Chapter 3
The total energy balance

The balance requirement
How it is satisfied
  role of the thermohaline circulation
  role of the wind-driven circulation
  role of the stationary waves
  role of the transient eddies
  role of the hydrologic cycle
Two kinds of eddy heat transports
Net Radiation

deficit

W m\(^{-2}\)
surplus
Eq 38° Pole

atmospheric transport

ocean transport

top of atmosphere

Earth’s surface

upward flux at Earth’s surface (ERA-40)
upward flux at Earth’s surface (ERA-40)
Annual cycle in insolation (top of atmosphere)
Climatological-mean subsurface T at Weather Ship N
July minus January surface air T
Conclusions concerning the annual cycle
the Earth system is out of balance in seasonal means
heat is stored in ocean mixed layer in summer
oceans reject heat during winter
oceanic heat storage damps the annual cycle in T
the response is lagged relative to the forcing
reflects differing heat capacities of atm. and ocean ML
Balance requirement: Poleward transport of energy
Atmosphere and ocean both contribute to the transport
Contribution from the Oceanic THC
Contribution from the wind-driven currents
Contribution from the wind-driven currents
Contribution from the wind-driven currents

annual-mean SST

departure from zonal average
Contribution from the wind-driven currents

blue (red) arrows show cold (warm) surface currents
The total oceanic transport ...

Note that the ocean contribution is strongest $\sim 18^\circ$ latitude. The wind-driven portion looks like it may be important.
Role of the mean meridional circulations

\[ MSE = c_p T + Lq + \Phi \] increases with height

hence, the Hadley cells transport MSE poleward

Ferrell cells transport MSE equatorward
Hadley cell analogous to this idealized schematic

MSE transport is toward the right.
Role of the eddies

\[ \frac{\partial}{\partial y} [v \ast T \ast] \]
The stationary wave contribution at low latitudes
cold air injected into tropics

surface winds
SLP contours
vertical velocity (color)

SST*
warm air exported in summer hemisphere

surface winds
SLP contours
vertical velocity (color)

SST departure from zonal averages

Wind arrows are transposed from upper panel
stationary wave contribution to poleward heat flux
ERA-40 Reanalysis
The stationary waves cool the tropics by injecting cool air in the trade wind belts. They warm the extratropics by importing warm, moist air along the western flanks of the subtropical anticyclones, mainly in the summer hemisphere. The transport is shallow and strongest ~20° latitude.
The transient eddy contribution
baroclinic waves dominate the transient eddy transport
baroclinic waves dominate the transient eddy transport
baroclinic waves are organized in “storm tracks”

\[ v'T' \]

850 hPa
DJF
850 hPa heat flux vectors
irrotational component
850 hPa isotherms
\[
\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) - [v] \frac{\partial}{\partial y}[T]\cos\phi
\]

\[
- \frac{1}{\cos\phi} \frac{\partial}{\partial y}[v \ast T \ast]\cos\phi - \frac{\partial}{\partial p}[\omega \ast T \ast] + [Q]
\]

**spherical geometry**

\[
\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) - [v] \frac{\partial[T]}{\partial y}
\]

\[
- \frac{\partial}{\partial y}[v \ast T \ast] - \frac{\partial}{\partial p}[\omega \ast T \ast] + [Q]
\]

**Cartesian geometry**

\[
\frac{\partial[T]}{\partial t} = [\omega]\left(\frac{\kappa[T]}{p} - \frac{\partial[T]}{\partial p}\right) + P + [Q]
\]

**simplified**
\[
\frac{\partial [T]}{\partial t} = [\omega] \left( \frac{\kappa [T]}{p} - \frac{\partial [T]}{\partial p} \right) + P + [Q]
\]
\[
\frac{\partial [T]}{\partial t} = [\omega] \left( \frac{\kappa [T]}{p} - \frac{\partial [T]}{\partial p} \right) + P + [Q]
\]

\[A\quad Q_L = -\sigma \omega\]

\[B\quad Q_{\min} + P = -\sigma \omega\]

\[C\quad Q_L + P_{\max} = -\sigma \omega\]

MMC effective at horizontal transport
Dynamic stability

\[
\frac{\partial [u]}{\partial t} = [v] \left( f - \frac{\partial [u]}{\partial y} \right) + G + F_x
\]

Static stability

\[
\frac{\partial [T]}{\partial t} = [\omega] \left( \frac{\kappa[T]}{p} - \frac{\partial [T]}{\partial p} \right) + P + [Q]
\]

Note the similarity in structure of the two equations
Role of the hydrologic cycle

\[ MSE = c_p T + Lq + \Phi \]

- \( c_p \) is the specific heat of dry air at constant \( p \), 1004 J/kg
- \( L \) is the latent heat of vaporization, 2.5 \( \times \) 10^6 J/kg
- \( q \) is the specific humidity, dimensionless
- \( \Phi \) is geopotential height
The mass balance for water vapor

\[ \frac{\partial W}{\partial t} + \nabla \cdot Q = E - P \]

\[ Q = \frac{1}{g} \int_0^{p_0} qV dp \]

the vertically-integrated moisture transport

\[ \overline{Q} = Q_M + Q_T \]

\[ Q_M = \frac{1}{g} \int_0^{p_0} \overline{qV} dp \]

\[ Q_T = \frac{1}{g} \int_0^{p_0} \overline{q'V'} dp \]
\[
\frac{\partial W}{\partial t} + \nabla \cdot Q = E - P
\]

Averaged over a season

\[
\frac{\partial W}{\partial t} = 0
\]

\[
\nabla \cdot Q = E - P
\]
\[ E - P \]

\[ \frac{\partial W}{\partial t} + \nabla \cdot \mathbf{Q} = E - P \]

\[ \nabla \cdot \mathbf{Q} \]

ERA-40 data
\[ \nabla \cdot \mathbf{Q} = E - P \]

Approximations (voluntary)

\[ Q = \frac{1}{g} \int_0^{p_0} qV dp \sim \hat{q} \hat{V} \frac{\delta p}{g} \]

\[ \nabla \cdot qV = q \nabla \cdot V + V \cdot \nabla q \]

\[ \nabla \cdot \mathbf{Q} \sim \nabla \cdot \mathbf{V} \left( \frac{q_0 \delta p}{g} \right) \]

“slab”

ignore advection

approximate slab wind by surface wind
Water vapor  
Annual-mean conditions  
(satellite data)
Boreal summer
in the zonal average

\[
\frac{1}{\cos \phi} \frac{\partial}{\partial y} [Q] \cos \phi = [E] - [P]
\]
Role of the MMC in water vapor transport
On land surface

\[ P - E = \frac{\partial}{\partial t} \text{Storage} + R \]

Land + Atmosphere

\[ \frac{\partial}{\partial t} \text{Storage} + R = -\nabla \cdot Q \]

Land-locked basin

\[ \frac{\partial}{\partial t} \text{Storage} = -\nabla \cdot Q = P - E \]
Closed drainage basin: $R = 0$
In waves that transport heat poleward the ridges and troughs tilt westward with height.
An interesting parallel
It’s not chance coincidence

The math is identical
NH wintertime stationary waves

geopotential height $Z^*$

Note westward tilt with height
NH wintertime stationary waves

\[ Z_{500} \]

Note westward tilt with height
Idealized baroclinic waves

tilt westward with height
The eddy flux of geopotential $[v^* \Phi^*]$

second order term in the meridional transport of moist static energy

directed opposite to transport of zonal momentum

work term in kinetic energy cycle
The eddy flux of geopotential \( [v^* \Phi^*] \) second order term in the meridional transport of moist static energy

compare

\[
c_p \, r(v, T) \, \sigma(v) \, \sigma(T) \quad \text{with} \quad r(v, \Phi) \, \sigma(v) \, \sigma(\Phi)
\]

\[
r(v, T) > r(v, \Phi) \quad \text{and} \quad c_p \sigma(T^*) > g\sigma(Z^*)
\]

factor of 3 \quad \text{factor of 3}
The eddy flux of geopotential $[v \Phi^*]$ is directed opposite to the transport of westerly momentum.

First prove that $[v \Phi^*] = [v_a \Phi]$ i.e., that the flux of geopotential is accomplished entirely by the ageostrophic component of $v$

$$[v_g \Phi^*] = \left[ \Phi^* \frac{\partial \Phi^*}{\partial x} \right] / f$$

$$= \frac{\partial}{\partial x} \left( \frac{\Phi^*}{2} \right) / f$$

$$= 0$$
The eddy flux of geopotential and the flux of zonal momentum are in the opposite direction
for stationary waves

\[
\frac{Du^*}{Dt} = \begin{bmatrix} u \end{bmatrix} \frac{\partial u^*}{\partial x} = fy_a^*
\]

\(D/Dt\) Lagrangian time derivative

multiplying by \(\Phi^*\), zonally averaging, and using the identity

\[
\begin{bmatrix} \Phi^* \frac{\partial u^*}{\partial x} \end{bmatrix} = -\begin{bmatrix} u^* \frac{\partial \Phi^*}{\partial x} \end{bmatrix} = -f[u^* v^*]
\]

we obtain

\[
\begin{bmatrix} v_a^* \Phi^* \end{bmatrix} = -\begin{bmatrix} u \end{bmatrix} [u^* v^*]
\]
for waves propagating zonally with phase velocity $c$
replace $-u$ by $-(u - c)$

Above $\sim 700$ hPa $u > c$
The eddy flux of geopotential \[ \nu^* \Phi^* \]
work term in kinetic energy cycle by which the eddies equatorward of the latitude circle do work on the eddies poleward of it.
momentum flux

geopotential flux
momentum flux

geopotential flux
Causality

Which is the better interpretation?
Do the eddies induce diabatic heating?

\[ [v^*T^*] \text{ keeps } \partial[T]/\partial y \text{ from reaching thermal equilibrium and in this sense it induces } \partial[Q]/\partial y \]
or does diabatic heating induce eddy heat transports

or

\[ \partial [Q] / \partial y \] causes poleward moving air to be warmer than equatorward moving air, thereby inducing \([ v \cdot T ] \)
It depends on how you think about it.
It depends on how you think about it.

but in any case, simple stirring does not produce poleward heat transports in the absence of diabatic heating.

Is it possible to have poleward eddy heat transports in adiabatic flow?
Here’s how it can happen

As air parcels move through their elliptical orbits in the meridional plane (right) they conserve potential temperature.

\[
\frac{DT}{Dt} = \sigma \omega
\]

The motion is adiabatic and air parcels have cyclic orbits so the waves have no effect upon the slopes of the isentropes in the meridional plane.
Eddy heat fluxes that have no effect on the temperature field! How can that be?
Eddy heat fluxes that have no effect on the temperature field! How can that be?

It’s simple: The eddies induce a mean meridional circulation in which the adiabatic cooling in the ascending branch cancels the warming due to the convergence of the eddy heat flux poleward of the storm track........
Here’s how it works.

For perfect cancellation \[ \sigma[\omega] = -\frac{\partial}{\partial y}[v^* T^*] \]

If the MMC are represented as the gradient of a streamfunction \( \psi \) with \( [\omega] = -\frac{\partial \psi}{\partial y} \) and \( [v] = \frac{\partial \psi}{\partial p} \) then the eddy heat flux is the streamfunction for the MMC i.e., \( \psi = [v^* T^*] \)
Next thing you know he’ll be telling us that the Ferrell cell is eddy-induced!
So how are these eddy heat fluxes supposed to induce a MMC?

It’s simple: you just need to consider the orbit of air parcels in the waves and how it varies with latitude and height.
Eddy orbits looking in the upstream direction in westerly background flow

\( u > c \)

\( [v^*T^*] \) assumed to be strongest in the middle of the domain

For fixed \( u \) and \( c \) the width of the orbit is proportional to \( \sigma(v) \) and the depth is proportional to \( \sigma(T) \) and the area proportional to \( [v^*T^*] \)
At longitudes of ridges in eddies

At longitudes of troughs in eddies

Ascent in ridges is stronger than descent in troughs because eddies on equatorward side of the pink box are stronger. Hence, there is ascent in Eulerian mean.