

One-sided, first-order:

$$\frac{\psi_j - \psi_{j-1}}{\Delta x} = \frac{\partial \psi}{\partial x} - \frac{\Delta x}{2} \frac{\partial^2 \psi}{\partial x^2} + \frac{(\Delta x)^2}{6} \frac{\partial^3 \psi}{\partial x^3} + O[(\Delta x)^3].$$

Centered, second-order:

$$\frac{\psi_{j+1} - \psi_{j-1}}{2\Delta x} = \frac{\partial \psi}{\partial x} + \frac{(\Delta x)^2}{6} \frac{\partial^3 \psi}{\partial x^3} + O[(\Delta x)^4].$$

One-sided, third-order:

$$\frac{2\psi_{j+1} + 3\psi_j - 6\psi_{j-1} + \psi_{j-2}}{6\Delta x} = \frac{\partial \psi}{\partial x} + \frac{(\Delta x)^3}{12} \frac{\partial^4 \psi}{\partial x^4} - \frac{(\Delta x)^4}{30} \frac{\partial^5 \psi}{\partial x^5} + O[(\Delta x)^5].$$

Centered, fourth-order:

$$\frac{4}{3} \left(\frac{\psi_{j+1} - \psi_{j-1}}{2\Delta x} \right) - \frac{1}{3} \left(\frac{\psi_{j+2} - \psi_{j-2}}{4\Delta x} \right) = \frac{\partial \psi}{\partial x} - \frac{(\Delta x)^4}{30} \frac{\partial^5 \psi}{\partial x^5} + O[(\Delta x)^6].$$