Energy Balance Modeling  

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Net SW = net LW

\[
\frac{(1-a)S_0}{4} = \sigma T_s^4
\]

all quantities are globally averaged

\[
\lambda_B = \frac{\Delta Q}{\Delta T}
\]

Climate FB parameter

For example, raise \( S_0 \) so \( \Delta Q \) goes up

assumed constant

planet warms so that \( \Delta F_{\text{out}} = \Delta F_{\text{in}} = \Delta Q \)

At equilibrium energy balance is restored

\[
\Delta F_{\text{out}} = F_{\text{out}}(T_s + \Delta T) - F_{\text{out}}(T_s)
\]

\[
= \sigma (T_s + \Delta T)^4 - \sigma T_s^4
\]

but what is \( \frac{\Delta F_{\text{out}}}{\Delta T_s} \)?

\[
\frac{\partial F_{\text{out}}}{\partial T_s} \Delta T_s = 4\sigma T_s^3 \Delta T_s
\]

\[
\lambda_B = 4\sigma T_s^3 = 3.75 \text{ W/m}^2/\text{K}
\]
This FB is negative which is funny
because $g > 0$

Define $\Delta g = -g$, called
"Signed Feedback" by Boer & Hu 2003

Planet w/ an atmosphere

Four = $\sigma T_e^4$ solve for $T_e$ "effective Temp"

and find what height it occurs at in
real atm.

This is the height where $T(z) = T_e$

it is just a construct
(nothing is really there)

However think back to class on Monday
synchronous

dark lit
Tidally locked
planet

Here $T_e \approx T_s$ $T_e \approx T_{\text{cloudtop}}$

Very cold atmosphere have very little
greenhouse effect
Alternative approach

\[ F_{\text{out}} = \varepsilon \sigma T_4^4 \dfrac{1}{\tau} \]

- Emissivity
- IR Transmissivity of Atmosphere
- \( \varepsilon_2 a \approx 0.42 \)

One-D (which is \( \phi \))

\[ \phi_i = i \Delta \phi_i \]

Note: not uniform in \( \phi \)

\[ T_i = \text{Temp of SURFACE at } i \]

\[ S_i = \text{solar in at } i \]

\[ S_i (1-\alpha_i(T_i)) = F_{\text{out}}(T_i) + \nabla \cdot F \]

- Absorbed SW
- Outgoing LW
- Divergence at side
- (Sign corrected after class)

\[ \nabla \cdot F = \text{net heat flux} \]

Not necessary but it is traditional

\[ \varepsilon \sigma \alpha T_4^4 \approx A + BT_4 \]

Simplified Sellers form

First continuous

\[ \nabla \cdot F = -\dfrac{1}{\cos \phi} \frac{1}{\partial \phi} \frac{\partial D \cos \phi}{\partial \phi} \dfrac{\partial T}{\partial \phi} \]

(Sign corrected after class)

Simplifying from page 88 of book which has an error anyway in eq. 3.14

the last T \( \rightarrow q(T) \)
\[ S_i(1 - \alpha_i(T_i)) = A + B T_i - D \left( \frac{\frac{d}{d\phi} D \cos \phi}{\cos \phi} \right) \bigg|_{\phi = \phi_i} \bigg|_{\phi = \phi_i} \]

Traditionally let \( x = \sin \phi \)

Magically
\[ \frac{1}{\cos \phi} \frac{d}{d\phi} D \cos \phi \frac{d}{d\phi} \to \frac{d}{dx} D(1-x^2) \frac{d}{dx} \]
Shortwave Feedback diagram

![Shortwave Feedback Diagram](Image)

Longwave Feedback diagram

![Longwave Feedback Diagram](Image)

This F.B. makes $\frac{\partial F}{\partial T}$ more negative or $\frac{\partial F_{in}}{\partial T}$ more positive.

This F.B. lowers B.
Review Energy Balance Modeling

At equilibrium, \( \Rightarrow \) critical assumptions
& global-mean
\( F_{in} = F_{out} \)
\( F_{out} = A + BT \) for planet without
\( F_{in} = (1 - \alpha) S \)
\( \lambda = \frac{\Delta Q}{\Delta T} \)
\( \Delta Q = \text{imposed forcing from GHG} \)
accomplished by changing \( A \) by \( \Delta A \)
\( \Delta A < 0 \) for \( +CO_2 \)

CASE 1: If no ice, \( \alpha \) indep of \( T \)

So \( \Delta F_{in} = 0 \) in response to \( \Delta Q \)
\( \Delta F_{out} \) (at instant of doubling) = \( -\Delta Q \)
\( \Delta F_{out} = 0 \) after adjusting

So \( \Delta A + B \Delta T = 0 \)

\( \Delta T = -\frac{\Delta A}{B} \)

\( \lambda = \frac{-\Delta A}{(1 - \Delta A)_B} = B \)

You can check on HW8 for 2x to 4x or no AFB

or more elegantly

from Taylor's series
\( F(T + \Delta T) = F(T) - \frac{\partial F}{\partial T} \Delta T + \frac{1}{2} \frac{\partial^2 F}{\partial T^2} \Delta T^2 \)

\( \Delta F \approx \frac{\partial F}{\partial T} \Delta T \)

\( \lambda \)

if no ice (which was case for our B.B. planet

\( \lambda = \frac{2F_{out} - F_{in}}{\Delta T} = B \)
CASE 2: Now with ice-α FB

the area of ice changes w/ ΔT

assume we are not near snowball or ice-free end members

\[ \frac{dx}{dT} < 0 \]

\[ F_{\text{in}} = (1-\alpha)S \]

\[ \frac{dF_{\text{in}}}{dT} = - \frac{dx}{dT} \]

\[ T = B + \frac{dx}{dT} < \text{ than } T \text{ for ice-free} \]

units are W/m²/°C

⇒ larger temp Δ for a given W/m² of forcing

ice-α Positive FB
At Equilibrium

for EBM at 2x CO₂ or greater

there is no ice

To double CO₂

We altered \( \text{Four} = A + BT \)

by reducing \( A \) by 2.1 W/m²

\[ \Delta Q = \Delta \text{Four} = \Delta A = 2.1 \text{ W/m}^2 \]

\[ \Delta T = 1.00 \text{ C} \quad 2x \rightarrow 4x \text{ change} \]

\[ \lambda_{2 \rightarrow 4 \text{ EBM}} = \frac{\Delta Q}{\Delta T} = 2.1 \text{ W/m}^2/\text{K} \]

or \[ \lambda_{\text{lw}} = \frac{\partial \text{Four}}{\partial T} = B \quad \text{it works!} \]

(since no \( \alpha - \phi \) EBM)

With ice at 1x

\[ \Delta T = 2.44 \text{ C} \]

\[ \lambda_{1 \rightarrow 2 \text{ EBM}} = \frac{\Delta Q}{\Delta T} = \frac{2.1}{2.44} = 0.86 \text{ W/m}^2/\text{K} \]

\[ = \frac{\partial \text{Four}}{\partial T} + \frac{2[3(1-\alpha)]}{\partial T} \]

\[ = \lambda_{\text{lw}} + \lambda_{\text{sw}} \]

\[ \lambda_\alpha = 0.86 - 2.1 = -1.24 \text{ W/m}^2 \]

Recall signed FB \( \Delta = -2 \) for better intuition