This relation follows from the steady-state continuity equation for \(M\). The first term on the rhs of (14) is the three-dimensional advective flux divergence of precipitation, and the second is the flux divergence owing to the terminal fallspeed relative to the air motion. From such a steady-state assumption significant error can be expected, at least locally; however, we are not able to evaluate local time derivatives using this dataset.

To compute the distribution of precipitation particles, they are assumed to follow an exponential formulation similar to that of Marshall and Palmer (1948):

\[
N(D) = N_0 e^{-\lambda D},
\]

where \(D\) is particle diameter, \(N\) is the number concentration for a given size interval, \(N_0\) is the zero intercept, and \(\lambda\) indicates the exponential decrease with increasing diameter of the number concentration. The resulting precipitation content is

\[
M = \frac{\pi}{6} \int_0^\infty \rho_0 D^3 N_0 e^{-\lambda D} dD,
\]

where \(\rho_0\) is the density of the precipitation; \(N_0\) is set to \(N_{OR}\) in rain and \(N_{OG}\) in ice precipitation. These values are presented in Table 1. Equation (16) is solved for the value of \(\lambda\). The \(y\)-intercept values for the rain \(N_{OR}\) and ice precipitation \(N_{OG}\) size distributions were set as indicated in Table 1 (the same values used for method 1). The density of all ice particles in the size distribution was set to 0.1 g m\(^{-3}\).

The value of \(N_{OG}\) is much higher than that of Gunn and Marshall (1958), or the value used by Rutledge and Hobbs (1983) or HRA, but it is the value suggested by the ice size distributions determined for summer MONEX precipitation by Gamache (1990). They are also suggested by the results of Part II and Black (1990). Concentrations of \(\approx 100\) L\(^{-1}\), and 2-3 g kg\(^{-1}\) were found in Hurricane Norbert.

When \(P \leq 0\), cloud content (the amount of condensed water that is not in the form of precipitation in units of mass per unit volume) is assumed to be zero, and \(P\) is the evaporation or sublimation rate. When \(P > 0\), precipitation production is given by

\[
P = \alpha (M_c - M_0) + M_c \int_0^\infty \frac{\pi}{4} N_0 e^{-\lambda D} E_c D^2 V_T dD,
\]

where \(M_c\) is cloud content, \(M_0\) is the autoconversion threshold, and \(E_c\) is the collection efficiency. The terms on the rhs of (17) are Kessler's (1969) autoconversion and collection. If \(M_c < M_0\), the first term is set to zero (in the developed hurricane the overall importance of autoconversion is small, but it is included here for completeness). The method 1 retrieval is completed by solving (17) for \(M_c\). Once the cloud content \(M_c\), precipitation content \(M\), the rainfall \(R\), and the wind \(V\) are determined, the bulk water budget is computed from the integration of (1).

In this study, \(E_c\) is set to 1 for rain. The value of \(E_c\) for ice particles is assumed to vary exponentially with temperature from 1 at \(T = 0^\circ\)C to 0.1 at \(T = -20^\circ\)C (Hobbs 1974). The value of 0.1 is the same as that used by Churchill and Houze (1984). At temperatures \(< -20^\circ\)C, \(E_c\) is assumed to remain at 0.1 for frozen precipitation. Autoconversion of cloud to precipitation was set to 0.001 s\(^{-1}\) and 0.0001 s\(^{-1}\) for water and ice, respectively, while the autoconversion thresholds for water and ice are both set to 0.0005 kg kg\(^{-1}\).

![Table 1. List of values for important parameters used in the computation of the water budget. Refer to section 2 for further explanation.](image)
b. Method 2

The methodology used to solve the water continuity equation by method 2 has been developed extensively by HA, HRA, Roux et al. (1984), and Roux (1985, 1988). Only the basic ideas and details that are different from the original version are given here.

The acceleration of the wind in a steady-state storm in a moving coordinate system is

\[ \mathbf{A} = \mathbf{V}_r \cdot \nabla \mathbf{V}_r, \]

where \( \mathbf{A} \) is the three-dimensional acceleration and \( \mathbf{V}_r \) is the three-dimensional storm-relative wind determined from the Doppler analyses. These accelerations are used in the thermodynamic retrieval of Roux (1988), which yields the distributions of three-dimensional temperature and pressure perturbation required by the anelastic momentum equation to account for the observed accelerations. The \( T \) and \( p \) perturbations are measured relative to an assumed sounding, plus a constant. The constant is determined by comparing the sum of the perturbation distribution and the assumed mean vertical sounding with the available in situ data. In this study, we use the retrieved \( T \) and \( p \) only to determine the saturation mixing ratio (\( q_s \)). Given \( q_s \), micrometeorological modeling yields \( q_p \) and \( q_c \) where the air is saturated or \( q_p \) and \( q_c \) where the air is unsaturated.

The micrometeorological retrieval used is that of HRA. The continuity equations for total water (the sum of vapor, precipitation, and cloud) and precipitation are solved simultaneously. The steady-state equations in three dimensions are

\[
-K_H \nabla^2 H \frac{q_T}{\rho} \frac{\partial}{\partial z} \left[ (\rho K_z) \frac{\partial q_T^*}{\partial z} \right] + \mathbf{V} \cdot \nabla q_T = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho V_T q_p) \]  

(19)

and

\[
-K_H \nabla^2 H q_p \frac{\partial}{\partial z} \left[ (\rho K_z) \frac{\partial q_p}{\partial z} \right] + \mathbf{V} \cdot \nabla q_p = -\frac{1}{\rho} \frac{\partial}{\partial z} (\rho V_T q_p) + P, \]  

(20)

where \( q_T \) is the total water (solid, liquid, and vapor) mixing ratio. The perturbation total water mixing ratio, \( q_T^* \), is given by

\[ q_T^* = q_s + q_c + q_p - q_{so}, \]

(21)

where \( q_{so} \) is the level mean saturation specific humidity. The perturbation value of \( q_T \) is used for vertical diffusion, since a value as high as 1500 \( \text{m}^2 \text{s}^{-1} \) for \( K_z \) would cause the vertical diffusion to dominate artificially over advection, but the large value is required to maintain a computationally stable and tractable solution to (19) and (20) (further discussion below). The total water content \( q_T \) is then

\[ q_T = q_s + q_c^* + q_p, \]

(22)

where \( q_c^* \) is the saturation deficit (\( q_c^* < 0 \)) or cloud mixing ratio \( q_c \) (\( q_c^* > 0 \)). The liquid and ice mixing ratio \( q_c \) are equal, therefore, to \( q_p \) if \( q_c^* < 0 \) and to \( q_c^* + q_p \) if \( q_c^* > 0 \). In this study (19) and (20) are solved simultaneously by successive relaxation.

Precipitation production is determined by the micrometeorological parameterization of Lin et al. (1983), with the Rutledge and Hobbs (1983) modification of the rainfall evaporation parameterization. Only a subset of the micrometeorological processes of Lin et al. are considered here since it is assumed that only one kind of precipitation (rain or ice) can exist at a given location, either rain (\( T \geq 0\)°C) or precipitation ice (\( T < 0\)°C). To keep the problem and its solution manageable, it is further assumed that only one particle type can exist anywhere within the subfreezing volume.

The higher values of \( N_{0G} \) will lower the computed reflectivities associated with a given content of precipitation compared to those obtained using the \( N_{0G} \) of HRA. When \( N_{0G} \) is smaller, the same mass of precipitation must be distributed in larger particles. It is quite possible that the lower value for \( N_{0G} \) used by HRA explains the high reflectivities observed when their ice collection efficiencies were set to values > 0.03. The higher value of \( N_{0G} \) used here would allow them to raise their collection efficiency. More cloud could then be converted to ice precipitation, thus lowering cloud ice contents and thereby reducing the very high 3-4 g kg\(^{-1}\) of cloud ice seen in their retrievals. The collection efficiencies of cloud by raindrops and precipitation ice were set as in method 1.

The type of precipitation ice particle chosen for method 2 was an aggregate of unrimed side planes, assemblages of plates, bullets, and columns, as in Locatelli and Hobbs (1974). The fallspeeds of individual particles were those of Locatelli and Hobbs (1974). This particle type was chosen because the lower fallspeeds allowed more precipitation to be carried from the updrafts on the left side of the storm to the right side of the storm before falling through the melting band, in better agreement with radar observations. Precipitation was nearly absent on the right side of the storm when graupel was assumed.

The numerical method used to solve (19) and (20) is a centered-difference scheme. Successive relaxation is used, and a diffusion term is therefore required to solve the problem (the diffusion term includes the center value in the discrete computation, which is required for numerical stability). The vertical diffusion constant \( K_z \) was set to 1500 \( \text{m}^2 \text{s}^{-1} \), the value employed by HRA, while the horizontal diffusion constant \( K_H \) was set to 15 000 \( \text{m}^2 \text{s}^{-1} \), a value ten times larger than theirs. These values were chosen to be as small as possible, while still allowing a solution of the three-dimensional
problem with a manageable number of grid points (every 1.5 km horizontally and 0.5 km vertically), in a practical amount of time. An isotropic diffusion, which would have made the problem tractable, had too large a vertical diffusion. With the mixing coefficients employed in this study, vertical diffusion did not dominate over vertical advection, but vertical vapor diffusion below the dry downdrafts was an important term in the lower troposphere.

Boundary conditions are required to solve (19)–(22). The total water content \( q_T \) was set equal to the sum of \( q_p \) (determine from observed radar reflectivity) and \( q_v \) [from temperatures and pressures retrieved by the Roux (1988) method] at the top boundary \((z = 12.5 \text{ km})\), above the upper boundary of the water budget volume. At the bottom \((z = 0 \text{ km})\), \( q_T \) was set equal to the sum of the retrieved \( q_s \) and \( q_p \) \((q_v = 0)\). The vertical derivatives of \( q_s \) and \( q_p \) were set to zero at the bottom boundary. Wherever there was inflow across the horizontal boundary, the value of \( q_T \) was also set equal to the value of \((q_p + q_v)\). The value of \( q_v \) was retrieved from radar reflectivity observed at the boundary. Wherever there was outflow, the normal derivative of \( q_T \) was set equal to zero. When the zero normal derivative condition was set everywhere on the lateral boundaries, convergence to a solution was much slower, and the result was much more unsaturated than indicated by in situ observations. The values of saturation specific humidity were determined from Doppler thermodynamic retrievals of temperature and pressure. Although the bottom and top boundaries of the retrieval were 0.0 and 12.5 km, respectively, the budget was computed from 0.5 to 12.0 km.

c. Axisymmetric budget

To evaluate the importance of asymmetries in the wind, thermodynamic, and microphysical fields, an axisymmetric budget was determined. For this purpose, the wind field used in methods 1 and 2 was degraded to an axisymmetric field. The mean of all the winds at a given height and a given radius from storm center was determined. These values were then interpolated to a Cartesian grid of the same size as used for methods 1 and 2. The vertical wind was computed. The temperature and pressure were then retrieved for this axisymmetric wind field. The method 2 retrieval was applied to the axisymmetric wind and thermodynamic analyses, using the same boundary conditions as in the method discussed in section 5b and the parameters as listed in Table 1.

6. Results

a. Definitions

The term “front” refers to the half-plane on the side toward which the storm is moving and which is bounded by the line that passes through the storm center and is perpendicular to the tangent of the storm track at the location of the storm center. The term “rear” refers to the other half-plane. For an observer facing frontward, “left” refers to anywhere to the left of the tangent line, and “right” to anywhere to the right of the tangent line. During the composite period Hurricane Norbert was moving toward a heading of 327° at a speed of 6 m s⁻¹. Consequently, front, rear, right, and left refers to points that have a bearing from

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**Fig. 2.** Vertical wind in meters per second at (a) 3 km and (b) 6 km. Storm center is at the center of the analysis. The y-axis represents a heading of 327°, the direction toward which the storm was moving. Quadrants are indicated as right front (RF), right rear (RR), left rear (LR), and left front (LF). The letters “U” and “D” indicate regions of updraft and downdraft.
the storm center of 237° clockwise to 57°, 57° clockwise to 237°, 327° clockwise to 147°, and 147° clockwise to 327°, respectively. The left front (LF), right front (RF), left rear (LR), and right rear (RR) quadrants refer to the intersections of the front, rear, right, and left halves of the storm.

b. Precipitation and radar reflectivity

Figures 2a,b show that upward vertical wind was more widespread on the left side of the storm, while downward vertical wind was most prevalent on the rear side. Directly to the front of the storm, the vertical wind was generally upward, while directly to the rear it was generally downward. One would expect, therefore, that condensation predominated on the left side of the storm, with maximum precipitation just downwind, while evaporation predominated on the rear side.

The azimuthally averaged radial wind in Hurricane Norbert was observed to be very weak (≤2 m s⁻¹) at all levels. Figures 3a,b show the azimuth–height (constant radius) distribution of radial wind at radii of 25 (near the radius of maximum wind) and 37.5 km (the outer radius of the water budget volume), respectively. Although there were small areas of outflow and inflow below 3 km of ≥10 m s⁻¹, outflow or inflow at most azimuths and heights was <4 m s⁻¹. The radial winds are consistent with a front-to-rear flow of ~10 m s⁻¹.

Fig. 4. Horizontal cross sections of the three-dimensional composite of radar reflectivity. Units are dBZ. Cross sections are at (a) 0.5 km, (b) 3 km, and (c) 6 km. Plot orientation same as in Fig. 2.
Fig. 5. As in Fig. 4 except for method 2 retrieved reflectivity.

Fig. 6. Radius–height mean radar reflectivity. Averaging is done for a given radius and height over all headings as seen from the storm center (full 360°). Radius = 0 corresponds to the storm center. The observed reflectivity is shown in (a) and the retrieved reflectivity of method 2 is shown in (b).

below 3 km and a rear-to-front flow of \( \sim 5 \) m s\(^{-1}\) above 6 km.

The observed reflectivity at 0.5 km shown in Fig. 4a was strongest on the left side of the storm at a radius of about 18 km, and a very large crescent-shaped area of 30 dBZ or higher covered most of the left. The method 2 precipitation content (Fig. 5a) was maximum in the left rear quadrant at radii greater than 20 km. The outer contours determined by method 2 were in significantly better agreement with observations than the inner contours surrounding the maximum. The most important difference was that the region of retrieved reflectivity \( > 45 \) dBZ was much greater than that observed, and its center was shifted slightly outward and downwind of that observed. Most of the 50% higher total precipitation from method 2 was associated with the higher-than-observed peak reflectivity. Attenuation of the radar signal by intervening precipitation may be the explanation for observed reflectivities being lower than those retrieved by method 2.

This argument concerning intervening attenuation is substantiated by the radius–height mean reflectivity distributions shown in Figs. 6a,b. At the surface the retrieved reflectivity (Fig. 6b) was about 2 dB greater than the observed reflectivity (Fig. 6a), except at radii < 20 km, where the retrieval clearly did not reproduce
the inner reflectivity maximum. At the 4-km level, however, the observed reflectivity appeared to be equal or slightly more than the retrieved reflectivity. The observing aircraft was located at 6 km. Attenuation of the 3-cm signal by intervening precipitation could explain this discrepancy. Attenuation is highly nonlinear and complex, and we did not attempt to correct for it. At 3 and 6 km (Figs. 4b,c, and Figs. 5b,c), the agreement between the observed and retrieved reflectivities was much better.

c. Cloud content

Cloud content at the 3-km level determined by method 1 was found mostly in the left front of the storm (Fig. 7a). This result is consistent with the diagnosis of strongest upward vertical wind in that quadrant. From the distribution of updrafts at this level (Fig. 2a), however, one would expect more cloud than computed in the left rear quadrant and at radii beyond 20 km. This discrepancy is explained by the diagnosis of precipitation production, which was dominated by the vertical precipitation flux divergence. Method 1 diagnoses high cloud content if the precipitation formation rate is high and the precipitation content is low, both of which were the case just upwind of the inner precipitation maximum (Fig. 4), where method 1 found the highest cloud contents. The precipitation flux divergence determined from reflectivity, however,
appeared to be too small, possibly because of the attenuation of reflectivity.

The cloud content at 3 km determined by method 2 (Fig. 7b) was still concentrated in the left front of the storm. The cloud contents, however, were generally higher than those of method 1, and more cloud was found in the left rear quadrant, which is also consistent with the Doppler analysis that indicated updrafts $>5$ m s$^{-1}$ in the upwind half of the left rear quadrant.

The method 1 cloud content at 6 km (Fig. 8a) was again concentrated in the front of the storm, and cloud contents were again very high at small radii on the left front side. Somewhat more cloud, however, was diagnosed in the left rear quadrant.

Method 2 (Fig. 8b) indicates the main band of cloud content at 6 km located $\sim 35-40$ km from the storm center to the left of the storm, and spiraling inward to about 25 km at its downwind end. At its upwind end, the band was located about 5-10 km outward from that diagnosed by method 1.

d. Condensation and evaporation

According to method 1, condensation at 3 km occurred mainly in the front left of the storm (Fig. 9a). One thin weak band of condensation was diagnosed in the left rear quadrant. The maximum condensation occurred just upwind of the inner precipitation max-

Fig. 9. Constant height evaporation and condensation at 3 km as determined using (a) method 1 and (b) method 2. Units are grams per cubic meters per hour. Positive values indicate rates of evaporation, while negative values indicate rates of condensation. Plot orientation same as in Fig. 2. The letters "C" and "E" indicate regions of condensation and evaporation.

Fig. 10. As in Fig. 9 except for 6-km evaporation and condensation.
imum. A region of strong evaporation was found outside the updraft region in the left rear quadrant, suggesting that evaporation in this quadrant was greater in the downdrafts than condensation was in comparable updrafts, which would be highly unlikely. The method 2 condensation at 3 km (Fig. 9b) was stronger in general and concentrated on the left side of the storm. Much less evaporation was diagnosed in the left rear quadrant. Evaporation of precipitation is computed by microphysical parameterization in method 2, rather than from the continuity of observed precipitation as in method 1.

Method 1 condensation at 6 km (Fig. 10a) was greater in the left rear updraft region than at 3 km, and strong evaporation was indicated just downwind of the precipitation maximum. The intensity of this evaporation probably resulted from error in the precipitation flux divergence, derived from reflectivity, which at 6 km may have been influenced by the melting level (5 km). The gradients at this height were affected by assumptions concerning the precipitation content and fallspeed just above and below that level. Method 2 is not affected by brightband error, since it does not rely on the observed reflectivity. Instead, it follows the Doppler-derived vertical wind pattern (Fig. 2a). Less net condensation was found at 6 km than at 3 km, in agreement with moist thermodynamics. In the main band, method 2 condensation at 6 km (Fig. 10b) was generally similar in structure to that determined by method 1, and condensation in the main band was roughly equal to that diagnosed by method 1. The method 2 condensation band spiraled inward more, the evaporation was less intense and more evenly spread just downwind of the updrafts, and an inner region of condensation was diagnosed on the right side of the storm.

e. Azimuthally averaged mean structure and advection

To understand the relationship of the hurricane eyewall to the surrounding regions of more stratiform precipitation, it is useful to determine the transport of cloud and precipitation outward from the eyewall. This section examines mean radius–height distributions of radial water advection, condensation, and evaporation. They indicate the spatial distribution of the terms contributing to the full storm bulk water budget summarized in Table 2.

The azimuthally averaged radial water flow in Hurricane Norbert was weak (<|2 m s⁻¹|) and atypical. At radii of 25–35 km, many hurricanes have mean radial inflow below the 1-km level ~ 10 m s⁻¹ and outflow near 12 km ~ 10 m s⁻¹. Strong (>10 m s⁻¹) azimuthally averaged outflow did not exist in the upper troposphere, and strong mean inflow was not found in the lower troposphere, except in the lowest 1.0 km (where the wind flow was not well documented by the airborne Doppler as a result of contamination by sea surface echo). Consequently, for significant water outflow to occur, radial wind needed to be positively correlated with water content. In Norbert the correlation was weak; hence, the net radial outflow of water was small.

Figure 11a shows the full-storm, azimuthally averaged mean radial advection of water (cloud and precipitation in both liquid and ice form) as determined by method 1. The mean advection was very small. Small amounts of outflow are indicated above about 1 km and inflow at low levels at the radius of the eyewall (21 km). Method 2, shown in Fig. 11b, shows somewhat more low-level outflow at the eyewall radius and somewhat less midlevel outflow at a radius of 30 km. The radial advection associated with the axisymmetric wind field (Fig. 11c) was even smaller.

The method 1 azimuthally averaged radius–height distribution of condensation had a maximum at a height of about 5 km (near 0°C), as shown in Fig. 12a. Evaporation occurred beyond the 25-km radius and was quite weak at smaller radii. Again the radar bright band is indicated as a source of error in method 1. Method 2, which depends on the distribution of Doppler-derived vertical air motion, shows a condensation maximum (Fig. 12b) at ~2–3 km and a value at 5 km in the eyewall region smaller than that diagnosed by method 1.

<table>
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<tr>
<th>Budget term</th>
<th>Method 1 (10⁶ kg h⁻¹)</th>
<th>Method 2 (10⁶ kg h⁻¹)</th>
<th>Axisymmetry (10⁶ kg h⁻¹)</th>
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updrafts and downdrafts are weaker. Saturation in weaker downdrafts can be maintained more easily. Also, most of the strong updrafts in the three-dimensional analysis were on the left side of the storm, and the rain they produced also fell mostly on the left side of the storm. The strongest downdrafts, however, occurred just downwind of the heavy rain. Sufficient evaporation of rain to maintain saturation was not possible. The result was a mean condensation at a given

at the same location by method 1. The computation of condensation from the axisymmetric wind field is shown in Fig. 12c. Its structure was very similar to that of the asymmetric storm; however, the total condensation was approximately halved throughout the domain.

The main difference between the three-dimensional and axisymmetric computations was in the evaporation in downdrafts. In the axisymmetric fields, the averaged

FIG. 11. Radius–height mean advection of water (cloud and precipitation) in the radial direction. Averaging is done as in Fig. 6. Positive contours indicate radial outflow, while negative contours indicate radial inflow. Single solid curve is the 0°C isotherm. Units are grams per square meter per second. Advection is shown for (a) method 1, (b) method 2, and (c) method 2 applied to the symmetric radius–height mean wind field. The letters “I” and “O” indicate regions of inflow and outflow.

FIG. 12. Radius–height mean evaporation and condensation. Averaging is done as in Fig. 10. Units are grams per cubic meter per hour. Positive values indicate the rate of evaporation, while negative values indicate the rate of condensation. Results are shown for (a) method 1, (b) method 2, and (c) method 3 applied to the symmetric radius–height wind field. The letters “C” and “E” indicate net condensation or net evaporation.
height and radius that was no longer directly proportional to the mean upward motion.

A larger total source of water vapor is thus required in the three-dimensional case. According to method 2, this vapor was supplied mainly by greater upward eddy transport of vapor through the bottom boundary, as seen in Table 2. Since the resolvable upward vapor flux through the bottom boundary was approximately equal to the radial inflow of vapor, the upward eddy transport must have been supplied by enhanced surface evaporation. The mean value of upward eddy flux (1.25 g m$^{-2}$ s$^{-1}$) is similar to that estimated by Hawkins and Imbembo (1976) for Hurricane Inez (1966).

f. Azimuthally averaged advection by quadrant

Although the full 360° azimuthally averaged radial water advection was small, its structure varied by quadrant. In the right front quadrant, method 1 (Fig. 13a) found low-level inflow of water up to about 3 km, with outflow above that level. The method 2 (Fig. 14a) computations were qualitatively similar but showed less outflow at upper levels, consistent with the lower cloud content estimations of that method in the right front quadrant (Fig. 7–8). In the right rear quadrant, methods 1 and 2 both show outflow below the 3 km level, with weaker but deeper inflow above (Figs. 13b and 14b). The upper-level inflow computed by method 2 was somewhat weaker. In the left rear quadrant, both methods indicate outflow from 1 to 4 km in height and inflow below 1 km and above 4 km (Fig. 13c and 14c). The low-level inflow computed by method 2 was greater because of the higher cloud and precipitation contents (Figs. 4–8). In the left front quadrant, methods 1 and 2 produced similar radial advection results except for the placement of maxima (Figs. 13d and 14d). Inflow occurred in the lowest 3 km with outflow above.

From the analyses for individual quadrants, a pattern emerges. The upper-level inflow in the rear quadrants indicates that the lighter precipitation particles, which are ejected from the eyewall updraft and did not fall out quickly as rain, were not detrained immediately from the storm core. Instead, they were carried slowly inward until they reached the front of the storm, where they were ejected at high levels in the upper-level outflow of the front quadrants, in good agreement with the trajectories presented in Part II.

g. Constant radius analyses of radial advection

To determine the height and bearing from storm center at which the most intense outward transport of condensate occurred, constant radius azimuth–height
analyses have been constructed by both methods at fixed radii: 25 km (just outside the radius of maximum wind) and 37.5 km (the radius closest to the exterior of the budget region). At radius = 25 km, both methods indicate that upper-level inflow occurred with a fairly intense maximum at 177° and 8 km in height (Fig. 15). A low-level outflow maximum existed about 40° downwind of the inflow maximum. The strongest exchange of water with the environment was in the left rear quadrant and in the halves of the right rear and left front quadrants that were adjacent to the left rear quadrant. At 37.5 km (Figs. 16a,b), methods 1 and 2 produced patterns that are qualitatively similar below the 9-km level. Above that level, method 1 again estimated more intense inflow and outflow, consistent with the higher diagnosed cloud contents (Figs. 7–8). The water outflow maximum from method 1 tilted strongly upwind with height, centered below the melting level at about 120° and above the melting level at 300°. The upper-level outflow maximum computed by method 2 was centered at 270° but was much weaker than that indicated by method 1. The outflow maximum aloft in the left front quadrant at r = 37.5 km must have been the region of maximum advection of ice particles generated in the eyewall region out to the region surrounding the eyewall. This result agrees well with Part II, where it was inferred that the stratiform rainband to the southwest of the storm center, outside the eyewall region, was supplied by ice particles that must have been generated in the eyewall updrafts and circulated around the storm following outward spiraling trajectories that exited the eyewall zone in the left front quadrant at the 7–9-km level (see Fig. 23 of Part II). This result shows that, although net radial advection of cloud and precipitation was small for the storm as a whole, a substantial outflow of ice nonetheless occurred at upper levels in the left front quadrant.

This localized outflow of cloud and precipitation from the eyewall played an important role in the stratiform precipitation process taking place in the region outside the eyewall. This process could not have been identified without decomposing a three-dimensional water budget.

It is further evident from Fig. 16 that the maximum inflow was centered over a broad region from 160° to 360° (the left side of the storm) below 2.0 km in both sets of results and that deep upper-level inflow was a maximum above the 0°C isotherm south of the center.

A basic pattern of inflow and outflow of hydrometeor mass emerges from these two analyses. The strongest low-level inflow occurred in the front of the storm, particularly in the left front quadrant, while the greatest upper-level inflow was also concentrated in the left front quadrant. As discussed in Part II, this upper-level outflow of ice particles occurred as part of the circu-
sinks do not sum exactly. Residuals $\sim 1-5 \times 10^9$ kg h$^{-1}$ in the total budget of each quadrant (Figs. 17a,b) resulted mainly from the conversion from Cartesian to polar coordinates to compute the final bulk budget out to a radius of 37.5 km. Smaller residuals (according to the methodology of this study, they are computational errors) of $\sim 10^9$ kg h$^{-1}$ were in the original Cartesian solutions of (19) and (20).

The full storm bulk budget rainfall computed by method 2 exceeded that observed (method 1) by 50%. The total condensation computed by the two methods was comparable; however, the method 1 evaporation was greater, while the method 2 rain was greater. The disagreement between the methods was greatest in the left rear quadrant of the storm, where the reflectivity was greatest, and where attenuation probably affected the method 1 estimates.

The bulk budgets for individual quadrants for both methods indicate that about 20% of the rain fell on the right-hand side of the storm. Method 1 indicates that 45% of the rain fell in the left rear quadrant of the storm, while method 2 estimates the left rear rain accounted for 70% of the total rainfall. Both methods find $\sim 50\%$ of the bulk condensation ($C$) occurred in the left front quadrant, and that the net bulk condensation ($C - E$) in that quadrant was very close to half the value of the full-storm value. There are, however, some important discrepancies. Method 1 computations in-

Fig. 15. Constant radius plot of radial advection of water (cloud and precipitation) for radius from storm center equal to 25 km. Units are grams per square meter per second, and positive (negative) values indicate radially outward (inward) advection. The azimuth (abissa) for a specific location indicated is the heading of that location as viewed from the storm center. Single solid curves are the 0°C isotherm. Advection shown was determined by (a) method 1 and (b) method 2. Storm quadrants are labeled as in Fig. 2. The letters "I" and "O" indicate inflow and outflow.

l. Bulk water budgets

Table 2 shows the full-storm bulk water budget, while Figs. 17a,b display the bulk budgets by quadrant for methods 1 and 2, respectively. Except for $R_T$, inflows (outflows) of water are defined to be negative (positive). The sign of the numbers in Table 2 is the same as that defined in section 2 and used in (10). The values for advection of vapor in method 2 are included in Table 2, but it should be noted that they are not part of the liquid/solid budget of (10).

The reader may note that the budget sources and
dicate that net condensation occurred in the front of the storm, while net evaporation occurred in the rear of the storm. Method 2 estimates that net evaporation occurred only in the right rear quadrant, while it finds that net condensation actually occurred in the left rear quadrant as well as in the front half of the storm. Possible factors contributing to these differences in spatial distribution of precipitation in the storm are

1) The assumption of steady-state storm structure. Nearly two hours were required to map the winds and radar reflectivity needed to construct the storm composite. The time between dual-Doppler scans of the right front and left front quadrants was nearly two hours, although radar reflectivity observations were obtained more frequently. The merger of these two quadrants may have produced errors in the vertical velocity near that boundary and, hence, errors in condensation diagnosed by method 2.

2) The strong attenuation of the X-band radar (section 6b).

3) Radar calibration. An error of ~4 dB produces a factor of 2 error in the diagnosed precipitation content. A calibration error would thus affect method 1 computations greatly. A factor of 2 error in precipitation content would produce approximately a factor of 2 error in bulk net condensation (since it is strongly related to the bulk precipitation flux divergence).

Strong bulk radial water advection (C4 of Marks and Houze 1987) was not found in the full-storm budget, nor in any of the storm quadrants, by either method. The bulk advection was well under 10% of the total condensate produced. Quite different behavior has been found in mature and intensifying storms with strong eyewall updrafts and strong mean upper-level outflow (e.g., Marks 1985; Marks and Houze 1987). In the decaying Hurricane Norbert, the mean eyewall radial outflow was by contrast much weaker. Outward radial advection approximately equaled inward advection.

The condensation, evaporation, and net condensation computed from the axisymmetric wind field were all significantly less than those computed by methods 1 and 2 for the three-dimensional wind field (Table 2). The condensation was approximately half, and the evaporation was about one quarter. The net condensation was also less than from either of the other two computations.

As discussed in section 4, the error in the mean vertical wind diagnosed in the budget volume of Hurricane Norbert could be as large as 0.25 m s⁻¹ (but was probably smaller). At worst such an error could mean a doubling or halving of the bulk net condensation diagnosed by method 2 for either the three-dimensional or axisymmetric wind fields. The same degree of error might be expected for method 1, since the error in area-averaged rainfall amount determined by radar reflectivity can be as large as a factor of 2, and the precipitation flux divergence determined from the radar reflectivity is the most important factor in determining condensation by that method.

If bulk net condensation actually was twice that computed, thus rendering asymmetric condensation a smaller fraction of the total, the storm asymmetry would still be thermodynamically important. Relative humidity retrieved at the 3.0-km level (not shown) on the dry side of the storm was as low as 60%, and this value was confirmed by aircraft in situ data. The dynamically retrieved and the aircraft-observed temperatures on the dry (right) side of Hurricane Norbert were also ~5°C warmer than on the saturated side.
TABLE 3. Vapor budgets computed from method 2 as well as from assuming that the storm is saturated everywhere. The budgets are computed for (a) the three-dimensional Doppler wind field and (b) for the corresponding axisymmetric wind field. Units are $10^3$ kg h$^{-1}$.

<table>
<thead>
<tr>
<th>Budget term</th>
<th>Method $(10^3$ kg h$^{-1}$)</th>
<th>Global saturation $(10^3$ kg h$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Three-dimensional Doppler wind field for method 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial vapor advection</td>
<td>$-13.36$</td>
<td>$-7.44$</td>
</tr>
<tr>
<td>Bottom vapor advection</td>
<td>$-32.88$</td>
<td>$-32.47$</td>
</tr>
<tr>
<td>Radial vapor advection extended to surface</td>
<td>$-46.60$</td>
<td>$-40.43$</td>
</tr>
<tr>
<td>(b) Corresponding axisymmetric wind field for symmetry method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial vapor advection</td>
<td>$-10.14$</td>
<td>$-10.01$</td>
</tr>
<tr>
<td>Bottom vapor advection</td>
<td>$-28.59$</td>
<td>$-28.34$</td>
</tr>
<tr>
<td>Radial vapor advection extended to surface</td>
<td>$-38.87$</td>
<td>$-38.78$</td>
</tr>
</tbody>
</table>

i. Water vapor convergence

The water vapor budget may also be determined using method 2. Table 3a shows this budget, as well as that determined by using the three-dimensional wind field and assuming the air was saturated everywhere (global saturation) at the retrieved temperature. Table 3b shows the same budget computed with the axisymmetric wind field. The bottom lines in Table 3a,b indicate the amount of moisture convergence estimated if the wind at 500 m was extended to the surface.

The results shown in Table 3a indicate that the net horizontal vapor convergence into the budget volume was quite small, no more than about one-fifth of the total net condensation. The vertical advection through the 500-m level, however, was ~one-half of the net condensation computed by method 2, and an even greater fraction of that computed by the global saturation method. This vertical advection was roughly equal to the estimated moisture convergence in the 0–500-m layer. Some storms have significantly more net inflow above the 1 km level. Microphysical retrieval methods should be applied to mature hurricanes such as Hurricane Emily (1987), which had deep azimuthally averaged (~5–10 km) inflow and intense mean outflow above that. We may then be able to determine if total moisture convergence into the eyewall is typically dominated by that occurring in the lowest kilometer. The upward eddy transport of vapor through the 500-m level (Table 2) was ~two-thirds the magnitude of the upward vapor advection, or 40% of the vapor transport through the 500-m level. The total upward transport of vapor through the 500-m level was about 80% of the total transport into the budget volume and much greater than the computed horizontal vapor advection by the resolvable wind from the surface to 500 m (the computations assume that the wind field

Fig. 18. Vertical profiles of the mean condensation and evaporation rates for the budget region bounded by a distance 37.5 km from the storm center. Units are grams per cubic meter per hour. In (a) net condensation (condensation–evaporation) computed by method 2 and by assuming the hurricane is completely saturated are shown by the solid and dashed lines, respectively. In (b) method 2 condensation and evaporation are shown by the solid and long-dashed curves, respectively, while the global-saturation condensation and evaporation are shown by the medium-dashed and short-dashed curves, respectively.
at the surface was the same as at 500 m). These results indicate that about 40% of the vapor converging into the region within 37.5 km of the center of Hurricane Norbert was actually evaporated from the sea surface within this same radius. Much of the evaporation likely occurred in the region of cooler, drier air diverging from the eyewall downdrafts. The results in Table 3b indicate that the axisymmetric budget assuming saturation was nearly the same as the budget that does not assume saturation. Essentially, the axisymmetric budget is saturated.

Figure 18a shows the bulk net condensation as a function of height determined from method 2 and from the global saturation method, and Fig. 18b shows the separate bulk condensation C and bulk evaporation E. In the lower troposphere, method 2 indicates much higher net condensation than does the saturated downdraft assumption, as a result of the highly unsaturated downdrafts indicated by the method 2 computations. Figure 18b shows that E estimated by the global saturation method was much greater than that estimated by method 2; below the melting level the ratio between the two methods ranged from 3 to 6. These figures indicate the role of unsaturated downdrafts in changing the bulk water budget.

<table>
<thead>
<tr>
<th>Radial interval (km)</th>
<th>18.5–37.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapor divergence (10^6 g s^-1)</td>
<td>-16.4</td>
</tr>
<tr>
<td>Vapor convergence rainfall (mm h^-1)</td>
<td>17.4</td>
</tr>
<tr>
<td>Computed rainfall (mm h^-1)</td>
<td>17.8</td>
</tr>
</tbody>
</table>

j. Comparison with earlier budget studies

Figure 19 and Table 4 show the method 2 vapor budget displayed in a manner consistent with that of Hawkins and Rubsam (1968) and Hawkins and Imbembo (1976); the budget of an 18.5–37-km (10–20 nautical mile) annulus around the storm center is shown. Method 2 results have been converted from height levels to pressure levels to allow direct comparison with the earlier figures. Problems associated with determining the winds at 500 m and below degraded our estimate of the full water budget below the 900-mb level.

Our budget is similar to that found in Hurricane Hilda (1964) by Hawkins and Rubsam (1968; see their Fig. 20 and Table 6). Their total convergence of vapor in the annulus from 18.5 to 37 km was 16.9 × 10^9 g s^-1 while ours is 16.4 × 10^9 g s^-1. The turbulent eddy flux through the 900-mb level in our budget was 5.0 × 10^9 g s^-1, which also compares well with their surface evaporation estimate of 4.0 × 10^9 g s^-1. The comparison is also quite favorable in the layer from 800 to 900 mb, where both methods show a horizontal vapor convergence of ~4-5 (×10^9 g s^-1).

Although their budgets appear similar, Hurricane Hilda was intensifying while Hurricane Norbert was dissipating. Hurricane Norbert was being strongly ventilated, however, with cool low-level inflow and warm low-level outflow (Part I). This ventilation was apparently associated with slow dissipation. We do not know the characteristics of the asymmetry of the structure of Hurricane Hilda. Perhaps strong ventilation of the type seen in Norbert was not present in Hilda, thus allowing a portion of the latent heat release to remain concentrated in the storm core.

The Hurricane Norbert vapor convergence in the 18.5–37-km annulus was approximately half that reported by Hawkins and Imbembo (1976) for Hurricane Inez (1966). Inez, however, was small and more intense, with much greater inflow. The greater intensity of Inez must have been maintained by the stronger vapor inflow.

7. Conclusions

The two hydrometeor budgets computed in this study suggest that Hurricane Norbert obtained its moisture primarily from the front of the storm, most of the condensation occurred on the left side of the storm, and most of the precipitation occurred in the left rear quadrant of the storm. Downdrafts downwind
of most of the precipitation were strong enough to maintain significantly subsaturated conditions, with relative humidities as low as 60%. Ice particles generated in the eyewall region on the left side of the storm circulated around the storm at upper levels and were advected out of the storm in the left front quadrant. Most of the vapor entering the budget volume entered through the bottom boundary (500 m), and it greatly exceeded the low-level horizontal vapor convergence. Either the inflow from the surface to 500 m, which could not be determined well from Doppler, was much stronger than that observed by the Doppler at the 1-km level, or surface evaporation directly under the budget region was comparable to moisture convergence. Such elevated levels of evaporation could have been due to the presence of dry air from the unsaturated downdrafts diverging near the surface.

The vapor budget has also been compared to the budgets of Hurricanes Hilda (1964) and Inez (1966) computed by Hawkins and Rubsam (1968) and Hawkins and Imbembo (1976), respectively. The budget of Hurricane Hilda was very similar to that of Hurricane Norbert, while that of Hurricane Inez was substantially different, probably because Inez was more intense. The similarity in bulk budget between Hurricane Norbert, a dissipating storm, and Hurricane Hilda, an intensifying storm, indicates the need to understand the role of hurricane asymmetries in a number of different cases.

The hydrometeor and water vapor budgets computed in this study were three dimensional. Previous budget studies have relied on the assumption of axisymmetry, since the wind and thermodynamic analyses were usually performed upon data obtained along a limited number of flight tracks. The present results illustrate how misleading the assumption of axisymmetry can be. The three-dimensional water vapor budget computed by method 2 indicated approximately twice as much net condensation (heating) as did computations based upon the corresponding axisymmetric mean fields. The axisymmetric budget was nearly saturated everywhere except in the eye, and therefore, the net condensation was approximately that given by the product of total upward mass flux and the vertical derivative of saturation specific humidity. This direct relationship of upward mass flux to total net condensation was not found in the asymmetric three-dimensional computations.

The documentation of the asymmetric structure of Hurricane Norbert has allowed the hydrometeor budget to be related to the cloud microphysical structure of the storm analyzed in Part II. The outward advection of ice particles from upper levels in the left front quadrant of the storm is consistent with ice particle trajectories computed in Part II to explain the stratiform rainband structure outside the eyewall region.

The asymmetric water budget exhibited a much greater upward vertical eddy flux of vapor than did the axisymmetric budget, particularly under the downdrafts. This result suggests that enhanced evaporation was occurring under the dry downdrafts. This eddy flux provided most of the extra vapor supply necessary to balance the greater net condensation in the asymmetric case. Such an evaporation enhancement process is reminiscent of the "mass recycling" discussed by Gray (1982), according to which air traveling to the center of the storm goes up in moist updrafts and down in dry downdrafts as it spirals toward the storm center, thereby increasing surface exchange. Gray expected mass recycling to be important at distances of greater than 1° from the storm center. In the decaying storm described here, much more air goes up and down in individual eyewall drafts, than goes up in the tangentially averaged eyewall updraft.

The water budgets examined here are a first step toward evaluating the energy budget of Hurricane Norbert. The high quality Doppler radar–observed wind fields also make possible an examination of the storm’s angular momentum budget. Doppler datasets obtained since Hurricane Norbert [e.g., those obtained in Hurricanes Gustav (1990); Claudette (1991); and Jimena (1991)] in the eastern Pacific should, moreover, allow the inner core of the decaying storm analyzed here to be compared with intensifying and mature storms.

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