Review from last time:

\[ S_0(\pi r^2)(1-\alpha) \]  amount of energy absorbed  \( (1) \)

\[ (4\pi r^2)\sigma T^4 \]  amount of energy emitted  \( (2) \)

\[ (1) = (2) \rightarrow \]

\[ T = \left( \frac{S_0(1-\alpha)}{4\sigma} \right)^{1/4} \]  \( \text{(Model A)} \).

We get 255 K = -18° C = 0° F
Actual: 15° C = 59° F.
The difference is the \textit{natural greenhouse effect}.

---

\(2) \text{ The Natural Greenhouse Effect}\)

Let’s re-examine the Earth in radiative equilibrium and now taking into account the \textit{atmosphere}.

We will assume the atmosphere is essentially transparent to visible radiation, but is a good absorber of radiation in the infrared band (where the ground emits most radiation.)

The atmosphere, to maintain equilibrium, must radiate this energy away: half goes down (to the ground) and half goes to space.
2a) Tools needed to create a greenhouse model:

i) emissivity.

The earth’s atmosphere is not a perfect emitter in the infrared. The emissivity, $\varepsilon$, is the ratio of radiation emitted by an object to the radiation emitted by a perfect emitter at the same temperature.

Hence,

$$\text{Radiation emitted} = \varepsilon \sigma T^4$$

[$\sigma T^4$ is the energy emitted by a perfect emitter.]

<table>
<thead>
<tr>
<th>Emissivity</th>
<th>Note:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Perfect emitter (&quot;black body&quot;)</td>
</tr>
<tr>
<td>$\sim 0$</td>
<td>Very poor emitter</td>
</tr>
<tr>
<td>0.76</td>
<td>Earth’s atmosphere</td>
</tr>
</tbody>
</table>

ii) Kirchhoff’s Law.

Emission efficiency $= \text{absorption efficiency}$.

2b) The equilibrium surface temperature with an absorbing atmosphere.

For equilibrium, we must have energy balance in the atmosphere and at the surface.
(Cartoon of the ground, the atmosphere, and space, sending and receiving radiation)

Surface: \[ \text{Energy in} = \text{Energy out} \]
\[ S_0(\pi r^2)(1-\alpha) + \varepsilon \sigma T_a^4 (4\pi r^2) = \sigma T_g^4 (4\pi r^2) \]

\[ \Rightarrow S_0(1-\alpha) = 4\sigma (T_g^4 - \varepsilon T_a^4) \quad (4) \]

Atmosphere: \[ \text{Energy in} = \text{Energy out} \]
\[ \varepsilon \sigma T_g^4 (4\pi r^2) = \varepsilon \sigma T_a^4 (8\pi r^2) \]

\[ \Rightarrow T_a^4 = \frac{1}{2} T_g^4 \quad (5) \]

Replacing \( T_a^4 \) in (4) with (5)

\[ \Rightarrow S_0(1 - \alpha) = 4\sigma T_g^4 \left(1 - \frac{\varepsilon}{2}\right) \]

or,

\[ T_g = \left(\frac{S_0(1-\alpha)}{4\sigma(1-\varepsilon/2)}\right)^{1/4} \] \quad (6), or Model B
Something in the atmosphere absorbs energy emitted from the earth’s surface:

- water vapor, CO2…..

Plug in $S_0 = 1367$, $\alpha = 0.3$, $\varepsilon = 0.76$, [into Model B]

$\Rightarrow T_g = 288 \text{ K} = 15^\circ\text{C} = 59^\circ\text{F}$. [so it works!!]

Basically, this is the natural greenhouse effect.