Energy Balance Modeling  Feb 9, 2011

Review

I "zero dim"

Black body Planet

\[ \text{Fin} = \text{Fout} \]

\[ \text{net SW} = \text{net LW} \]

\[ (1-\alpha) \frac{S_\odot}{4} = \sigma T_s^4 \quad \text{* all quantities are globally averaged} \]

\[ \lambda_B = \frac{\Delta Q}{\Delta T} \]

For example raise \( S_\odot \) so \( \text{Fin} \) goes up instantaneously hence \( \Delta Q = \Delta \text{Fin} \)

Assuming \( \alpha = \text{const} \)

Then planet warms so that \( \Delta \text{Fout} = \Delta \text{Fin} = \Delta Q \)

At equilibrium energy balance is restored and planet warms by \( \Delta T_s \)

\[ \text{Fout}(T_s + \Delta T_s) = \text{Fout}(T_s) + \frac{\partial \text{Fout}}{\partial T_s} \Delta T_s + \theta \Delta T_s^2 \]

\[ \Delta \text{Fout} = \text{Fout}(T_s + \Delta T) - \text{Fout}(T_s) \]

\[ \propto \frac{\partial \text{Fout}}{\partial T_s} \Delta T_s = 4 \sigma T_s^3 \Delta T_s \]

\[ \lambda_B = 4 \sigma T_s^3 = 3.75 \text{ W/m}^2/\text{K} \]
This FB is negative which is funny because \( \Delta B > 0 \)

Define \( \Delta B = -2B \) called "Signed Feedback" by Boer & Hu 2003

Planet w/ an atmosphere

\[ F_{\text{out}} = \sigma T_{e}^{4} \text{ Solve for } T_{e} \] "effective Temp"

and Find what height it occurs at in real atm.

\[
\begin{align*}
\text{Te} & \quad \text{Ts} \\
\text{z} & \quad \text{this is the height where } T(z) = T_e \\
& \quad \text{it is just a construct (nothing is really there)}
\end{align*}
\]

However think back to class on Monday

\[ F_{\text{out}} \] dark lit Tidally locked Planet

on equator

\[ T_e \approx T_s \] here here

\[ T_e \approx T_{\text{cloud top}} \] very cold atmosphere have very little greenhouse effect
Alternative approach

\[ F_{out} = \varepsilon(T_a^4 - T_s^4) \]

IR Transmissivity, \( \varepsilon \)

\[ \varepsilon T_a \approx 0.62 \]

One-D (which is \( \phi \))

\[ \phi_i = \frac{\Delta \phi_i}{x_i} \]

note not uniform in \( \phi \)

\( T_i = \) Temp \( \phi \) SURFACE at \( i \)

at Equilibrium

\[ S_e = \text{solar in at } i \]

\[ \alpha_i(T_i) = \frac{0.6}{0.3} \]

\( T_s < T_c \)

\( \alpha_i(T_i) = \frac{0.6}{0.3} \]  

\( T_s > T_c \)

\[ S_e (1 - \alpha_i(T_i)) = F_{FOUR}(T_i) + \nabla \cdot F_i \]

absorbed SW  
outgoing LW  
divergence at side  
(Sign corrected after class)

\[ S_p \]

NP

not necessary but it is traditional to \( \varepsilon 0.04 T_s^4 \approx A + B T_s \)

Simplified First continuous

\[ \nabla \cdot F = \frac{i L}{\cos \phi} \int \frac{D \cos \phi}{\phi} \frac{dT}{dT} \]

(Sign corrected after class)

Sellers Form

simplifying from page of book which has an error anyway in eq 2.17 the last \( T \rightarrow q(t) \)
\[ S_i(1 - \alpha_i(T_i)) = A + B T_i - D \frac{d}{d\phi} \left[ \frac{D \cos \phi}{\cos \phi} \right]_{\phi = \phi_i} \]

Traditionally let \( x = \sin \phi \)

Magically
\[
\frac{d}{d\phi} \left[ \frac{D \cos \phi}{\cos \phi} \right] = \frac{d}{dx} \left[ \frac{D(1 - x^2)}{d} \right] dx
\]