Angular Momentum Presentation
Week 2
2. Definitions and data sources

The global atmospheric angular momentum about the earth’s axis can be expressed as

\[ AAM = M_r + M_\Omega, \quad (1) \]

where

\[ M_r = \frac{a^3}{g} \iiint u \cos^2\phi \, d\phi \, d\lambda \, dp \quad (2) \]

is the entire atmosphere’s angular momentum associated with its motion relative to the rotating solid earth, \( a \) is the earth’s radius, \( g \) is acceleration due to gravity, \( u \) is zonal wind, and the integral is performed over all latitudes \( \phi \), longitudes \( \lambda \), and pressures \( p \). The angular mo-
momentum associated with the rotation of the atmosphere’s mass is

\[ M_\Omega = \frac{a^4 \Omega}{g} \int \int p_s \cos^3 \phi \, d\phi \, d\lambda, \]  

(3)

where \( \Omega \) is the mean rotation rate of the earth and \( p_s \) the surface pressure (Rosen 1993).

The conservation of angular momentum states that changes in AAM are related to the surface torque by the relationship

\[ \frac{d}{dt} \text{AAM} = T_m + T_f, \]  

(4)

where the mountain torque \( (T_m) \) and friction torque \( (T_f) \) are defined through the following relationships (White 1991):

\[ T_m = -a^2 \int \int p_s \frac{\partial H}{\partial \lambda} \cos \phi \, d\phi \, d\lambda, \]  

(5)

\[ T_f = a^3 \int \int \tau \cos^2 \phi \, d\phi \, d\lambda. \]  

(6)

Here, \( \tau \) is surface stress and \( H \) indicates the height of the sloping topography.
zonal averaged zonal wind $[u]$ (m s$^{-1}$)
\[ \Delta M = k \times \Delta (\text{L.O.D.}) \]

Courtesy of David Salstein
\[ \Delta M = k \times \Delta (L.O.D. ) \]

Courtesy of David Salstein
Note large seasonal cycle in angular momentum: factor of 2 between Jan/Feb and Jul/Aug

Courtesy of David Salstein
Breakdown into frequency bands


Intraseasonal ($\sigma=0.56$)

Seasonal ($\sigma=2.07$)

Interannual ($\sigma=0.75$)

Courtesy of David Salstein

Variance

$\left[ x \times 10^{25} \text{ kg m}^2 \text{ s}^{-1} \right]^2$

Latitude

North ($N$) 60 30 0 30 60 90
South ($S$) 90 60 30 0 30 60 90

Intraseasonal  Seasonal  Interannual

Courtesy of David Salstein
SOI = -Southern Oscillation Index; peaks correspond to El Nino events

Courtesy of David Salstein
Positive AAM anomalies in tropics coincide with El Nino events (e.g., 1997-98)

Courtesy of David Salstein
Mountain and friction torques

\[ T_{\text{mountain}} = -R^2 \int \int p_s \frac{\partial H}{\partial \lambda} \cos \phi d\phi d\lambda \]

\[ T_{\text{friction}} = R^3 \int \int \tau \cos^2 \phi d\phi d\lambda \]

\[ T_{\text{gravity-wave}} = \text{Frictional related to sub-grid scale Action in the atmospheric model} \]

\( R = \text{Earth radius}, p_s = \text{surface pressure}, H = \text{topographic height} \)

\( \tau = \text{frictional stress, related to winds and roughness (model)} \)

\( \phi = \text{longitude} \quad \lambda = \text{latitude} \)

Courtesy of David Salstein
Regional Sources of Mountain Torque Variability and High-Frequency Fluctuations in Atmospheric Angular Momentum

HAIG ISKENDERIAN AND DAVID A. SALSTEIN

MONTHLY WEATHER REVIEW

FIG. 8. Sea level pressure at 1200 UTC for (a) 8 March, (b) 15 March, and (c) 20 March 1996. Contours are every 6 kPa. Highs and lows discussed in the text are indicated by the letters H and L.

FIG. 9. (a) Difference in surface pressure between 1200 UTC 15 March and 1200 UTC 8 March 1996 (every 4 kPa contoured, zero line omitted). Positive contours are solid and negative dashed. (b) Same as (a) except between 1200 UTC 20 March and 1200 UTC 15 March. Shading indicates surface elevation of at least 1000 m.

FIG. 10.

[Graphs and charts related to the above discussions are not transcribed here.]
Global AAM Tendency and Torque

- Dotted line: Global AAM Tendency
- Solid line: Torque

Units: $(x \times 10^{13} \text{ kg m}^2 \text{s}^{-2})$

Timeline:
- 1 Nov 95 to 1 Nov 96

Graph shows fluctuations over time.
Mountain Torque Variance
(14-day moving windows)
FIG. 9. (a) Difference in surface pressure between 1200 UTC 15 March and 1200 UTC 8 March 1996 (every 4 hPa contoured, zero line omitted). Positive contours are solid and negative dashed. (b) Same as (a) except between 1200 UTC 20 March and 1200 UTC 15 March. Shading indicates surface elevation of at least 1000 m.
For steady state:
Integrated over the globe
Net torque on atmosphere = 0

In the absence of mountains
net frictional torque = 0

if the surface winds are nonzero, there must be regions of easterlies and westerlies
For steady state:
Integrated over the globe
Net torque on atmosphere = 0

In the absence of mountains
net frictional torque = 0

if the surface winds are nonzero, there must be regions of easterlies and westerlies
\[-2\pi R_E^3 \int_{\text{eq}}^{30^\circ} \left[ \tau_x \right] \cos^2 \phi \, d\phi = \frac{2\pi R_E \cos \phi}{g} \int_{30^\circ} [mv] \, dp\]

Torque equatorward of 30° = Transport across 30°
Another balance requirement:

\[-2\pi R_E^3 \int_{eq}^{30^\circ} [\bar{T}_x] \cos^2 \phi \, d\phi = \frac{2\pi R_E \cos \phi}{g} \int_{30^\circ} [\bar{mv}] \, dp\]

Torque equatorward of 30 ° = Transport across 30°

Poleward flux of AAM across 30° latitude in both hemispheres
Another balance requirement:

\[-2\pi R_E^3 \int_{eq}^{30^\circ} [T_x] \cos^2 \phi d\phi = \frac{2\pi R_E \cos \phi}{g} \int_{30^\circ} \overline{[mv]} dp\]

Torque equatorward of 30° = Transport across 30°

Poleward flux of AAM across 30° latitude in both hemispheres
\[ \frac{2\pi R_E \cos \phi}{g} \int_{30^\circ} \left[ m \nu \right] dp = \frac{2\pi \Omega R_E^3 \cos^3 \phi}{g} \int_{30^\circ} \left[ \nu \right] dx dp + \frac{2\pi R_E^2 \cos^2 \phi}{g} \int_{30^\circ} \left[ \nu \nu \right] dx dp \]

**Total**

\[ M_\Omega \]

**requires**

poleward

mass flux

\[ M_r \]
Decomposition of $M_r$

$$[uv] = [\bar{u}] [\bar{v}] + [u'] [v'] + [\bar{u}^* \bar{v}^*] + [u'^* v'^*]$$

Steady MMC  transient MMC  steady eddy  transient eddy

a.k.a. stationary wave
Decomposition of $M_r$

$$
[u \bar{v}] = [\bar{u} \bar{v}] + [u'] [v'] + [\bar{u}^* \bar{v}^*] + [u'^* v'^*]
$$
\[ \int_{p_0} \begin{bmatrix} u \\ v \end{bmatrix} dp \]
\[ [u^* \nu^*] \]
Transient eddy kinetic energy at 250 hPa. Filtered 6 – 2 days

December-February
one-point correlation maps; 500 hPa height; highpass filtered
Conclusions

In accordance with the balance requirements, there is a strong poleward flux of angular momentum across 30° latitude.

The flux is greater during winter when the surface westerlies are stronger.

The poleward flux across 30° is accomplished exclusively by the eddies.

Transient eddies and stationary waves both contribute.

Nearly all the flux occurs around the jet stream level (above 500 hPa).

Balance requirement for an upward flux of M equatorward of 30° and a downward flux poleward of 30°.
Vertical transport of angular momentum

\[ \frac{2\pi \Omega R_E^3}{g} \int -[\bar{\omega}] \cos^3 \phi \, dy + \frac{2\pi R_E^2}{g} \int -[\bar{u}\bar{\omega}] \cos^2 \phi \, dy \]

\[ M_\Omega \quad M_r \]

\[ \frac{2\pi R_E^2}{g} \int -[\bar{\omega}] (\Omega R_E \cos \phi + [\bar{u}]) \cos^2 \phi \, dy \quad + \quad \frac{2\pi R_E^2}{g} \int [\bar{u}^* \bar{\omega}^*] \cos^2 \phi \, dp \]

MMC term \quad Eddy term
The eddies are not the answer

Scaling arguments: extratropical eddy fluxes are too small by a factor of $Ro$

Tropical eddy fluxes are almost nonexistent

Extratropical eddy fluxes are upward; not downward

So it must be the MMC
“Spin down” of the circulation in a teacup
The zonally averaged equation of motion

**Spherical geometry**

\[ \frac{\partial [u]}{\partial t} = [v] \left( f - \frac{1}{\cos \phi} \frac{\partial}{\partial y} [u \cos \phi] \right) - [\omega] \frac{\partial [u]}{\partial p} - \frac{1}{\cos^2 \phi} \frac{\partial}{\partial \phi} \left[ u^* v^* \right] \cos^2 \phi - \frac{\partial}{\partial p} \left[ u^* \omega^* \right] + F_x \]

**Cartesian geometry**

\[ \frac{\partial [u]}{\partial t} = [v] \left( f - \frac{\partial [u]}{\partial y} \right) - [\omega] \frac{\partial [u]}{\partial p} - \frac{\partial}{\partial y} \left[ u^* v^* \right] - \frac{\partial}{\partial p} \left[ u^* \omega^* \right] + F_x \]

**Neglecting vertical advection by MMC; using G to represent eddies**

\[ \frac{\partial [u]}{\partial t} = [v] \left( f - \frac{\partial [u]}{\partial y} \right) + G + F_x \]
\[
\frac{\partial [u]}{\partial t} = [v] \left( f - \frac{\partial [u]}{\partial y} \right) + G + F_x
\]

MMC  dynamic stability  eddy  frictional
\[\propto \frac{dM}{dy}\]  forcing  drag

For long term “balance requirement” \( \frac{d}{dt} = 0 \)

\[
[v] = \frac{G + F_x}{f - \frac{\partial [u]}{\partial y}}
\]
at A and D \[ G = 0 \]

at B and C \[ F = 0 \]
at B \( d[u]/dy \sim 30 \text{ m s}^{-1} \) over 2000 km \( \sim 1.5 \times 10^{-5} \text{s}^{-1} \)

\[
f \sim 4 \times 10^{-5} \text{s}^{-1}
\]

\[
f - d[u]/dy \sim 2.5 \times 10^{-5} \text{s}^{-1}
\]

at C \( f - d[u]/dy \sim 10 \times 10^{-5} \text{s}^{-1} \) \( 4 \times \) larger than at B
Recalling that

\[ [v] = \frac{G + F_x}{f - \partial[u] / \partial y} \]

and \( F_x = 0 \) at B and C

and G is roughly equal and opposite at B and C.
Recalling that

\[
[v] = \frac{G + F_x}{f - \frac{\partial[u]}{\partial y}}
\]

and \(F_x = 0\) at B and C

and G is roughly comparable at B and C

it follows that \([v]\) is \(\sim 4 \times\) stronger at B than at C

i.e., that the Hadley cell is roughly 4 times as strong as the Ferrell cell