Time lapse animation daily fields, unfiltered 500 hPa height field

courtesy of David Ovens

http://www.atmos.washington.edu/~bsmoliak/teleconnection.html

Univ. of Washington Dept. of Atm. Sci.
Lowpass filter: 5-day running mean
Highpass filter: departure from....
Time lapse animation
lowpass filtered
500 hPa height field

courtesy of David Ovens

http://www.atmos.washington.edu/~bsmoliak/teleconnection.html
500 hPa height variance DJF

30 day low pass ($Z_{30}$)  
6–30 day band pass ($z_{int}$)  
6 day high pass ($z_{HP}$)

eddy heat flux
850 hPa

courtesy of Justin Wettstein
2 to 6.5 day highpass

Blackmon et al. JAS 1977
2 to 6.5 day highpass

$u'u'$

$v'v'$

$u'u'$

$v'v'$
Anisotropy of high frequency transients

\[ v'v' > u'u' \]
$u'$ contours
positive anomalies only

$v'$ colored shading
tan poleward

courtesy of David Ovens

http://www.atmos.washington.edu/~bsmoliak/teleconnection.html
2 to 6.5 day highpass
Using $u'v'$ as an example, consider the distributions of

$$
\bar{u}'\Phi', \bar{v}'\Phi', \bar{u}_a'\Phi', \bar{u}_g'\zeta'
$$

for a full length paper about this, see Lau and Wallace JAS 1979
The eddy forcing is difficult to interpret because of the eddy-induced ageostrophic circulation.

We can get around this problem by considering the eddy forcing of the vorticity field.
adiabatic cooling below
adiabatic warming below

storm track

induced ageostrophic circulation

direct effect of eddy forcing
\[-\frac{\partial}{\partial y}(\bar{v}'v')\] only

indirect effects of eddy forcing

after Hoskins, James and White JAS 1983
The balanced geostrophic response

Note that the eddies produce an effective *westward transport* of westerly momentum through the storm track.

We can see this effect more clearly by considering the vorticity transport by the eddies.

*after Hoskins, James and White JAS 1983*
The eddy covariance tensor

\[
\begin{pmatrix}
\overline{u'^2} & \overline{u'v'} \\
\overline{u'v'} & \overline{v'^2}
\end{pmatrix} =
\begin{pmatrix}
K & 0 \\
0 & K
\end{pmatrix} +
\begin{pmatrix}
M & N \\
N & -M
\end{pmatrix}
\]

\[
K = \frac{\overline{u'^2} + \overline{v'^2}}{2}
\]

\[
M = \frac{\overline{u'^2} - \overline{v'^2}}{2}
\]

\[
N = \overline{u'v'}
\]

\[
\hat{M} = \sqrt{M^2 + N^2}
\]

\[
\alpha \equiv \frac{\hat{M}}{K} \quad \text{dimensionless coefficient of anisotropy}
\]

\[
\psi = \frac{1}{2} \tan^{-1} \left( \frac{N}{M} \right) \quad \text{angle of major axis relative to x axis}
\]
One point correlation maps

Unfiltered

< 6 d

After Wallace and Blackmon, Large scale Dynamical Processes in the Atmosphere, 1983
Unfiltered

> 10 d

After Wallace and Blackmon, Large scale Dynamical Processes in the Atmosphere, 1983
After Wallace and Blackmon, Large scale Dynamical Processes in the Atmosphere, 1983
Feedback of transients upon the background flow

1. Determining how the eddies change the \( \bar{q} \) field

   Estimate \[
   \frac{\partial \bar{q}}{\partial t} = -\nabla \cdot \bar{q}' \bar{V}' = -\frac{\partial}{\partial x} q'u' - \frac{\partial}{\partial y} q'v'
   \]

2. Use invertibility principle (solving elliptic equation)

For barotropic flow we can use \( \zeta \) in place of \( q \) in (1) and (2) reduces to solving Poisson’s equation

\[
\frac{\partial \bar{\Phi}}{\partial t} = \nabla^{-2} \left( \frac{\partial \bar{\zeta}}{\partial t} \right)
\]
nondivergent eddies

\[
\zeta' u' = -M_y + N_x; \quad \zeta' v' = -M_x - N_y
\]

\[
\frac{\partial \zeta}{\partial t} = -\nabla \cdot \bar{V}' \zeta' = 2M_{xy} - M_{xx} + M_{yy}
\] (1)

but the features in the \( u'v' \) field tend to be zonally elongated along storm tracks. It follows that \( N_{xx} \ll N_{yy} \) so (1) can be rewritten as

\[
\frac{\partial \zeta}{\partial t} = -\nabla \cdot \bar{V}' \zeta' \approx \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} 2M + \frac{\partial}{\partial y} N \right)
\] (2)

or

\[
\frac{\partial \zeta}{\partial t} \approx -\frac{\partial}{\partial y} \left( \nabla \cdot \bar{E} \right) \quad \text{where} \quad \bar{E} \equiv (-2M, -N)
\] (3)
Features in the mean flow also tend to be zonally oriented, so

\[ \frac{\partial \vec{u}}{\partial y} \gg \frac{\partial \vec{v}}{\partial x} \]

Hence, (3) can be written as

\[ \frac{\partial}{\partial t} \left( -\frac{\partial \vec{u}}{\partial y} \right) = -\frac{\partial}{\partial y} \left( \nabla \cdot \vec{E} \right) \]

or

\[ \frac{\partial}{\partial y} \left( \frac{\partial \vec{u}}{\partial t} \right) = \frac{\partial}{\partial y} \left( \nabla \cdot \vec{E} \right) \]

It follows that

\[ \frac{\partial \vec{u}}{\partial t} = \nabla \cdot \vec{E} \]

where

\[ \vec{E} \equiv -\left( u'^2 - v'^2, \ u'v' \right) \]
The “E vector” in this horizontal (barotropic) flow, with sign reversed, traces the 2-dimensional flux of $u$ by the transient eddies. Analogous to the Eliassen-Palm flux vector in the meridional plane. Like the EP flux, it can also be identified with the group velocity.
\[ \vec{E} \equiv -\left( u'^2 - v'^2, \ u'v' \right) \]

involves the anisotropy of the eddies.
DJF 300 hPa. $u$ contours; E vectors

after Wallace and Lau, *Issues in Atmospheric and Oceanic Modeling* 1985
2.5–6 d highpass

after Wallace and Lau, Issues in Atmospheric and Oceanic Modeling 1985
time-longitude sections
30° to 60°N
1985-86

Edmund K.M. Chang,
J. Atmos. Sci. 1993
lag correlations

30° to 60°N

Edmund K.M. Chang,
J. Atmos. Sci. 1993
rate of downstream phase propagation

rate of downstream dispersion
(group velocity)
If the flux of wave activity is eastward along the storm track, it follows that the work term $u'\Phi > 0$ along the storm track at the 300 hPa level. Prove that this is true.

**Hint:** consider the ageostrophic contribution $u_a'\Phi$ as inferred from the balance of forces in the meridional direction on an air parcel moving through the waves from west to east.
Generalization of $E$-vector formalism to three dimensions

In analogy with

$$\frac{\partial \zeta^-}{\partial t} \simeq - \frac{\partial}{\partial y} (\nabla \cdot \mathbf{E}) \quad \text{where} \quad \mathbf{E} \equiv \left( \overline{v'^2} - u'^2, -u'v' \right)$$

we can write

$$\frac{\partial q^-}{\partial t} \simeq - \frac{\partial}{\partial y} (\nabla_3 \cdot \mathbf{E}) \quad \text{where} \quad \mathbf{E} \equiv \left( \overline{v'^2} - u'^2, -u'v', -\frac{v'\alpha'}{\sigma} \right)$$

after Hoskins, James and White JAS 1983
Idealized storm track showing $E$ vectors

\[ \nabla \cdot \vec{E} > 0 \]

$E$ vectors from below

$\bar{v}' \alpha'$

$\bar{u}' \bar{v}' +$

$\bar{v}'^2 > \bar{u}'^2$
After Lau and Holopainen JAS 1984

Transient eddy forcing of the climatological-mean flow

\[
\left\{ \frac{1}{f} \nabla^2 + f \frac{\partial}{\partial p} \left( \frac{1}{\sigma} \frac{\partial}{\partial p} \right) \right\} \frac{\partial \Phi}{\partial t} = D + R_1,
\]

Time-averaged geopotential tendency equation

\[
D = D^{\text{HEAT}} + D^{\text{VORT}},
\]

\[
D^{\text{HEAT}} = f \frac{\partial}{\partial p} \left( \frac{\nabla \cdot \nabla' \theta'}{S} \right)
\]

\[
D^{\text{VORT}} = -\nabla \cdot \nabla' \zeta'.
\]
Tendency induced by all transients: high frequencies dominate

After Lau and Holopainen JAS 1984
tendency induced by high frequency transients

at 300 hPa the tendencies induced by the heat and vorticity fluxes by the transient eddies oppose one another so the net forcing is rather small.

at 1000 hPa they reenforce one another so the eddy-induced tendency is much larger. Note how the heat fluxes dominate.

After Lau and Holopainen JAS 1984
More about the evolution of the transients
after Wallace and Lau, *Issues in Atmospheric and Oceanic Modeling* 1985
Note dominance of westward arrows: zonally elongated perturbations localized in climatological-mean jet exit regions

after Wallace and Lau, Issues in Atmospheric and Oceanic Modeling 1985
Why do transient disturbances become more elongated as we go toward lower frequency?

First consider the situation in the ocean in which there is no zonal background flow. All transient perturbations propagate westward for the beta effect. The phase speed is given by

\[ c = \frac{\beta}{k^2 + l^2} \]

and the frequency apparent to a fixed observer by

\[ \omega = \frac{k\beta}{k^2 + l^2} \]

where \( k \) is the zonal wavenumber (the inverse of wavelength) and \( l \) is the meridional wavenumber.

Longer waves (i.e., waves with smaller two-dimensional wavenumber \( k^2 + l^2 \)) propagate westward faster than shorter waves.
Why do transient disturbances become more elongated as we go toward lower frequency?

Now consider two disturbances with the same two-dimensional wavenumber $k^2 + l^2$.

One shaped like A and the other shaped like B.

A takes longer to pass the fixed observer, so it has a lower frequency. If a spectrum of waves is present, with some being zonally elongated, like A and some meridionally oriented like B. The more strongly we lowpass filter the data, the more the zonally elongated disturbances will be favored. In the limit of zero frequency, the wind perturbations will be nearly purely zonal.
zonal geostrophic surface velocity based on satellite altimetry

zonal geostrophic surface velocity based drifters

Maximenko et al. GRL, 2008
Why do transient disturbances become more elongated as we go toward lower frequency?

Now consider two disturbances with the same two-dimensional wavenumber \( k^2 + l^2 \)

One shaped like \( \text{A} \) and the other shaped like \( \text{B} \)

\( \text{A} \) takes longer to pass the fixed observer, so it has a lower frequency. If a spectrum of waves is present, with some being zonally elongated, like \( \text{A} \) and some meridionally oriented like \( \text{B} \). The more strongly we lowpass filter the data, the more the zonally elongated disturbances will be favored. In the limit of zero frequency, the wind perturbations will be nearly purely zonal.

The argument carries over to the atmosphere where \( \omega = k \left( u - \frac{\beta}{k^2 + l^2} \right) \)

In this case, the most slowly propagating disturbances will be the ones for which the term in parentheses is smallest, but for a prescribed spectrum of \( k^2 + l^2 \), the zonally elongated ones will still have the lowest frequencies.
10-30d bandpass DJF 500 hPa height

lag correlations with 500 hPa height at reference grid point

after Blackmon et al. JAS (1984b)
More about the evolution of the transients
after Blackmon et al. JAS (1984b)

5 days earlier

5 days later

after Blackmon et al. JAS (1984b)
Regressed onto the cosine coefficient of zonal wavenumber 2 on 50°N

As in left panel but 5 days later

Consecutive 5-day mean wintertime 500 hPa height

After Wallace and Hsu, JAS, 1983
Day 2

Barotropic vorticity equation

Basic state flow consisting of
superrotation

“Mountain” turned on on Day 0

Day 5

Day 8

Courtesy of B. J. Hoskins
Highpass: periods shorter than a week

Summary:

Phase propagation $\frac{-u}{\partial x} \frac{\partial}{\partial x}$ dominates; $\bar{v}^2 > \bar{u}^2$

10-30d bandpass

Eastward dispersion dominates; great circle routes
Energetics of the transients

Lorenz kinetic energy cycle: time mean vs. transients

Time mean includes both zonally symmetric flow and stationary waves.

Transients may be decomposed into frequency ranges.
If we assume that the time mean flow is nondivergent,

\[
C_K = -u'u' \frac{\partial \bar{u}}{\partial x} - u'v' \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) - v'v' \frac{\partial \bar{v}}{\partial y}
\]

and that \( \partial \bar{u} / \partial y \gg \partial \bar{v} / \partial x \)

Then

\[
C_K = \left( \bar{v}'^2 - \bar{u}'^2 \right) \frac{\partial \bar{u}}{\partial x} - u'v' \frac{\partial \bar{u}}{\partial y} = \vec{E} \cdot \nabla \bar{u}
\]

If \( \vec{E} \) is up the gradient of \( \bar{u} \), then the flux is downgradient and the transients are gaining KE at the expense of the time mean flow.
After Blackmon et al., JAS, 1984a
Indication of geographically-fixed patterns

Pattern moves with reference grid point

Suggests the existence of “teleconnection patterns”

After Blackmon et al., JAS, 1984a