Homework 1

1. Consider two water parcels at a pressure of 1 bar. Parcel A has a salinity of 34 ppt and a temperature of 32°C, and parcel B has a salinity of 25 ppt and a temperature of 0°C. Reading off the charts provided in class, both have equal density of 1020 kg m⁻³. Assume that both water parcels also have equal sound speed of 1500 m s⁻¹.

(a) If equal masses of water parcels A and B were mixed, would the mixture be more, less or equally dense as parcel B at 1 bar?

(b) If both water parcels are adiabatically compressed to a pressure of 100 bars, would parcel A now be denser, less dense, or equally dense as parcel B? What would the density of water parcel A be?

(c) If both water parcels are adiabatically compressed to a pressure of 100 bars, would parcel A be warmer, colder, or the same temperature as parcel B?

(d) Now consider a parcel C consisting of a balloon filled with dry air and enough lead ballast to have exactly the same density as parcels A and B at 1 bar. Would parcel C be denser, less dense, or equally dense as parcel B if adiabatically compressed to 100 bars?

For all parts, explain your reasoning.

2. For each of the following, identify whether the hydrostatic and/or the Boussinesq approximations would apply to the given flow, and briefly justify your answer.

(a) Wind-driven turbulent mixing just underneath the ocean surface.

(b) Sound propagation through the atmospheric boundary layer.

(c) Flow around the Space Needle during a wind storm.

(d) The Sumatran tsunami propagating across the Indian Ocean.

(e) Midlatitude atmospheric jet streams

(f) Jupiter’s Great Red Spot.

3. The temperature profile of the tropical troposphere is maintained by moist convection in the form of heavily raining cumulonimbus cloud systems. The strongest updrafts lift moist air from near the surface to an altitude higher than 15 km. They stay buoyant due to latent heating due to condensation in the rising, cooling updrafts. By the time they reach 15 km, essentially all of their water vapor has condensed into liquid water or ice, which precipitates out of the updraft. This air spreads out to define the temperature just below the tropopause.

(a) Suppose the air starts at a near-surface pressure of $p_s = 1$ bar, temperature of 302 K, and specific humidity (mass fraction of water vapor in the air) $q = 0.019$ kg kg⁻¹. Calculate its density at this pressure. Here and throughout this problem, ignore the ‘virtual’ effect of water vapor on the air density. Using a reference pressure $p_{ref} = 1$ bar, what is the potential temperature $\theta_s$ of the air?

(b) Suppose this air is lifted to the top of the cumulonimbus clouds at an ambient pressure of $p_t = 125$ mb, without any external input of heat. If the water all stayed as vapor, what would its potential temperature be? How about its temperature?
(c) If (more realistically) the water vapor condenses into cloud in the rising air parcel, this produces condensational heating of the rising air parcel. Suppose that as the air rises up to 125 mb, all of the water vapor is condensed into liquid water and rained out (ignore freezing for this argument). This provides a heat input $Lq$ per unit mass of air, where $L = 2.5 \times 10^6 \text{ J kg}^{-1}$ is the latent heat of vaporization. This heat input takes place gradually as the air cools, but for simplicity assume that the heat is all added at a representative pressure $p_Q = 700 \text{ mb}$. Argue that this will increase $\theta$ of the rising air parcel by $Lq/(c_p \Pi_Q)$, where $\Pi_Q = (p_Q/p_{ref})^{R/c_p}$.

What will its potential temperature $\theta$ now be at 125 mb? What will its temperature and density be?

(d) Show from hydrostatic balance that $d\Pi/dz = -g/(c_p \theta)$, where $\Pi = (p/p_{ref})^{R/c_p}$ is called the Exner function. Assuming that potential temperature has an average value $\theta = (\theta_t + \theta_s)/2$ across the troposphere, estimate at what altitude $z_t$ the air pressure will be $p_t$.

(e) Estimate the mean squared buoyancy frequency of the tropical troposphere. For air $\rho_0$ is proportional to $\theta^{-1}$, and $d(\ln \rho_0)/dz = -d(\ln \theta)/dz$, so use the discretized formula:

$$N^2 = \frac{g}{\theta}(\theta_t - \theta_s)/z_t$$