Homework Assignment # 3 solutions and discussion

*1. **Contouring exercise**: On the back of this page is a simplified map of sea level pressure as reported by weather stations across North America at 10 p.m. Eastern Time on 27 January 1999. (A real weather map would contain many more stations and other data in addition to pressure). Draw isobars (lines of constant pressure) on the figure, from 1012 mb up to 1028 mb, contouring every 4 mb (i.e. 1012, 1016, 1020, 1024 and 1028 mb). Label high pressure and low pressure centers with "H" and "L", respectively. **Hints**: Use a pencil first, then smooth the contours as you trace them in ink. The 1020 mb isobar is probably the easiest contour line to start your contouring with. Remember the guidelines listed on the back of the sheet handed out in quiz section. **Bonus**: Winds blow counterclockwise around low pressure centers and clockwise around high pressure centers, parallel to the isobars (we will explain this phenomenon in class later). The strength of the wind is proportional to the pressure gradient, which is represented on your drawing by how closely the isobars are spaced. Using this information, draw a few sample wind arrows on your figure; make the length of the arrow proportional to the wind speed.*

See the solution in class or posted outside the TA office. Also make sure you understand all the rules for contouring! Especially: label your contours; do not extend your contours too far from where you have data; and make sure all the values on one side of a isobar are greater than its value, while values to the other side of the contour are less.

2. a) I have 6 kg of dry air in a rigid box. The volume of the box is 5 m^3. Using the formula for density, ρ=mass/volume, calculate the density of the air in the box.

density = mass / volume = 6 kg / 5 m^3 = 1.2 kg/ m^3

b) Now I add some air (1.8 kg) to the box, without changing the volume. Calculate the new air density in the box.

density = (6.0 kg + 1.8 kg) / 5 m^3 = 7.8 kg / 5 m^3 = 1.56 kg/ m^3

c) Now consider a balloon containing the same mass and with the same volume as the box in part (a). Initially, the density inside the balloon is the same as inside the box. Now we raise the balloon several hundred meters. What happens to the balloon? Why?

If the air pressure outside the balloon is less than the air pressure inside the balloon, the walls of the balloon will experience a net outward force and the balloon will quickly **expand**. If the balloon expands, the air density inside the balloon will decrease because the mass of air inside the balloon does not change but the volume of the balloon increases (ρ=mass/volume). We also know that the air inside the balloon will **cool** during the expansion process because expansion of the balloon's walls requires work (read: energy), which comes from the internal kinetic energy of the molecules of air inside the balloon. So if the density of the air inside the balloon decreases, and the temperature of the air inside the balloon also decreases, we know from the ideal gas law that the pressure inside the balloon must also decrease as it expands ($p=\rho RT$).
Air pressure always decreases with height in the atmosphere. So as the balloon rises in the atmosphere, it experiences less pressure from the outside and the balloon must **expand** and **cool** until the pressure inside the balloon equals the pressure outside the balloon. At any

given height, the pressure inside the balloon must equal the pressure outside the balloon or the balloon will quickly expand or contract until this condition is met. Note that the reason the balloon expands is because *pressure* decreases with height, NOT because density decreases with height. The density inside the balloon and outside the balloon may be very different, but as long as the pressure inside the balloon and the pressure outside the balloon are the same, the balloon will remain the same size.

d) If the volume of the balloon increases to 5.8 m^3 in part (c), but the mass of air inside the balloon stays the same, what is the new density of air inside the balloon? Do you think this is greater than, less than, or equal to the density of air outside the balloon at this height? How about the pressure inside the balloon versus outside the balloon?

density = 6 kg / 5.8 m^3 = 1.03 kg/ m^3
We have already noted that the pressure inside the balloon must be the same as the pressure outside the balloon. Figuring out the air density inside the balloon versus the air density outside the balloon is a little trickier, and you actually do not know for sure which is higher unless you also know the temperature both inside the balloon and outside the balloon. But here is a good argument for why the density is probably greater inside the balloon: recall that the environmental lapse rate in the troposphere is about -6.5°C/km on average, but the dry adiabatic lapse rate is about -10°C/km. So if the balloon rises, say, 1 km, the air inside the balloon will be 10°C colder than it was at the surface (assuming that the air inside the balloon is relatively dry), while the temperature outside the balloon is probably only about 6.5°C colder than the surface air. Since the pressure inside and outside the balloon is the same, then the colder air (the air inside the balloon) will be less dense than the warmer air (the air outside the balloon). You can work this out for yourself using the ideal gas law for both gases:
p(inside)=ρ(inside)*R*T(inside)
p(outside)=ρ(outside)*R*T(outside)

since p(inside)=p(outside),
ρ(inside)*R*T(inside)=ρ(outside)*R*T(outside)
ρ(inside)=ρ(outside)*T(outside)/T(inside)

note that if T(outside) is greater than T(inside), then ρ(inside) will be larger than ρ(outside). You certainly did not have to do this calculation to get credit for this problem, but this is a good illustration of how density, pressure, and temperature are related.

e) Do you think the air inside the balloon at this height is colder or warmer than the air inside the box on the ground in part (a)? How do you know?

From the argument in part (c), we know that the balloon must have cooled as it rose and expanded. Again, the work of expanding the walls of the balloon requires energy, which comes from the internal kinetic energy (temperature) of the air inside the balloon. Since the box did not rise, it is at the same temperature as it started. So the temperature of the air inside the balloon is colder than the temperature of the air in the box. Make sure that you make the difference between *adiabatic cooling*, which is taking place *inside* the balloon, and the temperature decrease of the environment *outside* the balloon. The balloon does not cool because the temperature decreases with height in the troposphere, it cools because some of the kinetic energy of the air molecules is used to expand the walls of the balloon..

Things to remember:
--at the same pressure, warmer air is less dense than colder air (but if pressure changes you must have more information to know which is less dense!)
--at the same temperature, pressure is proportional to density (but if temperature changes you must have more information to figure out how pressure and density change!)
--pressure is a force, but density and temperature are not (they are just numbers that describe the physical state of a parcel or of the environment)
--as air parcels (or balloons filled with air) rise, they expand and cool. They are always at the same pressure as the air around them.

3. a) On Mars, the pressure at the surface is 6 mb. At what altitude in the Earth's atmosphere is the air pressure 6 mb? (Use a graph from the book of pressure versus height).

On Earth, the air pressure decreases rapidly with height and reaches a value of 6 mb at an altitude of approximately 35 kilometers. Note that the "quick and dirty" approximation of 1mb/8m decrease in pressure with height is only valid near sea level.

b) What does this imply about the total mass of the Martian atmosphere? Why?

Air pressure is the weight of the atmosphere above you. Assuming that the force of gravity on Mars is close to the force of gravity on Earth, this means that the total mass of the Martian atmosphere is the same as the total mass of the Earth's atmosphere above 35 kilometers. This is only 0.6 percent of the total mass of the Earth's atmosphere (6 mb / 1013 mb = 0.006 = 0.6 percent). So the Martian atmosphere has much less total mass than the Earth's atmosphere. The force of gravity is actually slightly less on Mars (about 70% of Earth's gravitational force). But even if you correct for the difference in gravity, the mass of the Martian atmosphere is still only about 1% of the mass of the Earth's atmosphere.
Note that DENSITY is mass divided by volume. It makes no sense to say that the Martian atmosphere is "less massive because it is less dense," unless you also know the volume (which in this case you don't). Also, when you say something is "thin" or "light", you need to define these terms or be more specific.

4. a) Professor Battisti finishes lecturing in Johnson 101 (at an altitude of 100 meters above sea level) and walks up to his office on the seventh floor of the Atmospheric Sciences building (at 180 meters above sea level). Using the following equation, compute the pressure change that he feels after walking up the stairs (assume that the density of air at both locations is about 1.2 kg/m^3, the acceleration due to gravity is 9.8 m/s^2, and that the air pressure inside Johnson 101 is 1013 mb). If you use the units given above, divide the last term by 100 to get an answer that is in millibars.

$$p_2 = p_1 + \rho g \Delta z/100$$

The difference in elevation between these two points is Δz=80m. If we let p_1 be the pressure in Professor Battisti's office and p_2 equal the pressure in Johnson 101, then:

$$p_1 = p_2 - \rho g \Delta z/100 = 1013 \text{ mb} - [(1.2 \text{ kg/m}^3) \times (9.8 \text{ m/s2}) \times (80 \text{ m})]/100$$

$$p_2 = 1013 \text{ mb} - 9.4 \text{ mb} = 1003.6 \text{ mb}$$

so the pressure change is -9.4 mb

Alternatively, you could leave the equation as originally stated and note that the elevation change would have to be negative. Either way, you know that the pressure in Professor Battisti's office *must be lower* than the pressure in the classroom because it is at a higher elevation. Many people calculated the correct magnitude of the pressure change, but got the sign wrong.

b) Looking back at your contour map from problem 1, estimate how far you would have to travel from Seattle to undergo the same change in pressure that you calculated in part (4a). Does pressure change more strongly with height or with horizontal distance?

On the map, Seattle has a pressure of 1024 mb. So we want to find somewhere on the map that is about 9 mb lower than this value, or 1015 mb. I found a pressure of 1015 mb in the middle of Canada that is about 900 km away. So the *horizontal* pressure gradient in this case is about 0.00001 mb/m (9mb/900km = 9mb/900000m = 0.00001 mb/m). The *vertical* pressure gradient from part (a) was 9.4 mb over 80 meters, or roughly 0.1 mb/m, which is much, much smaller than the vertical pressure gradient. Thus pressure changes much more strongly with height than with horizontal distance.

c) Since wind speed is proportional to pressure gradient, why doesn't the wind blow vertically?

The force of gravity is constantly acting on the Earth's atmosphere, yet the atmosphere does not fall to the ground. This is because there is a strong vertical pressure gradient everywhere on earth which produces an upwards force that exactly balances the force of gravity pulling down. This is called *hydrostatic balance*. If gravity were suddenly turned off, the massive vertical pressure gradient would cause a huge wind outwards to space. There is no force such as gravity acting in the horizontal direction on Earth, so any horizontal difference in pressure causes the wind to blow, which moves air from high pressure (more mass) to low pressure (less mass), reducing the pressure gradient. The constant action of the winds keeps the horizontal pressure gradients small, although we shall see soon that the coriolis force arising from the Earth's rotation actually makes the wind blow parallel to the isobars and not directly from high to low pressure. Many people talked about the coriolis force, fronts, air masses, or isobars in their answers, which do not explain anything about the vertical force balance.