Sensitivity of the Estimated Monthly Convective Rain Fraction to the Choice of Z–R Relation

MATTHIAS STEINER AND ROBERT A. HOUSE JR.

Department of Atmospheric Sciences, University of Washington, Seattle, Washington

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ABSTRACT

This study investigates the sensitivity of the estimated monthly convective rain fraction— that is, the percentage of the areal rain accumulation contributed by precipitation identified as convective—to variations of the Z–R parameters used in radar-based rainfall estimation. Accurate knowledge of the fractions of precipitation that are convective and stratiform is important for climatological studies estimating the heating of the atmosphere. Extensive datasets from two climatologically different precipitation regimes, Darwin, Australia, and Melbourne, Florida, are used. The potential uncertainty of using (i) an arbitrary choice of the power factor $b$ and (ii) either single or multiple Z–R relations (stratified by precipitation type) for converting radar reflectivity to rain rate is investigated quantitatively.

The analyses reveal that estimates of the monthly convective rain fraction are sensitive to the choice of Z–R parameters. A maximum sensitivity is found for precipitation regimes with an approximately equal mix of rainfall from convective and stratiform precipitation systems. For example, estimates of the convective rain fraction for monsoonal rainfall at Darwin may range from 30% to 80%, solely depending on the choice of Z–R relations, even though all of these Z–R relations are tuned to produce the same total rainfall. In contrast, for the highly convective, sea-breeze-triggered, multicellular storms around Melbourne, the estimates of the convective rain fraction may range from 80% to 100%.

Different approaches to how the appropriate parameters of the Z–R relation(s) may be obtained are discussed. Varying the Z–R parameters to maximize the correlation of the radar-estimated monthly rainfall at the gauge sites and the rain gauge accumulations does not reveal enough sensitivity to make any choice significantly better than a single Z–R relation for both convective and stratiform rain. Multiple Z–R relations may be justified, but apparently not on the basis of a convective–stratiform separation.

1. Introduction

The basic modes of precipitation are convective and stratiform (Houghton 1968; Battan 1973; Rogers 1979; Houze 1993). Most precipitation systems can be decomposed into these elements, although the distinction might not always be sharp. The physical grounds for making such a separation are based on the magnitude of the in-cloud vertical air motions (Houze 1993, chapter 6; Steiner et al. 1995). Differences in precipitation type may lead to distinctly different vertical profiles of the latent heat release to the atmosphere (Houze 1982, 1989; Mapes and Houze 1993, 1995). Implications of the different heating profiles in convective and stratiform regions for the large-scale circulation of the Tropics are discussed by Hartmann et al. (1984), DeMaria (1985), Lau and Peng (1987), and Mapes and Houze (1993, 1995). To estimate the effects of these different heating profiles on the large-scale circulation and climate, it is important to have accurate knowledge of the fractions of precipitation that are convective and stratiform (Simpson et al. 1988; Tao et al. 1993). These fractions may be estimated using radar data; however, they exhibit an uncertainty, which we seek to quantify. Steiner et al. (1995) investigated the uncertainty arising from the classification of radar echoes into convective and stratiform elements. In this paper, we focus on the sensitivity of the estimated monthly convective rain fraction—that is, the percentage of the areal rainfall accumulation contributed by precipitation identified as convective—to the choice of how radar reflectivity is converted to rainfall rate.

Radar enables three-dimensional observation of precipitation with excellent areal coverage and high resolution in space and time. The radar reflectivity factor $Z$ is physically related to rainfall rate $R$. However, the rain detected by the radar is not identical to the rainfall reaching the earth’s surface since the radar beam is generally well above the ground. Moreover, reflectivity measurements are complicated by beam geometry, range, and attenuation (Zawadzki 1984; Austin 1987; Joss and Waldvogel 1990). In addition, the relation between the...
radar reflectivity and rainfall rate is dependent on the raindrop size distribution, which may vary significantly from storm to storm and even within storms. Since the radar generally measures some distance above the ground, up- and downdrafts and size sorting due to wind shear may play a significant role, especially in convective rainfall (e.g., Battan 1976; Austin 1987; Atlas et al. 1995). Low-level growth of raindrops or evaporation may also affect the radar rainfall estimation.

Because of the above factors, it is common practice to use data from rain gauges to calibrate the radar rainfall estimates from reflectivity in the vicinity of the gauge. The more rain gauges there are in a network, the more points can be calibrated in the radar area. This adjustment with rain gauges can be made by means of matching mean accumulations of the radar rainfall estimates at the gauge sites with the gauge totals—this may be done on different timescales, ranging from instantaneous radar-estimated and gauge-based rain-rate pairs to hourly, daily, weekly, monthly, seasonal, or even longer-term accumulations (Hitchfeld and Bordan 1954; Smith et al. 1975; Wilson and Brandes 1979; Klazura 1981; Steiner et al. 1995)—by matching percentiles of reflectivity and rain-rate cumulative frequency distributions (Miller 1972; Calheiros and Zawadzki 1987; Atlas et al. 1990; Rosenfeld et al. 1993, 1994) or by correlating radar and rain gauge measurements (Ciach et al. 1997). Statistical methods such as multivariate analysis of rain gauge and radar data (e.g., cokriging) have also been explored to optimize the radar- and gauge-based rainfall estimation (Krajewski and Ahnert 1986; Krajewski 1987; Ciach et al. 1997).

For this study, we approximate the relationship between $Z$ (mm$^6$ m$^{-3}$) and $R$ (mm h$^{-1}$) by an exponential expression of the traditional form:

$$Z = AR^b,$$

where $A$ and $b$ are positive empirical constants. Typical values of the multiplicative factor $A$ may range from a few tens to several hundreds (Battan 1973), while the power factor $b$ is limited to $1 \leq b \leq 3$ (Smith and Krajewski 1993), with typical values ranging between $b = 1.2$ and $b = 1.8$ (Battan 1973; Ulbrich 1983). In a particular application of (1) to a 1-month set of radar data, we may use a fixed power factor $b$ and adjust the value of $A$ to guarantee that the radar-based rainfall estimates will agree with the available rain gauge data. This removal of the mean bias of the radar rainfall estimates at the gauge sites is commonly referred to as a radar-gauge adjustment. The resulting estimate of the convective rain fraction is not dependent on the multiplicative factor $A$, but it is dependent on the exponent $b$. Since the choice of $b$ is somewhat arbitrary, this results in an uncertainty of the estimated convective rain fraction.

An additional uncertainty arises because multiple $Z$–$R$ relationships (according to precipitation type) may be used to attempt to improve the radar rainfall estimation (Joss and Waldvogel 1970; Battan 1973; Austin 1987). Short et al. (1990) and Tokay and Short (1996) have suggested that separate $Z$–$R$ relations should be used for convective and stratiform rain. As far as the monthly rainfall accumulation is concerned, the point may be moot; Steiner et al. (1995) showed that estimates of the monthly areal rainfall are not sensitive to whether one or two $Z$–$R$ relationships are used, given that a proper rain gauge adjustment is made. However, using multiple $Z$–$R$ relations, the estimated convective rain fraction becomes a function of the power factor $b$ and the ratio of the multiplicative factors $A$. Thus, as long as one school of thought is that separate $Z$–$R$ relations should be used to convert reflectivity obtained in convective and stratiform regions to rain rate, there remains an additional component of uncertainty in the estimated fraction of the total rain that is convective.

The primary objective of this study is to determine the uncertainties in the radar-estimated convective rain fraction arising from (i) uncertainty in $b$ and (ii) whether one or two $Z$–$R$ relations are used. In addition, we attempt to find the optimal radar reflectivity–rain rate relationship to reduce that uncertainty. The analyses use data collected by the radar and rain gauge network at Darwin, Australia, and data collected by the Melbourne NEXRAD [Next Generation Weather Radar (Baer 1991; Crum and Alberty 1993; Crum et al. 1993; Teleseotsky 1995)] WSR-88D [Weather Surveillance Radar—1988, Doppler (Heiss et al. 1990)] radar in central Florida and the surrounding rain gauge networks (Table 1).

### 2. Data and methodology

**a. Darwin, Australia**

Darwin is located within the Northern Territory of Australia and exhibits a monsoonal climate with pronounced wet and dry seasons (Holland 1986; Keenan et al. 1988). The rainy season, which is characterized by “active” (low-level westerly winds) and “break”

### Table 1. Some characteristics of the radars used in this study.

<table>
<thead>
<tr>
<th>Site</th>
<th>Period</th>
<th>Radar</th>
<th>Wavelength</th>
<th>Peak power</th>
<th>Beamwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darwin, Australia</td>
<td>February 1988</td>
<td>NOAA/TOGA</td>
<td>5 cm</td>
<td>400 kW</td>
<td>1.65°</td>
</tr>
<tr>
<td></td>
<td>December 1993–March 1994</td>
<td>BMRC</td>
<td>5 cm</td>
<td>250 kW</td>
<td>1.0°</td>
</tr>
<tr>
<td>Melbourne, Florida</td>
<td>July–August 1993</td>
<td>NEXRAD</td>
<td>10 cm</td>
<td>750 kW</td>
<td>0.95°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WSR-88D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
periods (easterly winds) of the monsoon, normally begins during late December and extends through March (Drosdowsky 1996). The rest of the year has little rainfall. The precipitation seen on the Darwin radar, located at Berrimah, Australia (12°27′26″S, 130°55′31″E, about 20 km east-southeast of Darwin), is of the following three main types.

1) During active monsoon periods, the precipitation typically occurs in oceanic mesoscale convective systems (MCSs), which consist of ensembles of convective lines and stratiform regions (Mapes and Houze 1992, 1993). These systems are similar to the tropical oceanic MCSs observed in the GARP (Global Atmospheric Research Program) Atlantic Tropical Experiment (Houze et al. 1981) and the Monsoon Experiments (Johnson and Houze 1987).

2) During break periods, the precipitating cloud systems are less frequent and are typically of a continental origin. They may be intense and are often in the form of squall lines with trailing stratiform precipitation (Drosdowsky 1984).

3) Pronounced diurnally forced thunderstorms locally known as “hectors” occur over Bathurst and Melville Islands to the north of Darwin (Keenan et al. 1988, 1990; Simpson et al. 1993).

A variety of precipitation systems observed in the vicinity of Darwin are discussed by Keenan and Carbone (1992).

For this study, we use Darwin radar and rain gauge data collected during February 1988 and the 4-month period December 1993–March 1994. The February 1988 data were collected by the mobile National Oceanic and Atmospheric Administration (NOAA) Tropical Ocean Global Atmosphere (TOGA) Program C-band research radar, while the 1993–94 data were collected by a C-band radar installation of the Australian Bureau of Meteorology Research Center (BMRC). Table 1 provides some details about these radars. The results of February 1988 (28 days) are based on four radar volume scans per day (see Steiner et al. 1995). The 1993–94 data consist of volume scans every 10 min. They are divided into two subsets. The first consists of 25 days of radar information collected in December 1993 and January 1994, while the second has 20 days of data collected in February and March 1994 (Table 2). These subsets include only days with nearly complete records (i.e., less than 10% of radar volumes missing per day).

The rain gauge network consisted of 22–25 sites in the field of view of the radar (Keenan et al. 1988; Rosenfeld et al. 1993; Short et al. 1993; and Steiner 1996).

b. Melbourne, Florida

Summer rainfall in central Florida is produced by showers and thunderstorms (e.g., Neumann 1971; Court and Griffiths 1986). These storms are usually triggered by horizontal, low-level convergence resulting from sea breezes entering the peninsula from both sides and only rarely are associated with fronts or large-scale disturbances (Byers and Rodebush 1948; Burpee and Lahiff 1984; Blanchard and López 1985). The rainfall of these sea-breeze-triggered multicellular storms is often heavy, but usually lasts only for a couple of hours. Rainfall produced by occasional landfalling hurricanes may result in significant rain accumulations. In contrast to the more typical sea-breeze thunderstorms, hurricanes exhibit approximately equal fractions of convective and stratiform rain (e.g., Marks 1985; Marks and Houze 1987). The sensitivity of the estimated convective rain fraction to the choice in Z–R relation for hurricanes is thus expected to be very similar to that observed for monsoonal rainfall around Darwin. By selecting data that represent rainfall from mostly sea-breeze-triggered multicellular storms typical of summer rainfall in central Florida, we are able to contrast the sensitivities of the estimated convective rain fraction to the choice in Z–R relation for two very different climatic rainfall regimes.

The Florida data used in this study were collected during the months of July and August 1993 by the S-band Melbourne NEXRAD WSR-88D radar (located at 28°06′47″N, 80°39′16″W) and rain gauges located within the radar range. Tipping.bucket rain gauge data from the Kennedy Space Center and the St. John’s Water Management District networks were used for the rain gauge adjustment. Details about the gauges and their

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Table 2. Estimates* of areal mean rainfall and convective rain fraction for three time periods at Darwin, Australia, and two time periods at Melbourne, Florida.

<table>
<thead>
<tr>
<th>Site</th>
<th>Period</th>
<th>Days</th>
<th>A</th>
<th>Rain</th>
<th>Convective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darwin, Australia</td>
<td>February 1988</td>
<td>28</td>
<td>164</td>
<td>163 mm</td>
<td>59%</td>
</tr>
<tr>
<td></td>
<td>December 1993–January 1994</td>
<td>25</td>
<td>77</td>
<td>245 mm</td>
<td>51%</td>
</tr>
<tr>
<td></td>
<td>February 1994–March 1994</td>
<td>20</td>
<td>86</td>
<td>374 mm</td>
<td>48%</td>
</tr>
<tr>
<td>Melbourne, Florida</td>
<td>July 1993</td>
<td>20</td>
<td>809</td>
<td>45 mm</td>
<td>94%</td>
</tr>
<tr>
<td></td>
<td>August 1993</td>
<td>20</td>
<td>601</td>
<td>76 mm</td>
<td>94%</td>
</tr>
</tbody>
</table>

* The amounts were computed by using single gauge-adjusted Z–R relations with a fixed power factor $b = 1.25$ and multiplicative factor $A$, as shown above.
positions relative to the radar are discussed by Steiner (1996). As for the Darwin data, only days with nearly complete records (i.e., less than 10% of radar volumes missing per day) are used, resulting in samples of 20 days for each month.

c. Quality control and convective–stratiform separation

The data were first quality-control edited and then interpolated to a Cartesian grid, as discussed by Steiner et al. (1995). The radar data collected at Darwin during February 1988 were manually edited to remove ground clutter and other spurious echoes, while for the other time periods of interest (see Table 2), we used an automated algorithm developed at the National Aeronautics and Space Administration (NASA)/Goddard Space Flight Center.2 The quality-controlled reflectivity data in polar space are subsequently interpolated to a Cartesian grid with 2-km horizontal resolution. For the February 1988 Darwin data, the horizontal level at 3-km altitude MSL is used as basis for the convective–stratiform separation and surface rainfall estimation, while for the other time periods, a Cartesian-gridded base scan (0.4° elevation angle) is used. Each pixel in the domain containing precipitation echoes is identified as convective or stratiform according to the classification scheme described by Steiner et al. (1995); the remaining pixels are labeled as “no echo.” This convective–stratiform classification scheme is independent of which Z–R relationship is applied to convert radar reflectivity to rain rate. The surface rainfall rate is estimated based on the reflectivity value at each pixel using a Z–R relation according to the pixel’s classification of the precipitation type.

A horizontal domain of 240 km \( \times \) 240 km was used for the February 1988 Darwin data,3 while for the other time periods, the data within a radius of 150 km from the radar site were used. The differences in the 1988 and 1993–94 data are not important for the outcome of this sensitivity study. Moreover, the convective–stratiform separation is not a critical factor in this sensitivity analysis since every calculation using a different Z–R conversion is based on the same reflectivity patterns that have initially been classified into convective and stratiform elements. We expect that a slightly different separation algorithm would produce very similar results with regard to the presented sensitivity analysis.

d. Z–R conversion and rain gauge adjustment

The radar rainfall estimates are made by inverting Eq. (1) to obtain \( R \) from \( Z \) and modifying it on the basis of rain gauge data such that the mean bias at the gauge sites is removed. The corrected radar-estimated rainfall rate is then

\[
R_c = A^{-1/b} \left( \frac{R}{R_u} \right)^{Z^{1/b}} \bar{Z}^{1/b},
\]

where \( \bar{R}_c \) is the uncorrected mean rain rate estimated by the radar at the gauge sites, \( R_u \) is the mean rate based on the rain gauge measurements, and \( A \) is the multiplicative factor after the rain gauge adjustment. Using this adjusted Z–R relationship to convert radar reflectivity to rain rate produces rainfall estimates at the gauge locations that have the same mean rain accumulation as the rain gauges. The adjustment factor is determined without considering the amounts of rain contributed by the different types of precipitation (i.e., convective and stratiform). If two different Z–R relationships are used for converting convective and stratiform reflectivity to rain rate, the same correction factor is assumed for both relations to preserve the initially applied ratio between the convective and stratiform multiplicative factors.

\[
\frac{\bar{A}_{\text{conv}}}{\bar{A}_{\text{stra}}} = \frac{A_{\text{conv}}}{A_{\text{stra}}},
\]

For each choice of \( A_{\text{conv}}/A_{\text{stra}} \) and \( b \), a different estimate of the convective rain fraction is obtained. We explore the sensitivity of the convective rain fraction estimate over a wide range of \( A_{\text{conv}}/A_{\text{stra}} \) ratios (1/4, 1/2, 1/1, 2/1, and 4/1) and power factors \( b \) (1 \( \leq b \leq 3 \) in steps of 0.25). We assume the same value of \( b \) for both the convective and stratiform Z–R relationship. A ratio of \( A_{\text{conv}}/A_{\text{stra}} = 1 \) implies the use of a single Z–R relation. For each set of Z–R parameters, a separate gauge adjustment is made to remove the bias in the overall rainfall4 (i.e., ensure that the mean of the radar rainfall estimates at the gauge locations equals the mean of the gauge totals). However, this adjustment of the multiplicative factor(s) has no impact on the estimated convective rain fraction, which is determined by their ratio \( A_{\text{conv}}/A_{\text{stra}} \) and by the power factor \( b \).

\[\text{Rain gauge adjustments do not automatically assure improvements in the radar rainfall estimates (e.g., Cain and Smith 1977; Zawadzki 1984; Koistinen and Puhakka 1986). Consistent improvements can be achieved only when the systematic differences (i.e., bias) between the gauge and radar rainfall estimates are larger than the standard deviation of the random scatter of the gauge versus radar comparisons (Smith and Cain 1983; Joss and Waldvogel 1990; Smith 1990). This point was of no concern for the Darwin data; however, the Melbourne data did not satisfy this requirement.}\]
Fig. 1. Sensitivity of the estimated monthly convective rain fraction to the choice of Z–R relation for the data collected by the Darwin radar in February 1988. The different curves shown indicate lines of constant ratio $A_{\text{conv}}/A_{\text{stra}}$, as a function of the power factor $b$ of the Z–R relation(s). The gray-shaded area indicates the range of most likely values for $b$. The solid and open circles, labeled “S” and “D”, respectively, show the position of the single and double Z–R approaches discussed by Steiner et al. (1995) within the parameter space.

3. Sensitivity of estimated convective rain fraction to Z–R parameters

Table 2 contains the rainfall amounts and convective rain fractions for the five time periods of interest, computed using single gauge-adjusted Z–R relations with $b = 1.25$. Significantly more rainfall occurred during the three analyzed time periods at Darwin than during the two summer months of 1993 at Melbourne. The rainfall at Darwin had a significant proportion of stratiform precipitation. In contrast, summer rainfall in Florida was highly convective (Table 2). The differences in the estimated rainfall amounts and convective rain fractions among the three time periods analyzed for Darwin (and similarly for the two periods at Melbourne) indicate differences in the frequency of the various types of precipitation systems developing around Darwin rather than differences in how the data were processed.

a. Darwin, February 1988

The sensitivity of the estimated monthly convective rain fraction to the choice of Z–R relation for the data collected during February 1988 at Darwin is shown in Fig. 1. For each Z–R configuration, a gauge adjustment is made, so all the points in this figure exhibit the same areal mean rainfall amount (163 mm; Table 2). The estimated convective rain fraction, however, is sensitive to the selected Z–R relation(s). For a power factor $b = 1.5$, which corresponds to an assumption of exponential Marshall and Palmer (1948) raindrop size distributions (Waldvogel 1975a,b), an uncertainty of $b \pm 0.1$ results in a ±3%–4% uncertainty of the estimated convective rain fraction. Increasing the ratio of $A_{\text{conv}}/A_{\text{stra}}$ from 1/4 to 1/2, 1/1, 2/1, and 4/1 results each time in a 10% reduction of that estimate. The gauge-adjusted multiplicative factors of the Z–R relations generally decrease with increasing power factor $b$ (not shown).

Although Fig. 1 covers a range of $1 \leq b \leq 3$, the values of $b$ usually indicated for rainfall are in the range $1.2 \leq b \leq 1.8$ (e.g., Battan 1973; Ulbrich 1983), as shown by the shaded region. The solid curve, representing the results for a single Z–R relation ($A_{\text{conv}}/A_{\text{stra}} = 1$), indicates an estimated convective rain fraction of 49% at $b = 1.5$, the middle of that range; values of 61% and 41% are obtained for $b = 1.2$ and $b = 1.8$, respectively. The general uncertainty in $b$ (in the absence of independent knowledge of $b$) produces an uncertainty of about ±10% in the estimated convective rain fraction. From the work of Ciach et al. (1997) and Yuter and Houze (1997), the most likely value of $A_{\text{conv}}/A_{\text{stra}}$ is 1. However, as long as the discussion about using single versus multiple Z–R relation(s) is not resolved, we must assume an uncertainty of perhaps a factor of 2 for the ratio $A_{\text{conv}}/A_{\text{stra}}$. A factor of 2 uncertainty in this ratio increases the uncertainty in the estimated convective rain fraction over the range $1.2 \leq b \leq 1.8$ by about an additional ±10%. The total uncertainty of the estimated convective fraction over this range is then approximately ±20%. According to Fig. 1, the selection of Z–R relation(s) could lead to estimates of the convective rain fraction as low as 32% or as high as 75%. This implies that despite an accurate areal mean rainfall estimate, an inappropriate choice of Z–R relation can produce a serious under- or overestimate of the convective rain fraction. This result has major implications for the results of the TOGA Coupled Ocean–Atmosphere Response Experiment (COARE; Webster and Lukas 1992) and the Tropical Rainfall Measuring Mission (TRMM; Simpson et al. 1988), both of which aim to estimate the convective and stratiform components of tropical precipitation.

Steiner et al. (1995) found that using a single Z–R relation with a power factor $b = 1.25$ or using two Z–R relations, as suggested by Short et al. (1990), produced very similar monthly convective rain fractions—59% for the former approach and 60% for the latter (Table 6 of Steiner et al. 1995). The two different approaches are indicated in Fig. 1 by the solid (single Z–R) and open circle (double Z–R). This result can be understood in the light of the work of Waldvogel (1975b), who showed that the power factor $b$ of an average Z–R relation for a mixture of different precipitation types differs from the power factors of the Z–R relations for
rainfall separated by type. Thus, it is possible to find a single $Z-R$ relation that will produce very similar results to those obtained by using multiple $Z-R$ relationships—two in our case—which are optimized for certain rainfall types.

b. Other months from Darwin and Melbourne

Figure 2 shows the results for four additional 20–25-day samples. The two Darwin months (Figs. 2a,b) show an almost identical sensitivity of the estimated convective rain fraction to the choice of $Z-R$ relation(s). Moreover, the depicted sensitivities are almost identical to that shown in Fig. 1. This is the case despite differences in the datasets (data were collected by two different generations of radar installations at Darwin, with 3-km gridded level versus gridded base scan and 6-h versus 10-min time resolution; see section 2). The demonstrated sensitivity therefore appears to be characteristic of monsoon rainfall observed around Darwin.

In contrast, the data collected during July and August 1993 at Melbourne exhibit much less sensitivity of the estimated convective rain fraction to variations of the $Z-R$ relation (Figs. 2c,d). The curves have a different shape, as well as a greatly reduced range of estimated convective fractions. The differences between time periods at the same site are much smaller than the differences observed between sites. Thus, each rainfall regime exhibits a characteristic sensitivity to the selection of the $Z-R$ relation(s). The sensitivity is much smaller for rainfall in central Florida sea-breeze convection than in monsoonal convection at Darwin. The Florida rain occurs predominantly in convective-scale thunderstorms and showers, which have little stratiform rainfall, while rainfall at Darwin occurs in mesoscale systems, which have more equal contributions by convective and stratiform precipitation.

In Fig. 2c, the solid curve, representing the single $Z-R$ relation ($A_{\text{conv}}/A_{\text{stra}} = 1$), indicates a value of 90% for the estimated convective rain fraction at $b = 1.5$, the middle of the range of likely values for $b$. Fractions of

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5 Waldvogel (1975b) showed an example of midlatitude widespread (stratiform) versus thunderstorm rain, where the shift in the reflectivity–rain rate relations (separated by precipitation type) is in the opposite direction than the shift from stratiform to convective rainfall in the Tropics, as indicated by Short et al. (1990). While the power factor $b$ of the average $Z-R$ relation in Waldvogel’s case is larger than either one of those separated by precipitation type, the opposite appears to be true for the tropical case. An explanation for the different behavior of midlatitude versus tropical rainfall may lie in differences between the dynamic and microphysical precipitation processes determining the raindrop size distributions.
95% and 86% are obtained for $b = 1.2$ and $b = 1.8$, respectively. Thus, the general uncertainty in $b$ produces an uncertainty of $\pm 5\%$ in the estimated convective rain fraction. Including an additional uncertainty of a factor of 2 in the ratio $A_{\text{con}}/A_{\text{str}}$, the overall uncertainty of the estimated convective rain fraction over the range $1.2 \leq b \leq 1.8$ is approximately $\pm 10\%$. According to Fig. 2c, the choice of $Z-R$ relation(s) could lead to estimates of convective rain fraction for Melbourne as low as 80% or as high as 97%. The results for August 1993 (Fig. 2d) are very similar to those obtained for July 1993 (Fig. 2c). The implications of the sensitivity to the choice of $Z-R$ relation for a site with rainfall similar to that shown for Melbourne do not appear to be as grave as for a site like Darwin, with its more equal mix of convective and stratiform precipitation and its greater overall amount of rainfall.

4. Choosing the appropriate $Z-R$ relation(s)

In light of the significant sensitivity of the estimated convective rain fraction to the choice of $Z-R$ relation, it becomes very important to find the appropriate relationship for converting radar reflectivity to rain rate. The more convincing evidence we can furnish for a given combination of $Z-R$ parameters, the more the uncertainty in the estimated convective rain fraction can be reduced. There are different ways to seek information about the relation between the radar reflectivity and rainfall rate. For example, measurements of raindrop spectra may be explored for obtaining guidance with regard to selecting appropriate $Z-R$ parameters. However, $Z-R$ relationships that are derived from raindrop size distribution observations are very idealized. They reflect the variability of the raindrop spectra at ground level (or flight level for airborne observations), but they do not incorporate other factors of the radar rainfall measurement that might actually be more important, such as beam geometry, height above ground, and wind and range effects. Consequently, one should be cautious about a straight application of raindrop-spectra-derived $Z-R$ relations for a radar rainfall estimation.

Instead of, or in addition to, using observations of the raindrop size distribution, instantaneous gauge-based rain rates may be paired with collocated measurements of radar reflectivity. These $(Z, R)$ pairs may be used to determine the $Z-R$ relation (or relations, if the pairs are classified by precipitation type) by means of least squares fit or probability matching methods. The least squares fit method, however, appears to be more sensitive to timing errors than the probability matching approach (Rosenfeld 1995). In contrast to the drop-size-based approach, instrumental differences between the radar and rain gauge observations are incorporated here. However, the scatterplots of instantaneous $(Z, R)$ pairs generally exhibit little correlation.

Timing errors are less crucial if one considers pairs of radar-estimated and gauge-based rainfall accumulations rather than instantaneous $(Z, R)$ samples. Moreover, the use of time-integrated rainfall samples enables the incorporation of data collected by a variety of gauge types—that is, not just those resulting from highly time-resolved tipping-bucket rain gauges. We compare monthly radar rainfall estimates at the gauge locations with the corresponding gauge-based rain accumulations. For each combination of $Z-R$ parameters, we determine the correlation between the site pairs of radar-estimated and gauge-based accumulations. We could also have used a root-mean-square difference, and/or the rain accumulations might have been replaced by the monthly mean rates. Systematically varying the power-law parameters $A$ and $b$, we seek the maximum correlation between the site pairs of radar-estimated and gauge-based accumulations, which in turn would imply the appropriate $Z-R$ relation(s).

Figure 3 shows the correlation among the site pairs of radar-estimated (closest pixel to gauge location) and gauge-based totals within a parameter space defined by the power factor $b$ and the ratio of the multiplicative factors $A_{\text{con}}/A_{\text{str}}$, from the sensitivity analyses depicted in Fig. 2. The data collected at Darwin generally exhibit a much higher correlation of the radar-estimated rainfall at the gauge sites and the gauge totals than the data of Melbourne. This reflects the lesser difficulty of measuring rainfall in the more widespread monsoonal precipitation at Darwin, compared to the greater problems related to rain gauge sampling of the (point) measurement of rainfall originating from multicellular thunderstorms and showers in central Florida. Apart from the differences in the magnitude of the correlations shown in Fig. 3, little sensitivity to variations of the $Z-R$ parameters is found for either the Darwin or Melbourne data. Within each panel of Fig. 3, the difference between the maximum and minimum correlation is less than 0.15. Based on this analysis, there is no particular combination of $Z-R$ parameters better than any other. Moreover, there is no apparent practical advantage (or justification) of using a double instead of a single $Z-R$ approach for estimating monthly areal rainfall.

Systematically varying the parameters of the $Z-R$ relation(s) to find the best correlation among the radar-estimated rainfall at the gauge sites and the gauge totals thus does not appear to be a satisfactory approach for obtaining the appropriate reflectivity to rain-rate conversion. It may be that a monthly timescale is too long to make a clear case for any combination of $Z-R$ parameters. However, analyses by Ciach et al. (1997), who studied the root-mean-square error (instead of the correlation as we do) as a function of $Z-R$ parameter variations on the basis of daily rainfall accumulations, also show very little sensitivity and thus provide a weak basis for selecting any particular combination of multiplicative and power factors.

Although observations of the raindrop size distribution indicate that there are potentially significant differences from storm to storm and even within storms,
it may be that given the way radar observes rainfall, in particular on timescales longer than the duration of a storm, instrumental measurement characteristics tend to become dominant and may thus smear possible effects of differences in raindrop spectra (causing differences in $Z$-$R$ relation) beyond recognition. Similarly, storm to storm differences would also tend to disappear in a $Z$-$R$ scatterplot once data from multiple storms were overlaid. Finally, in light of the Yuter and Houze (1997) study, there appears to be no clear basis for a distinction between convective and stratiform $Z$-$R$ relations, at least in tropical oceanic rainfall. Based on the airborne microphysical data collected in a variety of storm systems during TOGA COARE, Yuter and Houze (1997) do not find evidence that small- and large-drop spectra correspond uniquely to a convective–stratiform rainfall classification, as suggested by Short et al. (1990) and Tokay and Short (1996). Yuter and Houze (1997) find that raindrop spectra dominated by small and large drops have different $Z$-$R$ relations; however, they are not readily distinguished in radar reflectivity data. It might be possible, though, to distinguish between small- and large-drop spectra in polarimetric radar data, for example, using differential reflectivity, which is related to the median drop diameter (Seliga and Bringi 1976).

Consequently, future efforts should be directed toward combining the information provided by drop spectra observations, radar (including polarimetric radar) and rain gauge data, and modeling to optimize the $Z$-$R$ relation(s) for a particular site and time period of observation.

5. Conclusions

Radar-based estimates of the monthly convective rain fraction exhibit uncertainty, which arises from (i) ambiguity in the classification of radar echo patterns into convective and stratiform elements (see Steiner et al. 1995) and (ii) the choice of how radar reflectivity is converted to rain rate. This study investigated the sensitivity of the estimated monthly convective rain fraction to variations of the $Z$-$R$ parameters. In particular, the sensitivity arising from (i) an arbitrary choice of the power factor $b$ and (ii) whether one or two $Z$-$R$ relations (stratified by precipitation type—i.e., convective and stratiform) are used was explored.

Extensive datasets from two climatologically distinct precipitation regimes—Darwin, Australia, and Melbourne, Florida—were used for this study. The radar echo patterns were classified into convective and strat-
iform areas, independent of the conversion of radar reflectivity to rainfall rate, and this classification remained the same for all the calculations using different combinations of \( Z-R \) parameters. The method applied to separate convective and stratiform rainfall was not a crucial factor in this study, and we anticipate that a slightly different classification technique would produce very similar results with regard to the presented sensitivity analysis.

The radar-based rainfall estimates were adjusted to the rain gauge observations within the radar coverage by removing the mean bias of the radar estimates at the gauge sites. This rain gauge adjustment ensured that the mean of the radar rainfall estimates at the gauge locations agreed with the mean of the gauge-based rainfall accumulations. However, this procedure had no impact on the estimates of the convective rain fraction for any particular combination of \( Z-R \) parameters. Single as well as double \( Z-R \) approaches (i.e., the use of different relationships for convective and stratiform rainfall) were tested by varying the parameters of the \( Z-R \) relation(s) over wide ranges.

The analyses revealed a significant sensitivity of the estimated monthly convective rain fraction to the choice of \( Z-R \) conversion. The sensitivity was greatest for a rainfall regime with approximately equal proportions of rainfall contributed by convective and stratiform precipitation (i.e., monsoonal rainfall at Darwin). A significantly reduced sensitivity was found for a rainfall regime that was highly convective with little stratiform precipitation (i.e., sea-breeze-triggered multicellular thunderstorms in central Florida). It is anticipated that a rainfall regime that is composed almost entirely of stratiform precipitation with little convective rainfall will also show significantly reduced sensitivity of the estimated monthly convective rain fraction to \( Z-R \) parameter variations.

At Darwin, the choice of \( Z-R \) relation led to estimates of the monthly convective rain fraction that fell within a range of 30%–80%. An inappropriate selection of \( Z-R \) conversion may thus result in a serious error in the convective rain fraction, even though all the choices of \( Z-R \) relation(s), because of a gauge adjustment, would produce the same area-wide total (convective plus stratiform) monthly rainfall accumulation. Stratiform precipitation often accounts for some 40% of the total precipitation in tropical convective regimes (e.g., Houze 1977; Cheng and Houze 1979; Gamache and Houze 1983; Leary 1984; Churchill and Houze 1984; Houze and Rappaport 1984; Wei and Houze 1987; Chong and Hauser 1989; Gage et al. 1994; Steiner et al. 1995).

Therefore, the choice of \( Z-R \) relation can be a major issue. In contrast, for a regime with highly convective multicellular storms such as central Florida during the summer, where the convective rain fraction was about 90%, this estimate was not so strongly sensitive to the choice of \( Z-R \) relation, but still led to convective rain fraction estimates ranging from 80% to 100%.

With no prior knowledge of the values of the \( Z-R \) parameters for a given site and rainfall regime, or with no additional independent (maybe based on drop spectra) guidance for selecting any combination of \( Z-R \) parameters, the uncertainty of the estimated monthly convective rain fraction—solely due to an arbitrary choice of how to convert radar reflectivity to rain rate—is on the order of \( \pm 20\% \) for rainfall regimes with an approximately equal mix of convective and stratiform precipitation. About half of that uncertainty is contributed by an arbitrary choice of the exponent \( b \) in Eq. (1), while the other half is contributed by the uncertainty as to whether one or two \( Z-R \) relations should be used. The latter part of the uncertainty was measured by assuming that the same exponent \( b \) applies to both convective and stratiform regions and selecting the multiplicative factor \( A \) in Eq. (1) be different for convective and stratiform rainfall, allowing the ratio \( A_{\text{conv}}/A_{\text{stra}} \) to vary by up to a factor of 2. The overall uncertainty of the estimated convective rain fraction is reduced to maybe \( \pm 10\% \) for rainfall regimes that are either predominately convective or stratiform. Particularly for a climatic rainfall regime such as Darwin, it is therefore necessary to find the best combination of \( A_{\text{conv}}/A_{\text{stra}} \) and \( b \).

We attempted to find the appropriate values of \( A_{\text{conv}}/A_{\text{stra}} \) and \( b \) by optimizing the correlation of monthly accumulated rain amounts measured by gauges and estimated (using the respective \( Z-R \) parameter combination) from radar data at the gauge locations. However, it was not possible to optimize the values of \( A_{\text{conv}}/A_{\text{stra}} \) and \( b \) because the correlation coefficient did not vary sufficiently from one combination to another. Thus, we found no particular combination of \( Z-R \) parameters to be better than another. From a practical point of view, and since there was no apparent justification for using separate \( Z-R \) relations for convective and stratiform rainfall, estimating monthly areal rainfall using a single \( Z-R \) relation seems logical. Moreover, this result is consistent with the results of Ciach et al. (1997) and Yuter and Houze (1997), who found no evidence supporting the use of multiple \( Z-R \) relations for a rainfall estimation based on convective–stratiform separated radar reflectivity patterns. Nonetheless, even after potentially ruling out the use of multiple \( Z-R \) relations, which would cut the uncertainty of the estimated monthly convective rain fraction in half, the power factor \( b \) in Eq. (1) remains to be determined accurately.

Future efforts need to focus on incorporating all the available information, which may include raindrop spectra observations, three-dimensional radar volume scans, polarimetric radar and rain gauge data, and modeling. This appears to be the way to optimize the radar reflectivity to rainfall rate conversion for a particular site and time period of observation, and thus to reduce the uncertainty in the estimated convective rain fraction caused by the choice of \( Z-R \) relation. In a previous study, Steiner et al. (1995) found that the accuracy of the convective–stratiform separation algorithm is better
than 10% (in terms of a pixel classification). The ratio of the multiplicative factors \( A_{\text{rew}}/A_{\text{ras}} \) and the power factor \( b \) both need to be determined within \( \pm 10\% \) to achieve an accuracy of the estimated convective rainfall fraction that would make it useful for model-based estimations of the vertical profile of heating associated with tropical precipitation (Tao et al. 1993).

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